







A N  
Intire SYSTEM  
O F  
ARITHMETIC:  
O R

ARITHMETIC in all its Parts.

CONTAINING

- |                         |                           |                            |
|-------------------------|---------------------------|----------------------------|
| I. <i>Vulgar.</i>       | IV. <i>Sexagesimal.</i>   | VII. <i>Lineal.</i>        |
| II. <i>Decimal.</i>     | V. <i>Political.</i>      | VIII. <i>Instrumental.</i> |
| III. <i>Duodecimal.</i> | VI. <i>Logarithmical.</i> | IX. <i>Algebraical.</i>    |

With the Arithmetic of Negatives, and Approximation  
or Converging Series.

The Whole intermix'd with *Rules New, Curious, and  
Useful*, mostly Accounted for in the *PREFACE*.

The *Algebraic Part* is rendered more *Plain and Easy*, than hath  
been done, by Instructive Rules and Examples *Literally and Numerally, in a  
Method New*: Solving Equations, Simple, Quadratic, Cubic, &c. several ways.  
And in the proper Places of this Work are *An Accurate Table  
of Logarithms* to 10000, and Rules to find those to 1000000000, and Natural  
Numbers to such Logarithms; with *the full Use of the Table* in Multiplication,  
Division, Involution, Evolution, and in the Solution of *all Cases of Compound  
Interest*, of which there are 24 Large and Exquisite Tables, (and one for the  
Valuation of Church or College-Leases of their Land) as also those of *Simple  
Interest and Discount*, with a new Method of finding the later and the pre-  
sent Worth of Money for Days.

Also Ample Definitions and Explanations of Numbers, Quan-  
tities and Terms used in all Parts of Arithmetic, in Alphabetical Order; ren-  
dering the Whole more Intelligible, and the Easier Learned.

With an APPENDIX, shewing the Mensuration of more Superficies and  
Solids than any Book wrote purposely on that Subject has exhibited.

This TREATISE, for Copiousness and Novelty of Matter and Method,  
far exceeding the most Perfect Arithmetic extant.

Necessary for all who would in a short Time, and with little Study, acquire a com-  
petent Knowledge of Numbers and Species, or would make any considerable  
Progress in the Mathematicks.

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By EDWARD HATTON, Gent. 

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The Second Edition, with Additions:

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LONDON: Printed for G. STRAHAN, at the Golden-  
Ball, over against the Royal-Exchange in Cornhill. 1731.







To the LEARNED

Dr JOHN KEILL, M. D.

*Fellow of the Royal Society, and Professor of  
Astronomy in the University of Oxford.*

**G**REATNESS and Goodness (you know, Sir) compleat the Rational Specie: By Greatness I mean true Magnanimity embellished with Learning and Science; by Goodness I suppose such an one as above, finished with an humble condescending communicative Temper or Disposition: And by how much any one excels in the former, by so much he is to be admired for the later Endowments. I need not tell the World how much the Qualifications and Acquirements of the first kind are your's, Sir: Your Learned Treatises of Philosophy, Astronomy, &c. in another Language, are sufficient Indications of that: And for the other, I have experienced your Condescending, Generous, and Diffusive Goodness, not only in your giving me an Opportunity of being known to you; but by the many Favours in so short a time received from you: Which propitious Providence having so ordered as to be the Consequence of my writing the following Book, I could neither justly nor naturally think of Dedicated it to any but Yourself.

AND altho' I am far from having intended this Treatise for the few of your Superior and Universal Genius; yet I



## DEDICATION.

am not without Hopes, but that upon an impartial and particular Perusal, even such as You may find some things Novel and Acceptable. And as for those who have not, but are desirous to acquire, the Knowledge of Arts Mathematical, (a Study so exceeding Pleasant and Useful) I have endeavoured to guide them at least to the Entrance of that large and unfrequented Path, in which you and some others of your most Illustriously Learned Royal Society have made so vast a Progress.

THE Preface enumerates some of those many Things which I take to be New, and not contained in any other Treatise of Arithmetic in our own Language: And for what is in others, I know you are a much better Judge than,

S I R,

*Your Obliged and most Humble Servant,*

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Edw. Hatton.



TO THE  
READER.

**H**AVING for many Years past spent my Leisure Hours in Mathematical Studies, and not meeting with any Treatise teaching in a Succinct Order, and Good Method, all the Parts of Arithmetic; and having made Emendations and Improvements in most, and investigated and discovered things new in many others: I supposed the Publication thereof would be accordingly received by the Studious in this Art. But to be a little more particular, I have, by way of Introduction, inserted very Useful and Copious Definitions of Numbers, and Explications of Terms Arithmetical, which will admit of the Whole being easier understood and sooner learnt. I have next given the most large and plain Numeration-Table; and Tables and Rules to assist the less knowing in Addition without Pricking and Carrying; and how to add Money, Weight, Measure, &c. by one Rule: also Tables of Weight, Measure, &c. in a more accurate Method, and more various than others: The Reason of the Way of Working in Subtraction and Multiplication; and in Multiplying by several Digits, I have given the Reason why the Products of all from the first are put one place more towards the left hand: Also a large Table for Beginners, and a new brief one of my own Contrivance for the more Skilled. In Division I have shewed four several

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Ways,

Ways, with the Reason of the Method of Operation, and how to find all the even Parts of any Numbers, which is applied in finding the Aliquot Parts of a Pound Sterling, &c. And besides the plain and best Way of Extracting the Square and Cube Roots of Numbers, I have given Rules how to do that of the Biquadrate by an Example altogether New, and must be esteemed Curious, especially to those who are not acquainted with Algebraic Canons. In Progression you have several things not to be found in other Books of this Subject; as Rules to find the Total of a Series produced by different Factors, and of the Changes to be rung on any Number of Bells, &c. with the Reason of the Abstruse Rules for finding the Totals of Series's whether the Ratio be Arithmetical or Geometrical: And have recommended this Part of Arithmetic to the Learner's Perusal, and not to be passed over, as is too common both in Books and Schools. And whereas others on this Subject give but four, or at most five kinds of Rules of Proportion, I have in this Treatise exhibited Twelve. The Rules of Practice are far more Numerous, Brief, and Methodical, than any done by another Hand: And here are many things more than common in Fellowship, as three several Ways of answering Questions, &c. as also in Alligation, Barter, and Equation of Payments.

In Decimal Arithmetic (besides the Hint given of a New Specie thereof) you have so many things truly Novel and Curious, especially in Reduction and Multiplication, as would be too prolix to enumerate; where you will find some Answers exhibited by short and accurate Methods quite contrary to the General Rules given for Addition and Subtraction; i. e. By Adding and Subtracting one and the same Number, as Units to Tens place, &c. and the contrary.

And in the Use of Decimals, besides the Way of Answering any Question by Decimals as exactly as by Vulgar Fractions (which to me is wholly new) you have the only Genuine Tables of Discount that I know extant, with a New as well as more Brief and Easy Method of Computing both Discount and Present Worth of any Sum for Days, (so much practised by Traders) the Foundation of which you will find in Algebra, Chap. 10. I have next shewed the Arithmetic of Duodecimals and Sexagesimals; the former used in Mensuration of Superficies and Solids, the later (chiefly) in Astronomy, whence called by some Astronomical: and these I have done in a new plain Method, and for the more difficult Cases. As to Political Arithmetic, I have said enough to shew its Nature and Manner of Process to discover what is required, illustrating the same by Examples, and have referred to others who have purposely and wholly treated thereon. In Logarithms I have omitted nothing that I could find necessary or deficient in other Authors; and believe I may with Veracity affirm, That no Book

of



*of Arithmetic affords so ample and plenary Instructions on that Head: And have endeavoured to render that part which has put many to a Ne plus ultra (as being somewhat less Intelligible, and not easily retained in Memory) I mean the Addition and Subtraction of the Logarithms of Decimals, plain to any of a moderate Capacity; and have enlarged in a familiar way on the Use of Logarithms in Multiplication, Division, Involution, Evolution or the Extraction of the Roots of all Powers; and in the Solution of all the Cases of Compound Interest: As also how Operations in Vulgar Fractions are performed by Logarithms, whereof I have given you a large Table actually and by Inspection to 10000 and made it subservient for Natural Numbers, tho' so great as 100000000, by help of a Table of Proportional Parts (there also inserted) or without it; and have taken the elaborate Pains to write both Tables all over with my own Hand, that I might the better answer for their Accuracy: In doing whereof, I have corrected 33 Errors in one of our last published Tables to 10000; and comparing the same also with Mr Norwood's 3d Edition of his Trigonometry, I have rectified his in 182 several places: so that I hope mine will be found very Correct. In Lineal Arithmetic I have performed my Operations three several ways, and have shewed how to make the Lines of Chords for the Mensuration of Arcs of Circles; the Line of half Tangents for Diameters, on those Great Circles represented thereby in Projections of the Sphere; and the making the Line of Numbers, commonly called Gunter's Line, to any Radius from a Table of Logarithms; and this both on a right Line and on the Ambit of a Circle: which I would not omit, in regard I found many of the Instrument-Makers that I discoursed on this matter, to be ignorant of it. What I have advanced in Instrumental Arithmetic, will be found not only New and Pleasant, but very Useful, particularly the new and plain Way of Working by Neper's Bones; but especially the Use of my New Circular Instrument contrived by me, whereby the Reduction of Coin, Weight, and Measure to Decimals, and the contrary, are speedily, easily, and accurately performed; not by guess, (as some Scales only have them) but actually and explicitly, as I have shewn by various Examples. And Multiplication, Division, and Extraction of Roots, &c. may be done by the two Lines of Numbers made to turn round one within the other, and are placed next the Limb or Edge of the Instrument, the Dimension whereof being so large as 33 Inches, I doubt not but it will be allow'd as the best and most compleat extant for all Arithmetical Uses. In Algebra I have previously given many more Numerous and Useful Definitions relating to this Analytical Art; than any Treatise of Arithmetic affords; and have throughout that profound Science illustrated the Nature and demonstrated the Truth of the Symbolical Operations*

tions by Numeral Examples, intermixing several things New, and my own Invention: particularly the Way of finding the Unciæ of Powers to the 15th, and the Powers of a Binomial to the 10th, as also the Unciæ of any Term or Member of any Power without the Knowledge of those of any previous Power. I have also shewed the Reason of the Process in Extracting the Roots from the respective Powers of a Binomial; and have likewise made the Algorithm of Surds very perspicuous by Examples in Numbers as well as Species, and have done the like in all the Rules of Algebra; and I have shewed not only the Solution of Simple Equations and those by various Positions, but Quadratical, and Cubical, each three different ways: In all which, as well as in Approximation or Converging Series, I have purposely designed to render the Manner of Solution intelligible to a mean Capacity, that being the principal thing in which the Learned Authors on this Subject have been deficient. The Arithmetic of Negatives I have fully accounted for, as will appear if to what is said under Negative Arithmetic, in the Alphabetical Explanation at the beginning of Algebra, you add what is in Sect. 3. of Chap. 7. in Sect. 2, 3, 4, 5. of Chap. 10. and under the last of my Examples of Converging Series, near the Close of this Treatise.

But notwithstanding the particular Regard of my former Labours, and especially of this, to promote a sort of Learning so Useful to the Public; yet so numerous are the capacious and prejudiced Readers of this our envious Age, that it would be Vanity in me to hope to escape their Censures. But if my Endeavours prove acceptable to the two Classes of Readers for whom they were chiefly intended, i. e. the Candid and Industrious Teacher and the Diligent and Studious Learner; I shall esteem it a good Step towards an ample Compensation for the uncommon Care and Trouble of this Performance, thus dedicated to the Service of the Public; and shall the less regard the Carps and rash Judications of pragmatistical and ungrateful Dispositions, who fancy there is nothing in Arithmetic beyond what they have acquired the Knowledge of: or if they learn any thing from a Book of this kind, will be the last in paying their Acknowledgment, and probably the first in decrying the Work, because it will instruct some to know more than they would have them, or (as we say) make others as wise as themselves. However, I would not be discouraged by the ill Conduct of such from imitating the most Perfect and Benign Pattern, who is kind to the Unthankful and the Evil: for whatever some Authors may have proposed in being so, I have disclosed my Thoughts this way, much more to promote the Proficiency of the Reader, than to indicate the Science of the Author.

And for the Encouragement of the Impartial and Ingenuous Student, who laying aside Pride and Prejudice, designs only Improvement in the Knowledge  
of

## The P R E F A C E.

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of what is contained in the subsequent Pages, I can assure him, that I have faithfully adapted this Work to the Capacity of the less Acute, and have offered to his Perusal many things which he will not find elsewhere; of which I have given some Instances, as above, though you have many more mentioned in the Index, which will be found Exasperations.

I have not in this Treatise (as is common where there are many Parts of Mathematics, &c. in one Volume) only touched on each kind; for you will find Vulgar Arithmetic, Decimals, Logarithms, and Algebra, as copiously insisted on, as in almost any Treatise wrote only upon some one of those Species: Nor can any thing be expected to make the whole more truly agreeable to the Title, which is a piece of Justice that all Authors owe to the Public, and cannot be denied but to have been fully observed by me, as I am not without Vouchers from good Hands, to confirm the Truth of, especially in my Merchant's Magazine, and my Index to Interest.

And for the Satisfaction of such as are cautious of buying the first Impression of a Book, because, say they, there may be Additions to future Impressions; I do hereby promise, that as I know of no Necessity for it, so I have no Thoughts of doing any thing farther on this Subject.

I have, besides what I promised in my Proposal to Subscribers, added an Appendix, which contains the best Way of measuring a greater Variety of Superficies and Solids, than any Book, tho' wrote purposely on that Subject, exhibits.

In fine, I am so sensible of the Care and Assiduity used to finish this Body of Arithmetic, so as to render it in some degree Compleat, that I hope I need not doubt of its being acceptable to the World; which hath already favoured me, by a kind Reception of my former Endeavours in this Way, altho' I had not bestowed near that Thought, which (in Gratitude) I have done on this Work, in order to present it, as near as I could, in Perfection.

For besides all that is abovesaid, I have not only comprised in this Treatise, the most material Tables, and other things in my Index to Interest (which shall never be but here re-printed) but have added Tables of Simple Interest at 4 per Cent. of Discount at 4 and 5 per Cent. and 16 Tables of Compound Interest at 3, 4, 5, and 8 per Cent. And the said Index (before the making these considerable Additions to it) was approved of, and recommended as the most copious, easy, and useful, Book that hath been written on that Subject, by the judicious Persons following; besides many others, as I have their own Hands by me to demonstrate.

S. Shepherd



## The PREFACE.

S. Shepherd Esq; Deputy-Governour,  
 Sir W. Chapman Bart. } then Di- } of the South-Sea  
 Sir Samuel Clarke, } rectors } Company.  
 Sir John Blunt Bart.  
 Sir G. Cook, first Prothonotary of the Court of Common-Pleas.  
 The Reverend Mr Whiston, sometime Professor of Mathematics in  
 the University of Cambridge.  
 Nath. Pigot, of the Middle Temple Esq;  
 P. Lacy, of the Inner Temple Esq;  
 Mr H. Jackson, of the Middle Temple, Sub-Treasurer.  
 Mr J. Frewen, of Lincolns-Inn.  
 Mr Humphry Ditton, Master of the Mathematical School at  
 Christ's-Hospital.  
 Mr S. Waters, Accomptant General }  
 Mr P. Sergeant, } Accomptants } to the East-India Comp.  
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THE

# THE INTRODUCTION:

*Containing an Alphabetical Definition of all kind of Numbers, and the Terms of Art used in Common Arithmetic explained; which the Reader may have recourse to, as he finds occasion.*

*Note, That the like for Algebra is inserted immediately before that Species of Arithmetic. Vid. Sect. 1. of Chap. X.*

**A** *Bundant Numbers,*] Are such as having their Aliquot Parts added together, the Sum exceeds the Numbers of which they are Parts: as 36 is an abundant Number, because its Aliquot Parts, 1, 2, 3, 4, 6, 9, 12, 18, added, makes 55, which is more than the 36.

*Abstract Numbers,*] Such Numbers in general, as have no Denomination annexed to them.

*Absolute Numbers,*] Such in reality as they appear to be, as 2, &c. is 2 Units contrary to 2 less than nothing (as in Algebra is explained) and to negative Indices of Logarithms: also it is that known Number in an Equation, which solely possesseth one side. See *Algebra*.

*Aliquot Parts,*] Are the even Parts of any Number, when there is no Remainder in the Division. See at the End of *Division*.

*Aliquant Parts,*] Such as will not divide another Number assigned without a Surplus or Rest: as 7 is an aliquant Part of 18 or 23, &c.

*Amiable Numbers,*] Are 2 such, as that the Sum of the Aliquot Parts of one, will make up the other Number alternately.

*Arithmetic*] In all its Parts (or in the most comprehensive Sense) is the Art of solving Questions by Figures, Lines, Instruments, or Symbols, as by the several kinds following doth appear.

*Arithmetical Complement,*] See Sect. 4. of Chap. VII. and *Complement* under C.

*Arithmetical Progression,*] See *Progression*, Chap. II. Sect. 2.

*Articles,*] In this Science, are Numbers divisible by 10 without Remainder; as 10, 20, 30, 100, 120, 130, &c. 2000, 4000, &c. the same with round Numbers or Decades.

*Artificial Numbers,*] Are Logarithms. See Chap. VII.



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*Antecedent Numbers.*] The first named of two that are compared together: As if it be said that some Numbers are in proportion as 3 is to 7; here 3 is the Antecedent, and 7 the Consequent.

*Binarys.*] Two Places of any Number, (as pointed for the Square Root.)

*Biquadrate.*] See *Powers* under *P*.

*Broken Numbers.*] Such as are commonly called Fractions, which see.

*Cardinal Numbers.*] The 9 Digits.

*Central Numbers.*] Such as have a Digit placed in the middle between alike Number of Reverse, Circulating, &c. Numbers, as 3457345, 12521, 78678, &c.

*Characteristic*] Of any Logarithm, is the Index, See *Chap. VII*.

*Circular Numbers.*] Are such as being squared, cubed, &c. the Figure in Units place of the Power, is the same with that of the Root; as 6 times 6 is 36, 5 times 5 is 25, &c. which are sometimes called Spherical Numbers.

*Circulating Numbers.*] According to some, are those Digits which are regularly repeated, whether the Number be of one and the same Digit repeated, as 222, 7777, &c. or of many as 494949, 372372, &c. And when these fall out in reducing Money, Weight, Measure, &c. to Decimals, you gain many Decimal Places with the less trouble.

*Commensurate Numbers.*] Such as one Number will justly measure; as 6 and 15 are measured by 3, &c. And the Fraction  $\frac{2}{3}$  is of like Value with  $\frac{4}{6}$ , because the Terms are commensurate by 3.

*Compleat Numbers.*] See *Perfect Numbers*.

*Complement-Arithmetical.*] Is the Remainder when any Logarithm is deducted from 10.

*Composed Numbers.*] The same as *Composit Numbers*; which see.

*Compound Fractions.*] Fractions of Fractions, as  $\frac{1}{2}$  of  $\frac{2}{3}$ , &c. See *Chap. II. Sect. 1*.

*Composit Numbers.*] Such as are measured by some other besides an Unit; as 297 is composed of 99 by 3, either of which measures 297.

*Common Multiples.*] See *Multiples* following.

*Consequent Numbers.*] The second of any two in a Proportion. See *Antecedent*.

*Contrait Numbers.*] The same as *Concrete Numbers*, (but is no proper Term.)

*Concrete Numbers.*] Such as have a Denomination annexed, as 5 *Lib.* 7 Yards, &c. *Cube*

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*Cube Numbers.*] The Product of Square Numbers by their Roots; as 8 is the Cube of 2, 27 of 3, being Products of 4 by 2, and of 9 by 3.

*Cubed Square and Cubed Cube Numbers.*] See *Powers* under *P*.

*Cube Root.*] See *Roots* under *R*.

*Decimal Fractions.*] Those whose Denominators are 10, or some Power of 10.

*Decades.*] The same with Articles, as above.

*Deficient Numbers.*] Those whose Aliquot Parts (added together) are less than the Numbers of which they are Parts; as 26 is a deficient Number, because its Parts 1, 2, and 13, make but 16.

*Differences, or Common Differences,* as relating to Logarithms.] See the 3d and 4th general Heads under *Seet. 4. of Chap. VII.*

*Digits.*] In Arithmetic, are Numbers of 1 Place, as 1 to 9 inclusive.

*Direct Proportion.*] Is when the 4th N<sup>o</sup>. (sought) is the Quote arising by dividing the Product of the 2d and 3d Number by the 1st.

*Dividends.*] Numbers which are to be divided. See *Division, Seet. 5. Chap. I.*

*Divisors.*] Numbers of Parts into which another Number is to be divided. See *Chap. I. Seet. 5.*

*Dividual.*] Separable or Divisible. Also that Part of a Dividend which is immediately under your Operation.

*Diminutive Numbers.*] Are the same as Deficient Numbers, which are explained above.

*D.*] From *Denarius* (a Penny): The Mark put over a Column of Pence in Books of Accounts.

Feet. Inch. 12<sup>ths</sup>

*Duodecimals.*] Fractions whose Denominators are 12, as 7 : 9 : 7 is 7 foot, 9 inches, (or 9 twelfths of a foot) and 7 twelfths of an inch, &c. Which are often marked by Measurers thus;

F.   I.   II.   F.   I.   II.   III.

7 : 9 : 7; and 48 : 11 : 3 : 10; and read the first, 7 foot, 9 primes, and 7 seconds; the second is, 48 foot, 11 primes, 3 seconds, and 10 thirds. See *Operations* thereby, *Chap. IV.*

*Duplicate, Triplicate, Sesquiplicate, &c. Proportion.*] See *Seet. 3. of Chap. II.*

*Equimultiples.*] See *Multiples.*

*Evenly even Numbers.*] Are those which even Numbers will measure by other even ones; as 48 is measured by 8, 6 times.

*Evenly odd Numbers.*] Are those which even Numbers measure by odd ones; as 40, which 8 doth measure by 5.

*Evolution*] The Extraction of Roots of Numbers, &c.

*Exponents*] Are Numbers (smaller Characters than those they stand next) which are placed near the upper part of a Root toward the right-hand, and shew what Power of that Root is expressed; as  $3^2$  is the Square of 3 or 9;  $3^4$  is the Biquadrate of 3 or 81. Also  $4^3$  is the third Power or Cube of 4, which is 64; the Exponents being 2, 4, and 3, standing higher than the others or Roots. So likewise  $15^5$  is the Sur-solid or fifth Power of 15;  $7^6$  is the squared Cube or sixth Power of 7;  $20^7$  is the second Sur-solid or seventh Power of 20, which is 1280000000. See *Powers* here and in *Algebra*.

*Exclusions*] Such Numbers in a Question, as being excluded, renders it less perplexed and the easier resolved; as might be shewn by several Examples.

*Factors*] Both the Numbers, which for the most part in Multiplication are called the Multiplicand and Multiplicator, (or Multiplier.)

*Figurate Numbers*] Such as do represent some Geometrical Figure, either Superficial or Solid. See those Numbers and Linear.

*Fractions*] Any Part or Number of Parts of an Unit *ad infinitum*: as  $\frac{1}{2}$  is Vulgar;  $\frac{1}{10}$ , &c. Decimal; and  $\frac{1}{12}$ , &c. Duodecimal. See Chap. II, III. and Sect. 10. of Chap. III.

*Geometrical Progression*] See *Progression*.

*Harmonical Proportionals*] See *Musical*, and Chap. II. Sect. 3. Head. II.

*Homogeneal Numbers*] Or Homogeneous, are those of the same nature; as the Indices of 2 Logarithms, if they are both affirmative, (without a Mark under) are Homogeneous; or they are so, if they are both negative, or have a Mark under thus,  $3$  and  $2$ , &c.

*Homologous*] Numbers or Terms in a proportion are said to be Homologous, when there is a Similitude between the Antecedents and Consequents; for as 3 to 9, so 7.21: here 3 is homologous to 7, as 9 to 21.

*Heterogeneal Numbers*] Those which are not of the same kind; as when one is negative, the other affirmative, as 3 and 2, contrary to homogeneal. Also mix'd Numbers, composed of a Whole and a Fraction, as  $2\frac{1}{2}$ : and in Surds, they are such as have different Radical Signs, as  $\sqrt{3}$  and  $\sqrt{4}$ . Vid. *Algebra*, Chap. X.

*Improper Fractions*] Such whose Numerator is equal to or exceeds the Denominator. See Chap. II. Incommen-



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*Incommensurate Numbers.*] Two such, as no one Number (except an Unit) will measure, contrary to commensurate; as 7 and 23, &c. cannot be measured by any one Number.

*Indices.*] Numbers which shew the Power of a Root; and in Logarithms they shew the Number of Places which the natural Numbers belonging to any Logarithm consists of, being the same with Characteristicks. See *Exponents*, and also Chap. IV.

*Ineffable Numbers.*] Are Surds, or irrational Numbers; which last see: Or (according to the Import of the Word) Numbers not to be expressed.

*Incomposits.*] Numbers which no other but an Unit will measure, being contrary to Composit, and are the same with prime Numbers, as 7, 11, 23, 29, 31, and hundreds more.

*Intire or whole Numbers.*] Any Number of Units or Ones; (but Fractions are one or more Parts of an Unit) so that *one* is the middle between infinite Units and infinite Parts: and tho' it is by some not allowed to be a Number, yet since it is the Foundation of all Numbers, whether whole or broken, (for 2, 3, 4, &c. are so many ones, and  $\frac{2}{3}$ ,  $\frac{1}{2}$ ,  $\frac{1}{3}$ , &c. are so many Parts of one) and since 1 is as properly half the Number of things which are but 2 in all, as 2 is half the Number which are four in all; it follows, that since 1 is not a Fraction, (or Part of 1, which would be a Contradiction to affirm) it must be a Whole, *i. e.* a whole Unit, or the Number 1, as properly as 2 is 2 Units, or the Number 2, 3 the Number 3, 4 the Number 4, &c. For Unity is as properly an Individual (or inseparable from Number) as these or any other. And if we take a Series of Numbers, as 1, 2, 3, 4, 5, here 1 in all respects is a Number according to its Nature, as any of the other Digits are agreeable to the same common Nature of Number, and 1 is one Term or Place in that Rank as well as the others: 'Tis true, 1 does not divide nor multiply, because it would be absurd to say that a Man had 1 Pound, Yard, Ounce, or the like, divided between himself; or that because I say, 1 Horse, 1 Field, &c. I should exclude the Number 1, because it does not make more or less than that Horse or Field, and so be contrary to the common Nature of all other Numbers. And as it multiplies and divides as much as it ought to do, making every thing once itself, and giving the Whole to one Person where no one is to have a share with him; so in Addition, &c. it does as all other Numbers do, *i. e.* augments or diminishes any other Number so much as itself amounts to: For  
if

if I add 1 to 5, it makes 6; that is, so much more than 5, as is the Number 1 added; as 2 added to 5, makes 7, which is itself (or N<sup>o</sup> 2.) more than the 5, &c. Besides, *one* in all Merchantile Affairs, as well as in the Eye of our *Laws*, and in common Conversation, is a N<sup>o</sup>. And 'tis far from being a parallel to a Point in Geometry, 10000 of which make no Magnitude, but twice one is what all allow to be a *Number*. And whereas some great Men, as *Euclid*, &c. have defined Number to be a Multitude of Units; it is highly probable they meant no more than this, That as Unity is but 1, and there are Millions of Millions, nay, infinite other Numbers; therefore, say they, that Number (in the general way of speaking, or for the most part without comparison) is a Multitude or Aggregate of Units.

*Integers,*] Are whole Numbers, compared with their Parts; as 1 Penny is an Integer composed of 4 Farthings, its Parts into which it is divided; 1 Shilling is an Integer compared to Pence or Farthings, of which it is composed. But an absolute Integer is an Unit of the highest Denomination of any Specie of Matter or Thing: as 1 Pound *Sterling*, &c. 1 Ton, 1 Circle, &c. and 2, 3, &c. of the like, are so many absolute Integers.

*Inverse Proportion.* See *Reverse*.

*Irrational Numbers, or Surds,*] Are such whose Roots cannot be accurately extracted, as being no Figurate Numbers: But then they are to be considered as Surds or otherways, when compared with the Power they are of. Thus 16 is a Surd, or irrational Number, if supposed the 3d Power of some other; but if you suppose it only the 2d Power, it is not Surd, but compleatly the Square of 4: and the like may be observed of others. And when we meet with Surd Numbers, (or those that are Rational, which are to be wrought with Surds) we mark them thus,  $\sqrt{7}$ ,  $\sqrt{7}$  is the way of expressing the Square Root of 7;  $\sqrt[3]{27}$  is the Expression of the Cube Root of 27, which is equal to 3, &c. But more of this under *Powers*, &c. in the Algebraic Part.

*Lib. or L.* (from *Libra* a Pound-weight, or 20 s.) The Mark put over a Column of Pounds.

*Lineal (or Linear) Numbers,*] Such as represent or are the Dimensions of a Line, Root, or Side of a Geometrical Figure. Thus if a Figure be an exact (or Geometrical) Square, containing 100 Foot, 100 is the Superficial Number, and the Side or Linear Number is

is 10: If it be a long Square (or Parallelogram) of the Content of 40 Yards, the Linear Numbers (or Sides) are 4 and 10, or 8 and 5.

*Lineal Arithmetic,*] Is that Science performed by Lines.

*Logarithms,*] Are Artificial Numbers, of great use in Mathematicks; the Invention of Lord *Neper*, Baron of *Merchiston*. See Chap. VII.

*Mixt Numbers,*] Whole Numbers and Fractions, (Vulgar or Decimal) which are placed together: as  $13\frac{1}{2}$ , 29.75, &c. Or any Number composed of Digits, or Digits and Cyphers that are not next the right-hand.

*Multiples,*] Are Numbers produced by the Multiplication of some known or assign'd Number: as 40 is a Multiple of 8 or 5, because either of those will divide 40 without Remainder, and 20 is a Multiple of 4 and 5.

*Multiples, or Equimultiples,*] Are Numbers multiplied by one and the same Number: as 12 and 28 are Equimultiples of 3 and 7, for  $12:28 :: 3:7$ .

*Multiples, or Common Multiples,*] Are when one and the same Number is produced by different aliquot Parts: as 40 is a common Multiple of 2, 4, and 5, because any of these divide it without Remainder, and because these multiplied one in another produce 40.

*Multiplicator,*] The same as Multiplier. See Chap. I. Sect. 4.

*Musical Proportionals,*] The same as Harmonical. See Chap. II. Sect. 3. Head. 11.

*Negative Arithmetic,*] See (N) near the beginning of Algebra, in which this kind is chiefly used.

*Number, or whole Number,*] Is one or more Units, expressible by the 9 Digits and 0, *ad Infinitum*; of which there are near 50 several Kinds mentioned in this Introduction. See *Intire*, &c. or a broken Number, is one or more Parts of an Unit.

*Numerator of a Fraction,*] The Number of Parts contained in it. See Chap. II.

*Ob.* (from *Obolus* a Halfpenny) The Mark put for two Farthings in old Writings.

*Perfect Numbers,*] Those which are equal to the Sum of their aliquot Parts; as 28 is equal to the Sum of its Parts 1, 2, 4, 7, 14, which is 28; and 1, 2, and 3, are 6.

*Parts Proportional,*] See 3d and 4th General Heads, §. 4. of Chap. VI.

*Plain Numbers,*] Are those which may be produced by multiplying some Number in another, as 6 the Product of 3 by 2, 8 the Product



# The INTRODUCTION.

duct of 4 by 2, &c. (but 7, 11, 13, &c. are no such.) These are the same with superficial Numbers, because the Content of a plain superficial Figure is produced by multiplying two Numbers, which are supposed its Length and Breadth together.

*Positive Numbers.*] The same with Affirmative. See *Absolute Numbers*, above.

*Powers of Numbers.*] Are the Root or first Power; the Square or second Power, produced by multiplying the Root in itself, &c. as follows.

|             | <div style="display: flex; justify-content: space-around; text-align: center;"> <div>Roots, or 1st Power.</div> <div>Square, or 2d Power.</div> <div>Cube, or 3d Power.</div> <div>Biquadrate, or 4th Power.</div> <div>Surfold, or 5th Power.</div> <div>Squared Cube, or 6th Power.</div> <div>The second Surfold, or 7th Power.</div> <div>The Squared Biquadrate, or 8th Power.</div> <div>Cubed Cube, or 9th Power.</div> <div>Squared Surfold, or 10th Power.</div> </div> |   |    |    |     |     |      |      |       |       |
|-------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---|----|----|-----|-----|------|------|-------|-------|
| 1st Example | 2                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | 4 | 8  | 16 | 32  | 64  | 128  | 256  | 512   | 1024  |
| 2d Example  | 3                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | 9 | 27 | 81 | 243 | 729 | 2187 | 6561 | 19683 | 59049 |

Each Power being produced by multiplying the next before it towards the left-hand by the Root.

And that these Terms of the Powers are really what they are called; if the 5th Power or Surfold (in the 1st Example 32, in the 2d 243) be squared or multiplied in itself, it produces the squared Surfold or 10th Power, for 32 by 32 gives 1024; or 243 by itself makes 59049. And the like may be observed of the rest: for a more full Account of which, see the Word *Powers* in the beginning of Algebra, Chap. VII.

*Promiscuous Numbers.*] Composed of Digits without any limited Order, as 7291, 3924, and thousands of others, as under *Mixt Numbers*. Proper

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- Proper Fractions.*] See Chap. II. Sect. 1.
- Proportion.*] The same with Ratio; its various kinds. See the beginning of Sect. 3. of Chap. II. of which some are *Subduple*, *Subtriple*, *Submultiple*; which Words see: It is marked with Points thus, 3. 4 :: 6. 8. if direct.
- Prime Numbers.*] Such as no other but Unity will justly divide. See *Incomposits*.
- Proportional Parts.*] See *Parts Proportional*.
- Progression.*] Numbers in Arithmetical and Geometrical Progression. See Chap. II. Sect. 2.
- Q. (from *Quadrans* a Farthing) the Mark put over a Column of Farthings.
- Quotient.*] See *Division*, Sect. 5. Chap. I.
- Roots of Numbers.*] The first Powers. See *Powers* above, and *Linear N<sup>o</sup>*
- Ratio.*] Is the Reason or Proportion that one Number bears to another. Or more particularly it is direct, Simple Ratio, Duplicate Ratio, Triplicate, &c. See Chap. II. Sect. 3. Or Ratio in Progression is the Rate by which the Terms increase or decrease.
- Rectangle.*] The same as Product, which is the Sum produced by multiplying 2 together; as 2 Times 4 is 8 = the Product or Rectangle.
- Reverse Proportion.*] Is when the 4th N<sup>o</sup>. (sought) is the Quotient arising from the Division of the Product of the 1st, and 2d, by the 3d Number.
- Reverting Numbers.*] Such as are composed of a like Number of successive Digits ascending and descending, as 345543, 7887, &c.
- Round Numbers.*] The same with Articles and Decads, which see.
- Sesquiplicate Proportion.*] See Head 12. of Sect. 3. of Chap. II.
- Similar Numbers.*] Are either 1st, Plain, which are such as form like or similar plain Figures; as a Plain of 21 formed by the Rectangle of 7 by 3, is similar to another of 84 formed by the Sides or linear Numbers 14 and 6; for as 6 to 14, so 3 to 7. Or, 2dly, Similar solid Numbers are such as form like or similar Solids (as Parallelopipedons, or 2 or 3 Dice joined to the end of each other) of different, but proportional Dimensions.
- S. (i. e. *Solidus*) the Mark placed over Shillings in a Column of Money.
- Spheric Numbers.*] The same with Circular Numbers, which see.
- Subduple Proportion.*] Numbers are in subduple Ratio, when the Antecedent contains the Consequent (or the contrary) twice; as 5 to 10, or 40 to 20, and the like.

C

*Subtriplic,*  
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*Subtriple.*] Numbers are in a subtriple Proportion, when the Consequent is 3 times the Antecedent, or the contrary; as 9 to 27, or 90 to 30, &c.

*Submultiple.*] Numbers are in a submultiple Ratio, when the Consequent contains the Antecedent, or the contrary, above three times without Remainder.

*Square Numbers.*] All such as are produced by multiplying any Number by itself, because that Product represents a Figure of four equal Sides.

*Surd Numbers.*] The same with Irrational, which see.

*Sexagesimal Numbers.*] Are such Fractions as have 60 for their Denominator, used in measuring Time, and the Motion of the Celestial Spheres, Geography, and Navigation: But they put not down the Denominator in these, no more than in Duodecimals, because it never alters, being always 60, as those are 12. Thus

51 : 30 : 45 : 17 : 21 : 59 are read 51 Degrees, 30 Minutes, (or 60th Parts of a Degree) 45 Seconds, (or 60th Parts of a Minute) 17 Thirds, (or 60th Parts of a Second) 21 Fourths, (or 60th Parts of a Third) and 59 Fifths, or 60th Parts of a Fourth, &c.

Of Operations in this kind of Fraction, the Work of Addition is easy: Adding up a Column of Minutes, Seconds, &c. and dividing by 60, putting the Remainder under the Line, and carrying the Quote to the next Column toward the left hand, &c. borrowing 60 always in Subtraction. And for multiplying Degrees, Minutes, Seconds, Thirds, &c. these may be done decimally, by reducing the lower Denominations into the Decimal of the highest, and then working as is shewed in Decimals, Chap. III. And by the same Rule and Division of Decimals, is the best way to divide these. But see Chap. V.

*Successive Numbers.*] Such as are composed of Digits which stand in a natural Succession ascending and Descending, as 34567, or 9876.

*Tabular Numbers.*] Such as are found in Tables, as of Logarithms, Interest, &c.

*Tarif Numbers.*] The same as Tabular Numbers. But the Term is most used in Tables of Customs, and Series of a Divisor by the 9 Digits.

*Terms.*] The two Parts of a Vulgar Fraction are called its Terms; and so are the several distinct Numbers in an Arithmetical and Geometrical Progression.

*Ternary*



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*Ternary Numbers.*] Such as have 3 places: Or even 3 places of any Number are so many Ternarys, as used in extracting the Cube Root.  
*Triplicate Proportion.*] See *Proportion*, or *Sett.* 3. of *Chap.* II. *Head.* 10. *Unit*, or *Unity.*] The Number 1. See *Intire Numbers*, above.  
*Unevenly odd Numbers.*] Those odd Numbers produced by the Multiplication of 2 odd Numbers together; as 63, which is 7 times 9; 45, which is 9 times 5, &c.

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### C H A P. I.

*Treatise of the several Parts of single Arithmetic (called Common Arithmetic in whole Numbers) containing Numeration, Addition, Subtraction, Multiplication, Division, and the Extraction of Roots, which are called Simple or Single Rules of Arithmetic, as being Fundamental Parts or Principles, by one or more of which, all Operations by Numbers are performed.*

#### SECT. I. Of Numeration of Intire Numbers.

**B**Y this first Part we are taught how to read or write down any Number proposed, by assigning a proper and natural Denomination for the Place of each Digit in any Line, (tho' composed of never so many of those Digits) and then by observing what Digit is there placed.

By a Digit we understand any single Figure possessing but one place, as 1 to 9 inclusive.

There is also a Cypher, (or 0) which standing alone, or next the left hand of any single Figure or Number of Figures in whole Numbers is of no value; it only serving to fill up places, in order to augment the value of the simple Figure or Figures, which are placed to the left hand thereof; which places might as well be supplied by a Point (.) or Hyphen (-) thus, if that had obtained a Custom, as the (0) hath.

By what is said it appears, that after we know the Character of the 9 Digits, (which almost every Child learneth so soon as it can read) there is nothing remaining to make one able to read any Num-

ber, but to consider by what Name the several Places in any Number or Rank of Digits is called: and that will appear, by what follows, very obvious.

| <i>The Denomination of each particular Place of any Number not exceeding 9 Places, or that of Hundreds of Millions.</i> | <i>Examples how the Course or order of the Places augments toward the left hand infinitely.</i>                          | <i>How each of the 9 Numbers are to be read in one Line or Number.</i>     |
|-------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------|
|                                                                                                                         | 9th place.<br>8th place.<br>7th place.<br>6th place.<br>5th place.<br>4th place.<br>3d place.<br>2d place.<br>1st place. | One Hund. and 11 Mill.<br>One Hund. and 11 Thous.<br>One Hund. and Eleven. |
| One Hundred Millions                                                                                                    | 1 0 0 0 0 0 0 0 0                                                                                                        |                                                                            |
| Ten Millions - - - -                                                                                                    | 1 0 0 0 0 0 0 0 0                                                                                                        |                                                                            |
| One Million - - - -                                                                                                     | 1 0 0 0 0 0 0 0 0                                                                                                        |                                                                            |
| One Hundr. Thousand                                                                                                     | 1 0 0 0 0 0 0 0 0                                                                                                        |                                                                            |
| Ten Thousand - - -                                                                                                      | 1 0 0 0 0 0 0 0 0                                                                                                        |                                                                            |
| One Thousand - - -                                                                                                      | 1 0 0 0 0 0 0 0 0                                                                                                        |                                                                            |
| An Hundred - - -                                                                                                        | 1 0 0 0 0 0 0 0 0                                                                                                        |                                                                            |
| Ten - - - - -                                                                                                           | 1 0 0 0 0 0 0 0 0                                                                                                        |                                                                            |
| One - - - - -                                                                                                           | 1 0 0 0 0 0 0 0 0                                                                                                        |                                                                            |
| Two Hundred Millions                                                                                                    | 2 0 0 0 0 0 0 0 0                                                                                                        |                                                                            |
| Twenty Millions - -                                                                                                     | 2 0 0 0 0 0 0 0 0                                                                                                        |                                                                            |
| Two Millions - - -                                                                                                      | 2 0 0 0 0 0 0 0 0                                                                                                        |                                                                            |
| Two Hundr. Thousand                                                                                                     | 2 0 0 0 0 0 0 0 0                                                                                                        |                                                                            |
| Twenty Thousand - -                                                                                                     | 2 0 0 0 0 0 0 0 0                                                                                                        |                                                                            |
| Two Thousand - - -                                                                                                      | 2 0 0 0 0 0 0 0 0                                                                                                        |                                                                            |
| Two Hundred - - -                                                                                                       | 2 0 0 0 0 0 0 0 0                                                                                                        |                                                                            |
| Twenty - - - - -                                                                                                        | 2 0 0 0 0 0 0 0 0                                                                                                        |                                                                            |
| Two - - - - -                                                                                                           | 2 0 0 0 0 0 0 0 0                                                                                                        |                                                                            |
| Three Hundr. Millions                                                                                                   | 3 0 0 0 0 0 0 0 0                                                                                                        |                                                                            |
| Thirty Millions - -                                                                                                     | 3 0 0 0 0 0 0 0 0                                                                                                        |                                                                            |
| Three Millions - -                                                                                                      | 3 0 0 0 0 0 0 0 0                                                                                                        |                                                                            |
| Three Hundr. Thous.                                                                                                     | 3 0 0 0 0 0 0 0 0                                                                                                        |                                                                            |
| Thirty Thousand - -                                                                                                     | 3 0 0 0 0 0 0 0 0                                                                                                        |                                                                            |
| Three Thousand - -                                                                                                      | 3 0 0 0 0 0 0 0 0                                                                                                        |                                                                            |
| Three Hundred - -                                                                                                       | 3 0 0 0 0 0 0 0 0                                                                                                        |                                                                            |
| Thirty - - - - -                                                                                                        | 3 0 0 0 0 0 0 0 0                                                                                                        |                                                                            |
| Three - - - - -                                                                                                         | 3 0 0 0 0 0 0 0 0                                                                                                        |                                                                            |

From

# SECT. I. Numeration of Intire Numbers.

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From the Scheme above it may be observed, viz. 1<sup>st</sup>. From the middle Column it is plain that every place towards the left hand in any Number is 10 times the Value or Number of the place next it towards to right hand; as 1, 2, or 3, in the 2d, 3d, or 4th places becomes 10, 100, 1000; 20, 200, 2000; and 30, 300, 3000, &c.

2<sup>dly</sup>. The Column next the left hand gives you the Name that any Number in any place (not exceeding the 9th) is to be called by: As the first or uppermost in each of the three Examples above, is the Hundreds of Millions place; and consequently the Numbers are in Value 1, 2, and 3 hundred Millions, because the Digits there placed are 1, 2, and 3. But instead thereof, if 4, 5, 6, &c. were put in those places, the Numbers must be 4 hundred Millions, 5 hundred Millions, or 6 hundred Millions, &c. the Value or Name depending on the place that any Digit possesseth in any Number.

3<sup>dly</sup>. From the third Column it is evident, that in reading a Number there is a Contraction or Abbreviation of the Words in the first Column to be used. Thus (in the third Example) we do not read the three first Lines 3 hundred Millions, 30 Millions, and 3 Millions; but do only name the word Millions after the last of the three Figures. Thus 3 hundred thirty three Millions, and so of the Thousands, we say 3 hundred thirty three Thousand, &c. 333.

By these Rules, any Number, tho' never so great, may be read, as is farther exemplified by the following Table of this Art of Numbering; which is much more copious and ample, than any Arithmetic that I know of exhibits.

The



Number of Intest. Numbers.

[illegible]

1. It may not be improper to inform the Reader, that we make use of the words Billions, Trillions, &c. to prevent the Confusion that would arise by the often repeating Millions of Millions, &c. And if it be enquired how much a Trillion (or the like) is; I look for the Units of Trillions at the top of the Table, and casting my Eye downward in that Line till I come to the lower end of it, I then look directly toward the left hand, and find it a Number consisting of 19 places, *viz.* 18 Cyphers and (1) next the left hand in the 19th place, and is found by involving (or multiplying) a Million in itself, and that Product by a Million. So that a Trillion is as much as to say, one Million of Millions of Millions. So likewise a Quartillion is 1000000 multiplied three times in itself, (called its Biquadrate) and consists of 25 places: A Quinquillion is 1000000 multiplied 4 times in itself, and consists of 31 places, (called the Sur-solid or fifth Power of 1000000, of which Powers I shall speak in a proper place.) All which appears plain by the foregoing Table: for Units of Quinquillions, casting your Eye downward to the end of that Column or Series, and thence directly towards your left hand, you find the Figure 31, which shews it consists of 31 places, *viz.* 30 Cyphers and (1) next the left hand. And thus may you find what number of places any Number in the whole Table consists of.

2. Or if you would know the Value of a Number consisting of any number of places, it is but looking for the places in the Column of places, and casting your Eye directly to the right hand, till you come to a Figure, which tracing upward in the same Series or Row, you have the Name or Value of the Number, consisting of any of the places given.

3. By the Table it appears also, that there is no more difficulty in reading a Number of 36, &c. places of Figures, than in a Number consisting of 6 Places. For when any large Number is required to be read, begin at Tens place, and tell 6 towards the left hand, over which make a 1: then continue to tell 6 more the same way, and over that make a 2: and so forward 6 more, and make a 3, (which will fall over the 19th place): So that having marked thus your whole Number given as in the Table above, I read, for example, the 34 Number in the Table thus.

103 Thousand,

103 Thousand, 456 Quinquillions,  
 789 Thousand, 876 Quartillions,  
 543 Thousand, 234 Trillions,  
 567 Thousand, 898 Billions,  
 765 Thousand, 432 Millions,  
 345 Thousand, 678.

And the 33 in the Table thus:—290760 Quinquillions,

4300 Quartillions,

1325 Trillions,

732100 Billions,

73 Millions,

212437

In short, the Digit being known, and the place it possesseth, we have nothing to do but first to mention the Digit, and then the Place. As 9 in Units place is 9 Units, 8 is 8 Units, &c. 9 in Tens place is 9 Tens or Ninety; 9 in Hundreds place is 9 Hundred, &c. So 365 is 3 in Hundreds place, 6 in Tens place, and 5 in Units place; or 3 Hundred, 60, and 5: And the like of any other Numbers.

The narrow Column toward the right hand of the rest, may serve to refer to the Table upon occasion. As the Minutes since the Creation to *Anno* 1716 inclusive, are 3006387360, which consisting of ten places, is numbered as the 7th Line in the Table, with respect to the Names of the places: So likewise it being computed (in the Use of Multiplication) that the Number of Diamonds of a quarter of an Inch square that would pave our Globe, supposing it even and all Earth, are 12813844858011648000; which consisting of 20 places, is numbered or read as the 17th in the Table. And the Value of those Diamonds being £. 1281384485801164800000, which consists of 22 places, it must be read as the 19th in the Table above.

Thus it appears, that the Table is abundantly large enough to afford an Example how the greatest Number that can arise from almost any Subject, may be numbered: and, in truth, I have made it so large to oblige some curious Persons by way of Speculation, more than for any absolute Necessity there is for it.

There is likewise another Way of expressing Numbers by Letters, used by the *Roman* Numerists, and in Accounts in our antient Records, &c. and may be termed Literal Numbers, or the Numeral Letters; which are these:



|      |    |      |     |        |        |
|------|----|------|-----|--------|--------|
| I    | 1  | XXV  | 25  | CC     | 200    |
| II   | 2  | XXX  | 30  | CCC    | 300    |
| III  | 3  | XXXV | 35  | CCCC   | } 400  |
| IV   | 4  | XL   | 40  | or CD  |        |
| V    | 5  | XLV  | 45  | CDXC   | 490    |
| VI   | 6  | L    | 50  | D      | 500    |
| VII  | 7  | LV   | 55  | DC     | 600    |
| VIII | 8  | LX   | 60  | DCC    | 700    |
| IX   | 9  | LXX  | 70  | DCCC   | 800    |
| X    | 10 | LXXX | 80  | CM, or | } 900  |
| XI   | 11 | XC   | 90  | DCCCC  |        |
| XII  | 12 | C    | 100 | M, or  | } 1000 |
| XV   | 15 | CX   | 110 | CIO    |        |
| XX   | 20 | CXLV | 145 | CIOD   | 1500   |

$$CIODCCXX = 1720.$$

## SECT. II. Of Addition of Intire Numbers.

1. **A**DDITION is a Rule whereby several (even tho' never so many) Sums are aggregated and contracted into one Sum, called the Total.

2. If the Numbers to be added be of one Denomination, or be abstract Numbers, it is properly called Single Addition; and in this case let the Name of the thing you add be what it will, you must for every 10 in Units place carry 1 to the Tens place, and for every 10 in that place, carry 1 to the Hundreds; for every 10 in the Hundreds place, 1 to the Thousands place, and so forward, if you have never so many places; because in Numeration it has been shewn, that 10 in Units is 1 in Tens place, &c.

|   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |
|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |

## EXAMPLES.

## EXAMPLES.

| L.<br>Sterling. | lb.<br>Weight. | Ells. | Hundr.<br>Weight. | Barrels.  |
|-----------------|----------------|-------|-------------------|-----------|
| 17642           |                |       |                   |           |
| 3769            | 9367           |       | 9763              | 1764      |
| 8458            | 14239          | 9762  | 872               | 86        |
| 4764            | 4763           | 2997  | 939               | 9         |
| 13              | 8324           | 3862  | 26                | 732       |
| 327             | 8              | 8764  | 147               | 876       |
| 97864           | 0934           | 329   | 9183              | 384       |
| 132837          | 37635          | 25714 | 20930             | 3851 Sums |

| L.   | s. | d.  |
|------|----|-----|
| 7463 | 19 | 7½  |
| 421  | —  | 10  |
| 833  | 4  | 6½  |
| 1008 | 17 | —   |
| 94   | 12 | 11½ |
| 9    | 8  | 4   |
| 363  | 10 | 10½ |
| 1371 | —  | —½  |
| 432  | 13 | 4   |
| 92   | —  | 11½ |
| 140  | 10 | 9½  |
| 832  | —  | 10  |
| 721  | 19 | 8   |
| 9367 | 3  | 11  |
| 431  | 10 | 10½ |
| 210  | 11 | 6   |
| 182  | 11 | 11  |
| 32   | 16 | 8½  |
| 1003 | —  | 7   |
| 47   | 1  | 11  |
| 82   | —  | 10  |
| 382  | 15 | 9½  |
| 1714 | 3  | 11  |

3. If the Numbers to be added are Contract, or have several Denominations, which depend on the Value of the same thing, or on the same Sum received and paid, &c. as the Column of Pounds, Shillings, Pence, and Farthings in the Margin; you must begin with the Farthings, and for every 4 of them carry 1 to the Pence, for every 12 Pence carry 1 to the Shillings, for every 20 Shillings carry 1 to the Pounds, &c. But because I would not have you point at every 12 in the Pence place, which sullies and fouls Books that are kept clean; I have therefore inserted the following short Table, that you may see how many Shillings is in any Column of Pence that can well be supposed to come to hand; which may serve till you have learned how to divide by 12: for then you need not the Table.

The TABLE used in Addition of Pence.

| d. | s.  | d. | s.  | d.  | s.   | d.  | s.   | d.  | s.   |
|----|-----|----|-----|-----|------|-----|------|-----|------|
| 12 | = 1 | 60 | = 5 | 108 | = 9  | 156 | = 13 | 204 | = 17 |
| 24 | = 2 | 72 | = 6 | 120 | = 10 | 168 | = 14 | 216 | = 18 |
| 36 | = 3 | 84 | = 7 | 132 | = 11 | 180 | = 15 | 228 | = 19 |
| 48 | = 4 | 96 | = 8 | 144 | = 12 | 192 | = 16 | 240 | = 20 |

Note,

*Note*, That  $\frac{1}{4}$  is 1 Farthing, or one 4th part of a Penny (or other thing)  $\frac{1}{2}$  is one half of a Penny, &c. and  $\frac{3}{4}$  is 3 fourth parts of a Penny or other thing that is placed before it.

And in consequence you find the Sum of the Farthings in the last Example to be 19 (or 4 Pence 3 Farthings): Put down the  $\frac{3}{4}$  (as you see in the Sum) and carry the 4 d. to the Units place of Pence, saying 4 and 1 is 5, and 9 is 14, and 1 is 15, and so on, you'll find when you come to the top of the Series 78 Pence. Put down the 8 on a piece of (by or) waste Paper, and carry the 7 to the Tens place of Pence, saying 7 and 1 is 8, &c. till you come to 18, (at the top of the Column) which put down to the 8, and it makes 188 Pence; which by the little Table is 15 s. and 8 over: Therefore put down the 8 Pence, and carry the 15 s. to the Units place of Shillings, saying 15 and 3 is 18, and 5 is 23, &c. to the top of the Column, when the Sum is 77. Put down the 7, and carry 7 to the Tens place, saying 7 and 1 is 8, and 1 is 9, &c. which at the top you find 19. Put down the odd 1, (which makes the 7 17 s.) and carry half of 18 or 9 to the Pounds, which sum up as by the Example of 1 Denomination, and you find the Sum. total 27168 l. 17 s. 8 d.  $\frac{3}{4}$ .

4. Hence it may be inferred, that any Numbers may be added, tho' of divers Denominations, as tho' they were of one Denomination or Name. So the Sum of the several Columns of Money foregoing, is 27159 l. 182 s. 184 d. 19 grs. or l. 27168 : 17 : 8 $\frac{3}{4}$ . for the 19 Farthings any one knows is 4 d.  $\frac{3}{4}$ , 184 Pence by the little Table is 15 s. 4 d. and 4 d. in the Farthings is 15 s. 8 d. 182 s. is l. 9 : 2 : — and the 15 s. in the Pence is l. 9 : 17 : — Put down the 17, and add the 9 l. to 27159, makes the Total l. 27168 : 17 : 8 $\frac{3}{4}$ .

This is much the best way; done without pricking and carrying from one Denomination to another, which one is apt to forget when he has a mind to run a Sum over a second time.

And by one and the same Method you add all kind of Denominations of Weight, Measure, Money, &c. And for reducing the Sums properly, (as that above is done) you will easily see how to do that by the little Table following of the Quarters of Hundreds, &c. in the Sum of the Pounds, &c.

|       | l.       | s.    | d.    | grs. |
|-------|----------|-------|-------|------|
|       | 27159    | : 182 | : 184 | : 19 |
| Add { |          |       | 4     | : 3  |
|       |          | 15    | : 4   | : —  |
|       | 9        | : 2   | : —   |      |
|       | l. 27168 | : 17  | : 8   | : 3  |



TABLE 1. Of Averdupois Weight.

| Ton | Hund. | qr.C. | Pounds | Ounces | Drams  |
|-----|-------|-------|--------|--------|--------|
| 1   | 20    | 80    | 2240   | 35840  | 573440 |
|     | 1     | 4     | 112    | 1792   | 28672  |
|     |       | 1     | 28     | 448    | 7168   |
|     |       |       | 1      | 16     | 256    |
|     |       |       |        | 1      | 16     |

TABLE 2. Of English Coin.

| Farth. | Penny | Groat | Shilling | d. | Noble | Angel | Mark | Pound |
|--------|-------|-------|----------|----|-------|-------|------|-------|
| 4      | 1     |       |          |    |       |       |      |       |
| 16     | 4     | 1     |          |    |       |       |      |       |
| 48     | 12    | 3     | 1        |    |       |       |      |       |
| 320    | 80    | 20    | 6        | 8  | 1     |       |      |       |
| 480    | 120   | 30    | 10       |    | 1½    | 1     |      |       |
| 640    | 160   | 40    | 13       | 4  | 2     | 1½    | 1    |       |
| 960    | 240   | 60    | 20       |    | 3     | 2     | 1½   | 1     |

These Tables are put into the most easy and useful Method that I could think of, and are more especially adapted for reducing *Averdupois Weight* (by the first Table) and *English Coin* (by the second Table) from any one Denomination to another, with one Multiplication or Division.

For the more easy adding this or any other Example of *Averdupois greater Weight*, I have inserted the following little Table; which shews the *Quarters* of Hundreds in a Column of Pounds.

A Table

| <i>A Table of Quarters of Hundreds in a Column of Pounds.</i> |       |     |       |     |       |
|---------------------------------------------------------------|-------|-----|-------|-----|-------|
| lb                                                            | qr.C. | lb  | qr.C. | lb  | qr.C. |
| 28                                                            | 1     | 140 | 5     | 252 | 9     |
| 56                                                            | 2     | 168 | 6     | 280 | 10    |
| 84                                                            | 3     | 196 | 7     | 308 | 11    |
| 112                                                           | 4     | 224 | 8     | 336 | 12    |

In the Example last above, having added each single Series or Row of Figures (as by the Rules for one Denomination foregoing) I find the Sum 2054 Ton, 176 Hundred, 24 Quarters, and 230 Pounds.

Then (as in the Example before) I must reduce this Sum properly, which is done thus: By the little Table last inserted, 'tis plain that in 230 lb. there are 8 qrs. and 6 lb. In the 24 Quarters are 6 Hundred; and in the 176 Hundred (by taking half except Units place, and adding the odd 10 to the Units) are 8 Ton, 16 C. the Sum of which is Ton 2063 : 4 : 0 : 6, done without pricking, or any Charge to the Memory. But for such as can do Division, the last little Table will not be of such necessary Use as it is to others.

| <i>A Table shewing the Ounces in a Column of Drams; or lb in that of Ounces.</i> |       |        |
|----------------------------------------------------------------------------------|-------|--------|
| 16=1                                                                             | 80=5  | 144=9  |
| 32=2                                                                             | 96=6  | 160=10 |
| 48=3                                                                             | 112=7 | 176=11 |
| 64=4                                                                             | 128=8 | 192=12 |

*An Example of adding Averdupois greater Weight.*

Ton. C. qr. lb

47 : 3 : 2 : 27

192 : 19 : — : 12

43 : 18 : 3 : 24

973 : — : 2 : 14

9 : 15 : 1 : 15

47 : 18 : 3 : —

55 : 9 : 2 : 23

34 : 16 : 2 : 25

84 : 19 : 3 : 17

179 : 16 : 2 : 12

84 : 8 : 1 : 25

73 : 18 : 3 : 24

234 : 17 : — : 12

Sum 2054 : 176 : 24 : 230

8 : 6

6 : 0 : —

8 : 16 : — : —

2063 : 4 : 0 : 6

*The next Example is done thus:*

Having summed up the Units, and then the Tens place of Drams, I find the Sum 191; the Ounces in like manner 177, and the Pounds 3234.

And by the little Table above (which any one may make or enlarge by adding 16 to itself, &c.) I find that the next Number to and less

less than 191, is 176, against which stands 11 Ounces; and 191 is 15 Drums more, therefore I put 11 Oun. 15 dr. down as you see. And the Sum of the Ounces being 177, I look for that in the Table, and find the next and less Number thereto to be 176; against which is 11. So that I put down lb 11: 1 Oun. as you see in the Margin, and as in the foregoing Examples; which 11 added to the Pounds, and the 1 to the 11 Ounces, &c. the Sum reduced by the little Table is lb 3245 : 12 : 15.

And thus I have given you sundry Examples and Rules, altogether new, for adding large Numbers of several Denominations, as tho' they were but one Denomination, without carrying any thing from one Name or Denomination to another: which new Method I take to be every way preferable to the common, especially where there are large Examples to be added of several Denominations. I shall give you farther Tables, whereby little Tables may be made for Addition; or where from Substraction and Reduction of any thing may be performed.

*An Example of adding  
Averdupois lesser  
Weight.*

| lb    | Oun. | dr. |
|-------|------|-----|
| 764 : | 15 : | 15  |
| 73 :  | 13 : | 12  |
| 92 :  | 10 : | 13  |
| 173 : | — :  | 7   |
| 39 :  | 14 : | 11  |
| 91 :  | 12 : | 10  |
| 475 : | 9 :  | 8   |
| 39 :  | 15 : | 12  |
| 99 :  | 11 : | 11  |
| 732 : | 4 :  | 14  |
| 74 :  | 14 : | 9   |
| 86 :  | 11 : | 13  |
| 37 :  | 7 :  | 10  |
| 75 :  | 13 : | 14  |
| 32 :  | 10 : | 8   |
| 124 : | 4 :  | 3   |
| 53 :  | 11 : | 14  |
| 176 : | 4 :  | 7   |

lb 3234 : 177 : 191

: 11 : 15 } Add.  
11 : 1 :

lb 3245 : 12 : 15 Sum.

TABLE 3. Wine-Measure.

| Ton | Pipes | Hogsh. | Tiercian | Gallons | Quarts | Solid Inches. |
|-----|-------|--------|----------|---------|--------|---------------|
| 1   | 2     | 4      | 6        | 252     | 1008   | 58212         |
| 1   | 2     | 3      | 126      | 504     | 29106  |               |
| 1   | 1     | 1      | 63       | 252     | 14553  |               |
| 1   | 1     | 1      | 42       | 168     | 9702   |               |
| 1   | 1     | 1      | 1        | 4       | 231    |               |
| 1   | 1     | 1      | 1        | 1       | 57     |               |

TABLE 4.



TABLE 4. Beer-Measure.

| Pints | Quart | Pottle | Gallon | Firkin | Kilderkin | Barrel |
|-------|-------|--------|--------|--------|-----------|--------|
| 2     | 1     |        |        |        |           |        |
| 4     | 2     | 1      |        |        |           |        |
| 8     | 4     | 2      | 1      |        |           |        |
| 72    | 36    | 18     | 9      | 1      |           |        |
| 144   | 72    | 36     | 18     | 2      | 1         |        |
| 288   | 144   | 72     | 36     | 4      | 2         | 1      |

Note, Ale-Measure hath 32 Gallons to the Barrel.

TABLE 5. Dry Measure. (36 Bushels to the Chaldron of Coals.)

| Last | Weys | Quarters | Bushels | Pecks | Gallons | Pottles | Quarts | Pints |
|------|------|----------|---------|-------|---------|---------|--------|-------|
| 1    | 2    | 10       | 80      | 320   | 640     | 1280    | 2560   | 5120  |
|      | 1    | 5        | 40      | 160   | 320     | 640     | 1280   | 2560  |
|      |      | 1        | 8       | 32    | 64      | 128     | 256    | 512   |
|      |      |          | 1       | 4     | 8       | 16      | 32     | 64    |
|      |      |          |         | 1     | 2       | 4       | 8      | 16    |
|      |      |          |         |       | 1       | 2       | 4      | 8     |
|      |      |          |         |       |         | 1       | 2      | 4     |
|      |      |          |         |       |         |         | 1      | 2     |

TABLE 6. Long-Measure.

| Barley-Corns. | Inch  | Foot | Yard | Pole | Furlong | Mile |
|---------------|-------|------|------|------|---------|------|
| 3             | 1     |      |      |      |         |      |
| 36            | 12    | 1    |      |      |         |      |
| 108           | 36    | 3    | 1    |      |         |      |
| 594           | 198   | 16½  | 5½   | 1    |         |      |
| 23760         | 7920  | 660  | 220  | 40   | 1       |      |
| 190080        | 63360 | 5280 | 1760 | 320  | 8       | 1    |

TABLE 7.

TABLE 7. Square-Measure.

| Square Mile | Square Acres | Square Rods | Square Poles | Square Yards     | Square Feet       | Square Inches |
|-------------|--------------|-------------|--------------|------------------|-------------------|---------------|
| 1           | 640          | 2560        | 102400       | 3097600          | 27878400          | 4014489600    |
|             | 1            | 4           | 160          | 4840             | 43560             | 6272640       |
|             |              | 1           | 40           | 1210             | 10890             | 1568160       |
|             |              |             | 1            | 30 $\frac{1}{4}$ | 272 $\frac{1}{4}$ | 39204         |
|             |              |             |              | 1                | 9                 | 1296          |
|             |              |             |              |                  | 1                 | 144           |

TABLE 8. Of Solid Measure.

| Solid Inches    | Feet Solid                 | Solid Yard               | Solid Pole | Rod    | Acre  | Mile |
|-----------------|----------------------------|--------------------------|------------|--------|-------|------|
| 1728            | 1                          |                          |            |        |       |      |
| 46656           | 27                         | 1                        |            |        |       |      |
| 7762329         | 4492 $\frac{8}{9}$         | 166 $\frac{175}{180}$    | 1          |        |       |      |
| 1963885176      | 1136507 $\frac{625}{1000}$ | 42092 $\frac{825}{1000}$ | 253        | 1      |       |      |
| 15711081408     | 9092061                    | 336743                   | 2024       | 8      | 1     |      |
| 254358061056000 | 147197952000               | 5451776000               | 32770584   | 129528 | 16191 | 1    |

TABLE 9. Of Troy-Weight.

| Pounds | Ounces | Pennywt. | Grains |
|--------|--------|----------|--------|
| lb     | 3      | dw.      | gr.    |
| 1      | 12     | 240      | 5760   |
|        | 1      | 20       | 480    |
|        |        | 1        | 24     |

TABLE 10. Apothecaries Weight.

| Grains | Scruple | Dram | Ounce |
|--------|---------|------|-------|
| gr.    | 3       |      |       |
| 20     | 1       |      |       |
| 60     | 3       | 1    |       |
| 480    | 24      | 8    | 1     |

TABLE

TABLE 11. Of Time.

| Year | Weeks            | Days              | Hours | Minutes | Seconds  |
|------|------------------|-------------------|-------|---------|----------|
| 1    | 52 $\frac{1}{4}$ | 365 $\frac{1}{4}$ | 8766  | 525960  | 31557600 |
|      | 1                | 7                 | 168   | 10080   | 604800   |
|      |                  | 1                 | 24    | 1440    | 86400    |
|      |                  |                   | 1     | 60      | 3600     |
|      |                  |                   |       | 1       | 60       |

TABLE 12. Of Motion.

| Seconds | Minute | Degree | Sine | Quadrant | Circle |
|---------|--------|--------|------|----------|--------|
| 60      | 1      |        |      |          |        |
| 3600    | 60     | 1      |      |          |        |
| 108000  | 1800   | 30     | 1    |          |        |
| 324000  | 5400   | 90     | 3    | 1        |        |
| 1296000 | 21600  | 360    | 12   | 4        | 1      |

The Tables foregoing are so easy, as to need little Explanation: I have put them in two Methods, a small matter differing. Table 1. which is of Averdupois Weight, is to be thus read; 1 Ton is 20 Hundred, 80 Quarters of C. 2240 Pounds, 35840 Ounces or 573440 Drums: Also 1 C. is 4 Quarters, 112 lb. 1792 Ounces, &c. reading from the right towards the left hand, all which is useful in Reduction.

And the same is to be observed in the second sort of Table, only here you read the contrary Way, from the right hand toward the left: As 1 Pound Sterling is 1  $\frac{1}{2}$  Marks, 2 Angels, 3 Nobles, 20 Shillings, 60 Groats, 240 Pence, 960 Farthings. One Mark is 1  $\frac{1}{2}$  (or 1 Angel and 1 Third) 2 Nobles 13 s. 4 d. 40 Groats, 160 Pence, &c. And the Numbers above shew that 4 Farthings is 1 Penny, 4 Pence 1 Groat, 3 Groats 1 s. 6 s 8 d. = 1 Noble, &c.

As to the Proof of Addition, it is either performed by Addition or Substraction,

E

Thus



Thus the Sum of the 3 upper Lines added to the Sum of the 2 lower, is equal to the Sum Total:  
 For the Sum of any Parts added to the rest of the Parts, must give the Sum Total of all the Parts which make the Whole.

Or the Sum of the two lowermost deducted from the Sum Total, leaves the Sum of the greater, or of the three uppermost Sum Total = 17198 Numbers.

And thus I hope I have laid down such plain and copious Rules, &c. as will be useful to the young Reader in adding Money, Weight, Measure, &c.

|                                          |       |        |
|------------------------------------------|-------|--------|
| Sum of the 3 uppermost                   | 16542 |        |
| Sum of the 2 lower Lines =               | 656   |        |
| <hr/>                                    |       |        |
| Sum Total, or Sum of }<br>the Sums — — } | 17198 | Proof. |
| The lesser deduct                        | 656   |        |
| <hr/>                                    |       |        |

Leaves the greater = 16542 Proof.

When large Sums of Money are (in Records, &c.) required to be expressed Literally, or in Numeral Letters, they may be wrote, &c. thus ;

| I.                          | s.     | d.              |  | I.                     | s.     | d.              |
|-----------------------------|--------|-----------------|--|------------------------|--------|-----------------|
|                             |        |                 |  | M.                     | C.     | X.              |
| MDCCLXXXVI :                | VI :   | IX ob           |  | I, VII, III, VI :      | VI :   | IX ob           |
| M <sup>10</sup> DCCCCXXIX : | X :    | II              |  | X, IX, II, IX :        | X :    | II              |
| M <sup>8</sup> CCXCIX :     | XI :   | IX              |  | VIII, II, IX, IX :     | XI :   | IX              |
| M <sup>20</sup> DLXXXVII :  | X :    | IX ob           |  | XX, V, VIII, VII :     | X :    | IX ob           |
| M <sup>10</sup> CLII :      | VI :   | VIII            |  | DX, I, V, II :         | VI :   | VIII            |
| M <sup>19</sup> DCCXLV :    | XIX :  | VI ob           |  | XIX, VII, IV, V :      | XIX :  | VI ob           |
| M <sup>60</sup> CCXXXI :    | XI :   | IV              |  | LX, II, III, I :       | XI :   | IV              |
| <hr/>                       |        |                 |  | <hr/>                  |        |                 |
| M <sup>631</sup> DCLXXXII : | XVII : | — ob            |  | DCXXXI, VI, VIII, II : | XVII : | — ob            |
| viz. 631682 :               | 17 :   | — $\frac{1}{2}$ |  | or 631682 :            | 17 :   | — $\frac{1}{2}$ |
| Sums Total                  |        |                 |  | Sums Total             |        |                 |

To add the first Column 'tis easy till you come to the Pounds, and there add together 1 $\frac{1}{2}$  all under 10, to each Ten there add all Tens

Tens under 100; to those add what is under 1000, and the Thousands carry'd to the rest.

*Note,* The M, C, and X are Thousands, Hundreds, and Tens, the Titles of the Columns they stand over, as to place.

 SECT. III. *Subtraction of Intire Numbers.*

**T**HIS is a Rule whereby the Excess or Difference between two Numbers is discovered; that is, we find out by this part of Arithmetic how much one of two Numbers given, is greater or less than the other. Thus if 15 be compared with 20, we find it lesser by 5.

Hence it appears, that in Subtraction there are always 2 Numbers given, from whence a third is discovered.

Of the 2 Numbers given, the greater I call the *Compound Number*, (as being composed of the lesser Number and the Difference between those given.) The lesser I call the *Subtrahend*, it being that which is to be taken from or drawn out of the greater. And when that is done, there ariseth the third Number, which may be termed the Remainder, Excess, or Difference between the Numbers given, and is the Number sought for in this Part of the Art of Numbering.

As in Addition, so in Subtraction the Numbers given are either of one simple Denomination, or else they are one or both of several Denominations or Names; for the Numbers given are Abstract or Concrete.

I shall next illustrate what is said, by examples of one and more Denominations in the common Way of Working; and 2dly, in another.

*Examples of one Denomination, or of Abstract Numbers.*

| <i>Example 1.</i> | <i>Example 2.</i> | <i>Example 3.</i> |
|-------------------|-------------------|-------------------|
| Lent l. 71032     | Borrowed l. 19721 | Received l. 9211  |
| Received l. 9735  | Paid l. 8957      | Disbursed l. 1973 |
| <hr/>             | <hr/>             | <hr/>             |
| Remainer l. 61297 | Remainer l. 10764 | Rem. = l. 7238    |

*Examples of several Denominations, (or of Contract Numbers.)*

| <i>Example 4.</i>                | <i>Example 5.</i>                           |
|----------------------------------|---------------------------------------------|
| Gained l. 1192 : 12 : 3          | Debtor l. 10132 : 10 : 4 $\frac{1}{4}$      |
| Lost l. 287 : 14 : 7             | Creditor l. 975 : 15 : $\frac{3}{4}$        |
| <hr/>                            | <hr/>                                       |
| Remaineth gain'd l. 904 : 17 : 8 | Remaineth Dr l. 9156 : 15 : 3 $\frac{1}{2}$ |
| E 2                              | <i>Example</i>                              |

## Example 6.

|                                                                                                              |                                   |
|--------------------------------------------------------------------------------------------------------------|-----------------------------------|
| Received as an Agent or Factor, to be applied<br>and disbursed as <i>per</i> Order of my Employer—           | } l. 9000 : — : —                 |
| Paid <i>A. B.</i> for Goods bought ——— l. 2072 : 13 : 4                                                      |                                   |
| For a Ship with Rigging ——— 936 : 19 : 2                                                                     |                                   |
| For Custom to the King ——— 97 : 15 : 3 $\frac{1}{4}$                                                         |                                   |
| For Insuranc ——— 72 : 3 : 6                                                                                  |                                   |
| For Freight ——— 100 : 11 : 9 $\frac{1}{4}$                                                                   |                                   |
| In all ———                                                                                                   | 3280 : 3 : 1                      |
| Which Sum of the Payments being deducted<br>from the l. 9000, the Remainer (in the Factor's<br>hands) is ——— | } l. 5719 : 16 : 11 $\frac{1}{4}$ |

## Example 7.

|                                                                                                         | Weight                     | Value                          |
|---------------------------------------------------------------------------------------------------------|----------------------------|--------------------------------|
|                                                                                                         | C. gr. lb                  | l. s. d.                       |
| Bought of <i>A. B.</i> Sugars                                                                           | 193 : 3 : 25               | 386 : 19 : 4                   |
| of <i>C. D.</i> more                                                                                    | 323 : 3 : 17               | 532 : 18 : 6 $\frac{1}{4}$     |
| Bought in all                                                                                           | 517 : 3 : 14               | l. 919 : 17 : 10 $\frac{1}{4}$ |
|                                                                                                         | Weight                     | Value                          |
|                                                                                                         | C. gr. lb                  | l. s. d.                       |
| Sold <i>E. F.</i> 25 : 1 : 14 for                                                                       | 59 : 17 : 6                |                                |
| to <i>G. H.</i> 126 : 3 : 25 at                                                                         | 315 : 12 : 10              |                                |
| to <i>J. K.</i> 94 : 2 : 14 at                                                                          | 235 : 17 : 6 $\frac{1}{4}$ |                                |
| to <i>L. M.</i> 87 : 3 : 20 at                                                                          | 180 : 13 : 4               |                                |
| Sold in all                                                                                             | 334 : 3 : 17               | at 792 : 1 : 2 $\frac{1}{4}$   |
| Remains unfold <i>C.</i>                                                                                | 182 : 3 : 25               | 127 : 16 : 7 $\frac{1}{2}$     |
| Which unfold Goods cost me                                                                              | ———                        | 364 : 18 : 10 $\frac{1}{4}$    |
| From which last Sum deducting the next above<br>it, the Remainer is gained (already) by this<br>Account | } ———                      | 237 : 2 : 2 $\frac{1}{4}$      |

## Directions for performing the foregoing Operations.

IN the first Example you are to take the lower Line from the upper, or the lesser Number 9735 from 71032. Begin at Units place, and say 5 from 2 cannot be, but 5 from 10, which you must borrow, and add to 2; that is, 5 from 12, and there remains 7. Then having put the 7 under the Line, say 1 borrowed and 3, (the Figure in Tens place of the Subtrahend) is 4, from 3 (in the compound



pound Number) cannot be; but from 13 (borrowing 10 as before) leaves 9. Which put down, as you see, and say 1 borrowed and 7 (the Figure in Hundreds place of the Subtrahend) is 8 from 10 borrowed (because you cannot take 8 from 0) and there rests 2. One borrowed and 9 is 10 from 1 (in the Thousands place of the compound Number) cannot be, but 10 from 11 leaves 1, which put under, as you see. And lastly, 1 borrowed from 7 in the upper Line, and the Remainder is 6. So the whole Sum remaining is 61297. And thus any Operation of one Denomination is performed.

Then for several Denominations I shall instance, 1<sup>st</sup>, in the fifth Example. You must always begin with the least Name, here Farthings; and say 3 Farthings from 1 cannot be, but borrowing 1 Penny or 4 Farthings, add to the 1 Farthing makes 5; so 3 from 5, and there rests 2 Farthings: put under as in the Example, and say 1 Penny borrowed from 4 Pence in the upper Line, and there rests 3 *d*. Then say 15 *s*. from 10 *s*. cannot be; but 15 from 20 *s*. which you borrow, and the 10, *viz.* from 30, and there resteth 15. Then say 1 borrowed and 5 is 6, from 12, rests 6 Pounds, &c. as in the former Examples.

The sixth Example is performed by first adding together the Sums laid out, and then deducting that Sum, which is *l*. 3280 : 3 : 1 from the 9000. Thus 1 *d*. from 12 you borrow, leaves 11 *d*. 1 you borrowed and 3 *s*. is 4 from 20 *s*. which borrow, and the rest is 16; which put under and deduct the 1 borrowed from 10, &c. as before.

Lastly, in the seventh Example, you are to deduct the Hundreds, Quarters, and Pounds sold from those bought: to perform which, you must say 17 Pounds from 14 *lb*. cannot be, but 17 from 28 *lb*. which you borrow (that being a Quarter of a Hundred, which is the next Denomination) leaves 11; which added to the 14 *lb*, makes the 25 which you see. Then 1 Quarter of a Hundred borrowed and 3 is 4, from 7 (that is, 4 Quarters or 1 Hundred borrowed and added to the 3 Quarters standing in the upper Line) and the rest is 3 Quarters; which put in the Remainder, saying 1 (that is, 1 Hundred weight) borrowed and 4 (in the Sum of the Sales) is 5, from 7 in the Hundreds bought, leaves 2; which put down, and so proceed, as in subtracting one Denomination: and the quantity of Sugar remaining unfold you find to be *C*. 182 : 3 : 25. And the Value of what is sold deducted from the whole Value bought, the Remainder is *l*. 127 : 16 : 7½ (which is the Sum only wanting

wanting to make up the Cost of all that was bought.) Which Sum remaining deducted from the prime Value of what Sugar remains unfold, viz. 182 C. 3 gr. 25 lb. which costs £. 364 : 18 : 10½ the remaining Cash is £. 237 : 2 : 2½ which is got by this Trading, thus far.

*Note*, Tho' the Subtrahend, as here £. 127 : 16 : 7½ stands over the Compound Number £. 364 : 18 : 10½, it is as easily deducted as if the lesser Number stood under the greater, as is most usual.

By the Rules and Examples above, I suppose any one may perform Subtraction after the common way.

But the Work may be effected without saying *such a Figure from such a one cannot be*, and *1 borrowed*, &c. as in the Margin.

For where any Digit in the compound Number is less than that standing under it in the Subtrahend, mark the next toward the left-hand with a Point, and put over it a Digit, which is 1 less, as here over 4 in the Number given put 3; over 3, 2, &c.

|         |   |    |    |    |    |   |
|---------|---|----|----|----|----|---|
|         |   | 10 | 10 | 10 | 10 |   |
|         |   | 4  | 6  | 1  | 2  | 3 |
| From    | 1 | 5  | 7  | 2  | 3  | 4 |
| Take    |   | 8  | 9  | 5  | 6  | 5 |
| Refts = |   | 6  | 7  | 6  | 6  | 8 |

Then where the Figures put over the pricked ones (or those given) are still less than those respectively in the Subtrahend, place 10 partly over, but a little toward the left hand; then you have the very Numbers at large, from whence Subtraction is to be made. Thus 7 from 12 refts 5; 5 from 13 refts 8; 6 from 12 refts 6; 5 from 11 leaves 6; 9 from 16 leaves 7; and 8 from 14 leaves 6.

Now I have only inserted the Figures over those pointed, and the Tens to explain the Rule; but 'tis certain, the Work may be as easily performed mentally, by supposing every Figure as afore-said less by the 1 (supposed 10) which is borrowed from the next towards the left hand of the Number given to make Subtraction from. For whenever you borrow 10, that 10 is presumed to be taken from the next Figure towards the left hand; and of consequence therefore that is 1 less, (1 in any Series being 10 in another next it toward the right hand.) Thus in the Example, 7 from 12 refts 5; where the one Ten borrowed makes the Figure in Tens place a Unit less, i. e. the 4 is but 3. And when you say 5 from 13 refts 8; this 1 borrowed is actually taken from the next Digit 3, and leaves it but a 2, and so 2 becomes but a 1, 7 a 6, and 5 a 4, which is all supposed as you perform the Work, without putting down any thing but the Remainder.

So

### SECT. III. *Subtraction of Intire Numbers.*

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So also in divers Denominations, the Unit borrowed being supposed one of the next superior Denomination, must leave that a Unit less, as is explained in the Margin. Where 2 s. 6 d.

1 gr. in the upper of the two Lines given, being less than 17 s. 9 d.  $\frac{1}{4}$  in the lower

Line; therefore 4 Farthings

taken from the 6 d. makes the  $\frac{1}{4}$   $\frac{1}{4}$ , and 5 d. resteth;

12 d. borrowed from the 2 s.

and put over the 5 d. makes 17 d. and 1 s. resteth; 20 s. taken from the 3 l. makes the said 1 s. to be 21 s. and 2 l. resteth, &c. as in one Denomination; then I have it plain to my sight, that  $\frac{1}{4}$  from  $\frac{1}{4}$  leaves  $\frac{3}{4}$  or  $\frac{1}{2}$ ; 9 d. from 12 and 5 or 17 d. leaves 8; 17 s. from 21 leaves 4 s; 7 l. from 12 l. leaves 5; 9 l. from 10 leaves 1 l. and 3 l. from 6 l. leaves 3, without borrowing and paying. Which Method with Use will make the Work easier and shorter than the common Way; I mean, when only the two Sums given are wrote down, and the rest performed in the Mind, as Example.

This is done without the least mentioning,

borrowing or paying, and with more Ease and Expedition; and may serve also to

shew the Nature and Reason of the common Way of Working.

|                                                              | <i>l.</i> | <i>s.</i> | <i>d.</i> |
|--------------------------------------------------------------|-----------|-----------|-----------|
| The Rent-Roll of my Estate } (suppose) is <i>per annum</i> — | 1765      | 12        | 3         |
| Repairs, Taxes, &c. deduct.                                  | 279       | 15        | 9         |
| My clear Estate <i>per ann.</i> =                            | 1485      | 16        | 6         |

The Proof of Subtraction is either by *Addition* or *Subtraction*: For in the last foregoing Example, the Remainder *l.* 1485 : 16 : 6 being added to the Subtrahend, *l.* 279 : 15 : 9, the Sum is the compound Number given, or *l.* 1765 : 12 : 3; which proves the Operation to be right.

Or by *Subtraction*. If from the *l.* 1765 : 12 : 3 you take the Remainder *l.* 1485 : 16 : 6, there will then remain *l.* 279 : 15 : 9 = the Subtrahend given.

SECT.



SECT. IV. *Multiplication of Intire Numbers.*

**M**ULTIPLICATION is a Rule which shews the Number of Units produced by making one assigned Number any Multiplicity of times itself.

Or 'tis that Part of Arithmetic of such admirable Extent, that no Number or Quantity can be so great, but another greater may be discovered: For by it not only the Number of single Sands that would compose an Heap as big as our Earth, but even one to extend to the starry Firmament are within the limits of this part of the Numeric Art to compute.

It is an Abridgment of the Work of Addition, performing that in a Minute, that Addition would require some Hours to do.

Thus if 8766 were to be multiplied by 20000, (or to know how many Hours are in 20000 Years) 'tis only to double 8766, which is 17532, and to that place the four Cyphers toward the right hand will make it 175320000 the Answer, (which is done in less than half a Minute.) But how long it would require to put down 8766, 20000 times; or 20000, 8766 times; and then add all those Numbers together; I leave the Reader to judge, and consequently of the Excellency of this Rule, which is justly called Multiplication.

As in the last Section, so in this Part there are two Numbers given to find a third: To instance, as above, 8766 and 20000 are given, and 175320000 is the Number produced by or resulting from the Multiplication.

Of these 3 Numbers 1 (commonly the greater of the 2 given) is called the Multiplicand, the other of those given is the Multiplier, and both together they are called Factors; and the Number arising therefrom after Multiplication, is called the Product, or (in Geometrical Operations) the Rectangle.

The Relation these three Numbers have to each other, is, That the Product containeth either of the Factors so often as the other Factor contains a Unit: and consequently

As 175320000. to 8766 :: so 20000. to 1. Or

As 175320000. to 20000 :: 8766. 1. &c. But this by the by, till we arrive at the Rules of Proportion.

The whole Business of Multiplication consists in these three things.

1. In knowing mentally and readily what any two Digits multiplied together produce.

2. In

2. In the right placing those Products. And,
3. In collecting those particular Products into one general Product.

Hence it follows, that a Table of the Products of one Digit by another be composed, which, before you can proceed farther in this most pleasant and useful Science, must be learned by heart: thus

The first Column is what is absolutely necessary to get by heart.

The second Column for the most part, that is, in multiplying by 10 and 11, does not charge the Memory: for to multiply any Number by 10, is only to place a Cypher to the right hand of the Multiplicand, (for 1, tho' a Number, as augmenting and diminishing another by Addition and Subtraction, yet it neither multiplies nor divides.)

And to multiply any Digit by 11, is only to repeat the Digit as in the Example. And I have added the Multiplication of the Digits by 12, because that being got by heart, saves the trouble of making a whole Line, and of adding two Lines together: for when you have that by heart, you can multiply or divide by 12 (a Number much used, as being the Pence in a Shilling, the Inches in a Foot, &c.) as by one single Digit.

| <i>The Multiplication Table.</i> |    |     |     |
|----------------------------------|----|-----|-----|
| 3 times                          | 3  | is= | 9   |
|                                  | 4  | =   | 12  |
|                                  | 5  | =   | 15  |
|                                  | 6  | =   | 18  |
|                                  | 7  | =   | 21  |
|                                  | 8  | =   | 24  |
|                                  | 9  | =   | 27  |
| 4 times                          | 4  | =   | 16  |
|                                  | 5  | =   | 20  |
|                                  | 6  | =   | 24  |
|                                  | 7  | =   | 28  |
|                                  | 8  | =   | 32  |
|                                  | 9  | =   | 36  |
| 5 times                          | 5  | =   | 25  |
|                                  | 6  | =   | 30  |
|                                  | 7  | =   | 35  |
|                                  | 8  | =   | 40  |
|                                  | 9  | =   | 45  |
| 6 times                          | 6  | =   | 36  |
|                                  | 7  | =   | 42  |
|                                  | 8  | =   | 48  |
|                                  | 9  | =   | 54  |
| 7 times                          | 7  | =   | 49  |
|                                  | 8  | =   | 56  |
|                                  | 9  | =   | 63  |
| 8 times                          | 8  | =   | 64  |
|                                  | 9  | =   | 72  |
| 9 times                          | 9  | =   | 81  |
| 10 times                         | 2  | =   | 20  |
|                                  | 3  | =   | 30  |
|                                  | 4  | =   | 40  |
|                                  | 5  | =   | 50  |
|                                  | 6  | =   | 60  |
|                                  | 7  | =   | 70  |
|                                  | 8  | =   | 80  |
|                                  | 9  | =   | 90  |
| 11 times                         | 2  | =   | 22  |
|                                  | 3  | =   | 33  |
|                                  | 4  | =   | 44  |
|                                  | 5  | =   | 55  |
|                                  | 6  | =   | 66  |
|                                  | 7  | =   | 77  |
|                                  | 8  | =   | 88  |
|                                  | 9  | =   | 99  |
| 12 times                         | 2  | =   | 24  |
|                                  | 3  | =   | 36  |
|                                  | 4  | =   | 48  |
|                                  | 5  | =   | 60  |
|                                  | 6  | =   | 72  |
|                                  | 7  | =   | 84  |
|                                  | 8  | =   | 96  |
|                                  | 9  | =   | 108 |
|                                  | 10 | =   | 120 |
|                                  | 11 | =   | 132 |
|                                  | 12 | =   | 144 |

The wonderful Productions arising from the placing Numbers, are many and surprizing: I shall here insert an Instance, where, by placing only 9 Figures to the best advantage about 5 Digits given, constitutes all the Parts of the common Table that are absolutely necessary to be inserted for the more apprehensive part of Readers.

*A most brief Multiplication Table, viz.*

|                                                            |   |                                                            |   |                                                               |  |
|------------------------------------------------------------|---|------------------------------------------------------------|---|---------------------------------------------------------------|--|
| Two of these multiply, standing against the Numbers given. |   | Two of these given to be multiplied, or one to be squared. |   | Two of these add, respecting the given Numbers in the middle. |  |
|                                                            |   |                                                            |   |                                                               |  |
|                                                            |   |                                                            |   |                                                               |  |
|                                                            |   |                                                            |   |                                                               |  |
|                                                            |   |                                                            |   |                                                               |  |
|                                                            | 1 |                                                            | 9 | 40                                                            |  |
|                                                            | 2 |                                                            | 8 | 30                                                            |  |
|                                                            | 3 |                                                            | 7 | 20                                                            |  |
|                                                            | 4 |                                                            | 6 | 10                                                            |  |
|                                                            | 5 |                                                            | 5 | 0                                                             |  |

The large Table is so plain, that it needs no Explanation, as shewing that 8 times 8 is 64, 8 times 9 is 72, and the like of the rest.

And this Table is very near as easy; for you must always look for the Digits to be multiplied, in the middle Column: As suppose I would know what 8 times 6 makes; against 8 in the middle Column is 30 towards the right hand, and against 6 in the middle is 10 in the right-hand Column, the Sum of which is 40; and those standing against the others, in the left-hand Column, are 2 and 4, which multiplied together, make 8: so that 8 times 6 is 48.

And to square any of the middle Numbers, as suppose 7, double 20, which stands against it on one side, and multiply that by itself, which stands against 7 on the left-hand side; so 20 doubled is 40, and 3 squared makes 9: so that 7 times 7 is 49, &c.

So much for the Tables: I shall now proceed to shew how the Operation of Multiplication is performed. And,

2. The Application of the Rules in solving Questions, which may serve, instead of Reduction descending, to shew the Use of Multiplication.

*Proposition*



# SECT. IV. Multiplication of Intire Numbers.

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*Proposition 1.* To multiply a Number by another, which consists but of one place.

*Example 1.*

Multiplicand 79533  
Multiplier ..... 8

*Example 2.*

Multiply — 175945  
by ..... 9

Product 636264

Answer = 1583505

*Prop. 2.* To multiply any Number by another consisting of two, three, or more places. A shorter way for Example 1. See *Multiplication of Decimals.*

*Example 1.*

Multiply 74963  
by ... 75

*Example 2.*

47625  
357

*Example 3.*

76472  
2475

374815

333375

382360

524741

238125

535304

142875

305888

Product = 5622225

Product = 17002125

152944

Product = 189268200

*Prop. 3.* To multiply when one or more Cyphers are in the Multiplier, but not at the end.

*Example 1.*

Multiply.... 57234  
by.... 5203

*Example 2.*

97532  
32004

171702

390128

114468

195064

286170

292596

297788502

3121414128

*Prop. 4.* How to do when one or more Cyphers are next the right hand of one or both of the Factors.

*Example 1.*

9769 } Factors.  
1000 }

*Example 2.*

7399 } Factors.  
400 }

*Example 3.*

34000 } Factors.  
2300 }

9769000 = Product = 2959600 = Product. = 102

68

Product = 78200900

F 2

Rules

*Rules for performing the Operations above.*

*In Example 1.]* Say 8 times 3 (the Units place of the Multiplend) is 24; put the 4 under the Line, and carry the 2 Tens to the next Product, saying 8 times 3 is 24, and 2 carried is 26; put 6 in Tens Place of the Product, and carry 2 as before, saying 8 times 5 is 40, and 2 carried is 42; put the 2 under the Line as you see, and carry 4 (Tens) saying 8 times 9 is 72, and 4 carried is 76; put the 6 under, and carry the 7, saying 8 times 7 is 56, and 7 carried is 63, which (being the Produce of the last Figure) put all down, and you see that 8 times 79533 is 636264. And so much for Directions to multiply and carry the Tens from one particular Product to another; the Reason of which, and of placing the several Products a degree towards the left-hand, I shall shew by and by.

*In Example 2. of Prop. 2.]* You find by the Rules above, that 7 times 47625 is 333375; and then you multiply 47625 by 5 (the next Figure in the multiplier) saying 5 times 5 is 25: place that 5 under Tens place of the former Product, and proceed as before; and when you come to the 3, put 5 of the Product 3 times 5, under the Tens place of the last Line or Product; and so continue, if you had never so many Figures in the Multiplier, as you see in the other Examples.

*In Proposition 3.]* You have Examples, that when one or more Cyphers are in the middle of the Multiplier, you must (in beginning to multiply by the Digit next the Cyphers towards the left-hand thereof) place the first Figure of the Product not under Tens place of the former Product, as before; but put it so many places extraordinary towards the left-hand as there are Cyphers: the Examples thorowly explain the Meaning.

*In Prop. 4.]* It is shewn, that whenever Cyphers are to the right-hand of either or both Factors, you need only to multiply by the significant Figures or Digits, and then place all the Cyphers in one or both the Factors towards the right-hand of that Product; and when 10, 100, 1000, &c. is the Multiplier, you need only to put the Cyphers towards the right-hand of the Multiplicand, and that is your Product, as by the first Example of this Proposition.

*The Reason of the Method in the Process of the Work of Multiplication, by several Digits in the Multiplier.*

I shall give an Instance in the Work of the second Example of the second Proposition foregoing; and shall explain this by three Examples depending on each other, the second of which, supposing the Cyphers left out, is an Abbreviation of the first; and the third Example shews how much the Number of Figures in the second Example is lessened by carrying the Tens from the Product of every two Digits to that of the next, &c. and not putting the whole down, which reduceth 15 Numbers into 3.

| <i>Example 1.</i> | <i>Example 2.</i> | <i>Example 3.</i>       |
|-------------------|-------------------|-------------------------|
| 47625             | 47625             | 47625                   |
| 357               | 357               | 357                     |
| <hr/>             | <hr/>             | <hr/>                   |
| 35                | 35                |                         |
| 140               | 14   0            |                         |
| 4200              | 42   00           |                         |
| 49000             | 49   000          |                         |
| 280000            | 28   0000         |                         |
|                   | <hr/>             | 333375=Sum              |
| 250               | 25   0            |                         |
| 1000              | 10   00           |                         |
| 30000             | 30   000          |                         |
| 350000            | 35   0000         |                         |
| 2000000           | 20   00000        |                         |
|                   | <hr/>             | 238125=Sum              |
| 1500              | 15   00           |                         |
| 6000              | 6   000           |                         |
| 180000            | 18   0000         |                         |
| 2100000           | 21   00000        |                         |
| 12000000          | 12   000000       |                         |
|                   | <hr/>             | 142875=Sum              |
| <hr/>             | <hr/>             | <hr/>                   |
| Sum=17002125      | 17002125          | 17002125                |
| or Product        |                   | Sum Total<br>or Product |

In the first of these Examples you have the whole Work of the Multiplication as the Figures stand, the whole being put down without Carrying or Abridgment; as 7 times 5 is 35, 7 times 20 is 140,



140, 7 times 600 is 4200, &c. Then 50 times 5, 20, 600, 7000, and 40000 make the next 5 Lines: And 300 (the third Figure in the Multiplier) times 5, 20, 600, 7000, and 40000 make the last 5 Lines or Numbers in the Operation.

In the second Example you see how the Work stands when the unnecessary Cyphers are cut off, and thrown out of the Account. And

In the third Example you see that the Sum of the Numbers separated from the Cyphers do respectively make the Numbers, and fall in the same order as in the second Example of *Prop. 2*. So that here you see not only the Reasons why the Tens are carried from one single Product to another, when 1 Line or Number is only made instead of 5: but you also see plainly the Reason why the Units place of every Line stands under Tens place of the Product or Line of Figures preceeding.

Thus much for the Theory of Multiplication, I shall next shew,

## II. The Use and Application of Multiplication.

I. *Of Money.*] Pounds are reduced immediately into Shillings, Pence, or Farthings, by multiplying the given Pounds by 20, 240, or 960, as *per Table 2.* in Addition.

*Example,* in 7873 l. how many Shillings? &c.

|                                     |                |                                                 |                                                  |
|-------------------------------------|----------------|-------------------------------------------------|--------------------------------------------------|
| 7873 Pounds<br>20 Shill. in l. i. } | Multi-<br>ply. | <i>Example 2.</i><br>l. 7873<br>240 d. in l. i. | <i>Example 3.</i><br>7873 l.<br>960 qr. in l. i. |
| <hr/>                               |                | <hr/>                                           | <hr/>                                            |
| <i>8b.</i> 157460 = the Answer.     |                | 31492<br>15746                                  | 47238<br>70857                                   |

Pence for Answer 1889520 Farth. 7558080 Answer.

*Note,* These Shillings may be reduced into the Pence by multiplying them by 12; and these Pence into the Farthings by multiplying by 4. And the like of any other Pounds, Shillings and Pence.

*Quest. 2.* In l. 7873 : 18 : 11½  
how many Farthings?

Multiply 7873 by 960, as in the third Example; to the Product whereof add 911 = the Farthings in 18 s. 11 d. ½ and the Sum is the Answer, as *per Margin.*

|                                    |
|------------------------------------|
| l. 7873 : 18 : 11½                 |
| 960                                |
| <hr/>                              |
| 47238                              |
| 70857                              |
| <hr/>                              |
| 7558080                            |
| Add . . . 911 qr. in 18 s. 11 d. ½ |
| Sum = 7558991 = Answer.            |

2. Of

# SECT. IV. *Multiplication of Intire Numbers.*

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2. *Of Averdupois Weight.*] By the first Table in Addition it appears that Tons are reduced into Hundreds by multiplying them by 20; into Quarters of Hundreds by 80; into Pounds by 2240; into Ounces by 35840, &c.

*Example,* In 85 Ton how many Ounces?

Ounces in 1 Ton = 35840  
Tons given 85

3. *Of Liquid Measures.*] In 157 Ton of Wine, how many Quarts? By the third Table in Addition, you see that in 1 Ton are 1008 Quarts: therefore the Tons being multiplied by 1008, gives the Product 158256 Quarts.

157 Ton. | Answ. Oun. 3046400  
1008 = Quarts in 1 Ton.

1256  
157

158256 Quarts, Answer.

4. *To reduce Time.*] How many Minutes may we say it is since the Creation, supposing the Years, according to Sacred Chronology, to be 5716?

By the eleventh Table in Addition, there are in 1 Year — — } 525960 Minutes } Multiply.  
5716 Years }

315576  
52596

5. *To reduce Square or Superficial Measure.*] How many Diamonds, of 16 in a square Inch, will pave the Globe we live on, supposing it a compleat Sphere or Globe all of Earth?

368172  
262980

3006387360 Minutes for Answer.

The Ambit (or Circumference) of the Earth, according to Mr Norwood's measured Degree of near 70 } — 25040  
*English Miles* in a Degree, is Miles — — }

A fourth of which is Miles — 6260

Which multiplied by the Earth's Diameter = 7967

4382

3756

5634

The Product is the superficial Content of a Circle, } 4382  
whose Circumference is that of the Earth, viz. — } 49873420  
Product

Product brought over — — — 49873420

4

Which multiplied by 4, produceth the convex Area  
of the Earth (or whole superficial Content) in Square } 199493680  
Miles — — — — —

Which multiplied by the square Yards in one square  
Mile, which are (as per Table the 7th) — — — } 3097600

119696208

139645576

179544312

59848104

The Product is the square Yards on the Earth = 617951623168000

Which multiplied by the Inches in a square } ..... 1296  
Yard, which are — — — — —

3707709739008

5561564608512

1235903246336

617951623168

Produceth the square Inches on the } = 800865303625728000  
Earth's Surface — — — — —

Which multiplied by the Diamonds in 1 square Inch, viz. .... 16

4805191821754368

800865303625728

Produceth the Number of Diamonds } 12813844858011648000  
which answers the Question. — — — } 100

And the Value of the Diamonds (as  
afore said) that would pave the Globe  
at 100 l. each Diamond, is Pounds } 1281384485801164800000  
Sterling. — — — — —

This Value is numbred as the 16th in the Numeration Table.

By all which Examples it appears, that in this kind of Reduction,  
which some call Reduction descending, one general Rule is to be  
observed, viz.

Multiply any Number of any Denomination by any Number of  
Units of a smaller or inferior Name, that make a Unit of that given,  
and the Product shews how many of the later are contained in all  
the former.

For



For Proof of Multiplication, see at the end of *Division*.

Note, That many new and very brief Rules and Examples for Multiplication may be seen in *Decimals*.

SECT. V. *Division of Intire Numbers four several Ways.*

**D**IVISION is that Part of Arithmetic by which a Number given is divided, separated, or distributed into any Number of Parts assigned.

Hence 'tis plain, that it is the Reverse of Multiplication; for as that produceth an Increase, so this produceth a proportionable Decrease: And consequently the Truth of any Product of Multiplication, is proved by dividing it by one of the Factors, of which more hereafter.

Division performeth the Work of many Subtractions (as Multiplication does that of many Additions) by a few Figures and in a small Time; which Subtraction would require an incredible deal of Time, Figures and Paper, to effect. To instance in the former Numbers: If it were proposed to find out how many Years are in 175320000 Hours, there being 8766 Hours in 1 Year, therefore to do this by Subtraction, would require that you deduct 8766 so often

8766) 175320000 (20000  
 from 175320000, till nothing remain,  
 which would be after 20000 Deductions. But the Work of Division

shews the Answer with that Brevity which you see in the Margin.

It appears from hence, That in Division there are always 2 Numbers given, and a third sought for. The Numbers given, are, 1<sup>st</sup>, That which is required to be divided, which we call the *Dividend*. 2<sup>dly</sup>, The Number of Shares, Parts, or Portions, into which the said Dividend is assigned to be parted or divided, which we call the *Divisor*. 3<sup>dly</sup>, The Part or Share arising from the Work of Division (being the Number sought for) we call the *Quotient*, (from *quotiens*, i. e. how many times) which shews how often the Divisor is contained in the Dividend.

And if the Dividend contain somewhat more than the Quotient expresseth, (but not so much more as the Divisor amounts to) that Surplusage is called the Remainer or Remainder.

So that the Relation which the two Numbers given and that sought (supposing no Remainer) have to each other, is, That the Dividend contains either the Divisor or Quotient so often as the

other doth a Unit: For as a Unit to the Divisor :: so the Quotient to the Dividend.

Or as a Unit to the Quotient :: so the Divisor to the Dividend; and consequently,

The Product of the Divisor and Quotient is equal to the Dividend by 1; any of which Considerations proves the Truth of the Work of Division.

*The Operation of Division consisteth in these five things.*

1. In considering how many places toward the left hand of the Dividend the Divisor can be taken from: If from the like Number of Places that are in the Divisor, then,

2. To consider how often the first Figure in the Divisor can be had in the first of the Dividend towards the left hand. But if the Divisor cannot be taken from the like Number of Places with itself from the Dividend; then to ask how often the first Figure in the Divisor (as before) can be had in the two first places towards the left hand of the Dividend; in either of which Cases,

3. We put the Answer in the Quotient, (as you see in the Example.)

4. By that Figure so put in the Quotient, we multiply the Divisor, and put the Product under that part marked out in the Dividend.

5. We deduct that Product from the said marked Part or Dividual, and put the Remainder under a Line, as in the Examples.

*Proposition 1.* To divide by any single Figure or Digit.

*Example 1.*

|                |   |   |                 |          |   |                 |
|----------------|---|---|-----------------|----------|---|-----------------|
| <i>Divisor</i> | 7 | ) | <i>Dividend</i> | 87654321 | ( | <i>Quotient</i> |
|                |   |   |                 |          |   | 12522045        |
|                |   |   |                 | .....    |   |                 |
|                |   |   |                 | 7        |   |                 |
|                |   |   |                 | —        |   |                 |
|                |   |   |                 | 17       |   |                 |
|                |   |   |                 | 14       |   |                 |
|                |   |   |                 | —        |   |                 |
|                |   |   |                 | 36       |   |                 |
|                |   |   |                 | 35       |   |                 |
|                |   |   |                 | —        |   |                 |
|                |   |   |                 | 15       |   |                 |
|                |   |   |                 | 14       |   |                 |
|                |   |   |                 | —        |   |                 |
|                |   |   |                 | 14       |   |                 |
|                |   |   |                 | 14       |   |                 |
|                |   |   |                 | —        |   |                 |
|                |   |   |                 | 032      |   |                 |
|                |   |   |                 | 28       |   |                 |
|                |   |   |                 | —        |   |                 |
|                |   |   |                 | 41       |   |                 |
|                |   |   |                 | 35       |   |                 |
|                |   |   |                 | —        |   |                 |
|                |   |   |                 | 6        |   | Remainder       |

*Example*

*Example 2.*

8) 1743219 (217902

.....

16

14

8

63

56

72

72

019

16

3 remains.

*Example 3.*

9) 90123456 (10013717

.....

9

0012

9

33

27

64

63

15

9

66

63

3 remains

*Prop. 2.* To divide by 2, 3, or more Figures in the Divisor.*Example 1.*

21) 7334565 (349265

.....

63

103

84

194

189

55

42

136

126

105

105

0

*Example 2.*

136) 1234567 (9077

.....

1224

1056

952

1047

952

95 refts.

*Example 3.*

9876) 1234567 (125

...

9876

24696

19752

49447

49380

67 refts.

*Prop.*



*Prop. 3.* To divide by a Divisor having Cyphers in the first, second, &c. places thereof.

*Example 1.*

*Example 2.*

*Example 3.*

350)987654(2821 | 12000)987654(820 | 1000)976432 = Answ.

1— 70

1— 96

1— 1—

287

27

Quote = 976

280

24

Remains 432

76

Rests 3654 to divide by 12000

70

65

35

Rests 304

*Rules for performing the Work of the three last Propositions:*

Those by 1 Figure under *Prop. 1.* are so easy, that any one may divide, observing the five general Rules for proceeding. To instance in *Example 2.* I ask how often 8, the Divisor, can be had in 17, the two first Figures to the left hand of the Dividend, (because I cannot have 8 in one place of the Dividend) the Answer is 2 times; which 2 I put in the Quotient, and say 2 times 8 (the Divisor) is 16; which placing under the 17, and taking the 16 from 17 the Remainder is 1. To which bring down the next Figure 4 in the Dividend, (and as you bring down any Figure to a Remainder, mark it with a Point under, lest you bring it down twice) and then say how often 8, the Divisor, can be had in 14, the Answer is 1; which put in the Quotient, and say 1 time 8 is 8, which put under the Deduct from 14, and the Remainder is 6; to which bring down 3, (the next Figure in the Dividend) and ask how often 8 in 63, the Answer is 7, which put in the Quotient; and so go on to the end of the Work, observing this Rule, That if after you have brought any Figure down to a Remainder, you cannot have the Divisor in it; you must put a (0) in the Quotient, and bring down the next Figure, as in the last Figure but one of the Example we are upon.

*In*

*In the Examples to Prop. 2.* It is almost as easy to divide by 2, 3, or 4 Figures as 1, observing chiefly this Rule; as in Example 2. of this Prop. I ask how often 1 (the first Figure in the Divisor) may be had in 12, (the two first Figures in the Dividend) the Answer would be 12 times; but you must never take it above 9, that is, you must never put above 9 in the Quotient at one time. And also observe, that you do not put that 9 in the Quotient till you have tried on a piece of waste Paper whether 9 times 136 do not exceed 1234, (the first part of your Dividend) which finding it less, may consequently be taken from it: So 9 times 136 is 1224, from 1234, and there rests 10; to which bring down the next Figure (5) in the Dividend, and consider how often 136 (the Divisor) can be had in 105, the Answer is (0) which put in the Quotient, and then bring down the next Figure 6 makes 1056. Then say how often 1 (the first of the Divisor) can you have in 10, (the two first of that part of the Dividend, because that has one place more in it than is in the Divisor:) If you should say, 1 may be had 9 times or 8 times in 10, you will find that the Divisor multiplied by either of those Figures will exceed 1056, and so cannot be taken from that Number. Wherefore finding that 7 times 136 will be less than 1056, I put 7 in the Quotient, and multiplying 136 by 7, it produceth 952; which take from 1056, and the Remainder is 104: To which bring down the 7 (the last in the Dividend) makes 1047, and say how often 1 (the first of the Divisor) can be had in 10 (the two first of the said 1047) the Answer is 7 times, (for it will bear no more, without making the Product of the Divisor thereby to exceed the 1047) so 7 times 136, which is 952, deducted from 1047, the Remainder is 95: So that I find I can have 136 in 1234567, 9077 times, and 95 rests. By observing these Rules you'll easily see how to perform the Work of this Example, and by this any other; and I have been particular in giving Rules for that end.

*In the Examples under Prop. 3.* When Cyphers are in the first, second, &c. places of the Divisor, cut off the Cyphers, and as many Figures towards the right hand of the Dividend, and divide those remaining towards the left hand of the Dividend, by those that remain towards the same hand in the Divisor: and when the Work is ended, put the Figures cut off the Dividend to the right hand of the Remainder.

*And for the third Example, to divide any Number by 10, 100, 1000 10000, &c.* You have nothing to do but only cut from the right hand of the Dividend so many places as there are Cyphers at the end of the Divisor: so those remaining towards the left hand of the Dividend,

Dividend are the Quotient, and those cut off towards the right hand are the Remainder.

But there is

*A Second Way of Division:*

Where by omitting to put down the several Products, and deducting gradually as you multiply each Digit, you do it with near half the Figures.

Thus the third Example under *Prop. 2.* 9876)1234567 (125  
is done as in the Margin. For I say how often 9 in 12, the Answer is 1; then instead of saying once 9876 is so much, and putting it under 12345, I say 1 time 6 from 15, (where 10 is added to the 5 in Units place of 12345) and there rests 9, which put under, and saying 1 time 7 and 1 borrowed is 8 from 14 rests 6; 1 time 8 (in the Divisor) is 8, and 1 borrowed is 9, from 13 (borrowing 10 as before) and there rests 4, which put under, and say 1 time 9 (in the Divisor) is 9, and 1 borrowed is 10 from 14, and there rests 2. Then to the Remainder 2469 bring the next Figure in the Dividend, viz. 6, and say how often 9 in 24, the Answer is 2; then proceed as before, 2 times 6 is 12 from 16 rests 4, 2 times 7 is 14, and 1 is 15, from 19, rests 4; 2 times 8 is 16, and 1 borrowed is 17, from 26, and there rests 9, (you borrowing 2 to add to the 6 is 26) 2 times 9 is 18, and 2 borrowed is 20, from 24, leaves 4, or 4944; to which bring down the 7 in the Dividend, and proceed as before.

I desire this Method last mention'd may be well understood, for that the subsequent Examples will be performed by it, as being the shortest and easiest Way, tho' some prefer this

*Third Way of Division.*

Divide 1234567 by 136.

In this kind of dividing, the Divisor is put under so much of the the Dividend as it can be taken from; for 136 can't be taken from 123, therefore I put it under 1234, the first part of the Dividend, and then ask (as in the second Example of *Prop. 2*) how often 1 in the Divisor can be had in 12 in the Dividend, the Answer is 9 times; which 9 put in the Quotient as usual,

$$\begin{array}{r}
 109 \\
 10045 \\
 1234567 \quad 9077 \\
 13666 \\
 133 \\
 1
 \end{array}$$



usual, and say 9 times 6 in the Divisor is 54, from 54, (borrowing 5 to add to the 4 standing over the 6, and as you mention 6 and 4, dash them out with your Pen) there rests 0; 9 times 3 in the Divisor is 27, and 5 borrowed is 32, from 33 (borrowing 3 Tens to add to the 3 standing over the 3 in the Divisor) and there rests 1, which put over the 3, as you see. Then remove your Divisor a degree towards the right hand, as in the Example, and consider how often 136 you can have in 105, (which is 10 that remained, and 5 next the 4 in the Dividend) the Answer is 0, which put in the Quotient, and taking the next Figure, *viz.* 6, into the 105, makes 1056; say how often 136 in 1056, or 1 in 10, the Answer is but 7 times; then 7 times 6 is 42, from 46, and there remains 4, which put over the 6, (dashing it out, and also the 6 in the Divisor) saying 7 times 3 is 21, and 4 borrowed is 25, from 25, (borrowing two Tens) and there rests 0, which put down over the 5, saying 7 times 1 is 7, and 2 is 9, from 10 leaves 1, which put over the Cypher as you see, dashing out the Figures in the Divisor as you multiply them, and of the Dividend as you deduct therefrom. Then remove the Divisor, and proceed as before, and as you see in the Example.

But there are two things which render this way of Division inferior in Estimation to the second, *i. e.* the repeating the Divisor for every Figure put in the Quotient; and also the cancelling the Figures, makes it very difficult to examine your Work in case of a Mistake.

*The Illustration and Rationale of the Work of Division.*

I shall instance in admitting that 9876543210 were to be divided by 45678.

*A Fourth*

*A Fourth Way of Division; or, The Operation of Division illustrated.*

| <i>Products of the Divisor.</i> | <i>Divisor repeated</i> |                                       |
|---------------------------------|-------------------------|---------------------------------------|
| by 1 = 45678                    | 45678                   | (9876543210 (200000 = first Quotient. |
| 2 = 91356                       |                         | 9135600000                            |
| 3 = 137034                      | 45678                   | 740943210 — 10000 = second Quote.     |
| 4 = 182712                      |                         | 456780000                             |
| 5 = 228390                      | 45678                   | 284163210 — 6000 = third Quote.       |
| 6 = 274068                      |                         | 274068000                             |
| 7 = 319746                      | 45678                   | 10095210 — 200 = fourth Quote.        |
| 8 = 365424                      |                         | 9135600                               |
| 9 = 411102                      | 45678                   | 959610 — 20 = fifth Quote.            |
| <i>The Tarif.</i>               |                         | 913560                                |
|                                 |                         | 46050 — 1 = sixth Quote.              |
|                                 |                         | 45678                                 |
|                                 |                         | 216221 = Sum or General Quotient.     |
|                                 |                         | resteth 372                           |

In the beginning of this Part or Section, Division is said to be the Work of many Subtractions, and so it is plain: But then we are taught here how to go a nearer way to work than to deduct the Divisor singly; for in the first Operation above, we deduct 200000 times the Divisor at one time, (which are all the 100000's of the Divisor that are contained in the Dividend.) At the second Working we deduct 10000 times the Divisor, (which is all the 10000's of the Divisor that is in the Dividend.) The third time we take 6000 times the Divisor from the Dividend, then 200 times, then 20 times, and then 1 time the Divisor from the Dividend: So that by this Art of Division you deduct at 6 times what by Subtraction would require 216221 times to perform.

Now to know how many times the Divisor to deduct the first time, I consider what part or places of the Dividend I can take the Divisor from, and find it the five first towards the left hand; therefore I mark that, by putting a Point under it, and I consider how often

often the Divisor can be had in those five places, or how often 4 (the first of one) in 9 (the first of the other) and find it 2 times. Now to know what Denomination to give this (2) I consider what place that Figure which I made the Point under possieth, which being the Hundred thousands, therefore this 2 is 200000: so that multiplying the Divisor by 200000, and deducting the Product from the Dividend, the Remainder is 740943210, which is an absolute new Dividend, to be divided by 45678 the Divisor. So having repeated the Work by the same Rules, at last a Number less than the Divisor remains, so the Work is ended.

And thus by dividing the whole into 6 distinct Dividends, there ariseth 6 Quotients; the Sum of which is the general or true Quotient.

But because Brevity is most commendable in this Art, therefore all superfluous Figures being omitted, as the repeating of the Divisor, the Cyphers at the ends of the Quotients and Subtrahends, the Work is then the same as in the two first Propositions: And if the whole Subtrahends be omitted to be put down, deducting as you gradually multiply the Divisor by the Figure put in the Quotient, the Work is then contracted as much as may be, and will stand as in the Example under the second Way of Division.

The Tarif shews you by Inspection how often the Divisor can be had in each Dividend (without trial or guessing) and the Digits towards the left hand shew what must be put in the Quotient.

### *The Use of Division.*

Farthings, Pence, or Shillings, are immediately reduced into Pounds, by dividing the given Number by 960, 240, or by 20, (as by the second Table in Addition.)

In 157460 Shillings how many Pounds? Cut off Units place, and take half the remaining Figures toward the left hand; and where the Number to be halved is odd, take the less half, and put 10 to the next Figure, &c. Thus  $\frac{1}{2}$  of 157460 is  $l. 7873$ .

**In 1889520 Pence how many Pounds?**

In 7558080 Farthings how many Pounds?

240) 1889520 (7873 Answer.

$$\begin{array}{r} \underline{209} \\ 175 \\ \underline{072} \\ 0 \end{array}$$

960) 7558080 (7873 Answer.

$$\begin{array}{r} \phantom{00} \overline{) 7558080} \\ \underline{838} \phantom{00} \\ 700 \phantom{00} \\ \underline{288} \phantom{00} \\ 0 \end{array}$$

H

*Note.*



*Note*, Farthings are reduced into Pence by dividing by 4, and Pence into Shillings by dividing by 12.

In 7558991 Farthings, how many Pounds, Shillings, Pence, and Farthings?

4) 7558991 (1889747 (157478 (7873 : 18 : 11  $\frac{1}{4}$  Answer.

$$\begin{array}{r}
 \begin{array}{r}
 \dots\dots\dots 12) \dots\dots\dots 20) \\
 \hline
 35 \\
 \hline
 35 \\
 \hline
 38 \\
 \hline
 29 \\
 \hline
 19 \\
 \hline
 31
 \end{array}
 \quad
 \begin{array}{r}
 \hline
 68 \\
 \hline
 89 \\
 \hline
 57 \\
 \hline
 94 \\
 \hline
 107 \\
 \hline
 11 \text{ Pence rest.}
 \end{array}
 \end{array}$$

Shillings are made Pounds by the above Rule of halving.

3 Farthings rest.

*Example 5.* In 3046400 Ounces, how many Ton?

Ounces in a Ton }  
 per Table 1. in } 35840) 3046400 (85 = Ton Answer.  
 Addition, }  $\begin{array}{r} 1 \\ \hline 17920 \end{array}$

*Example 6.* In 158256 Quarts, how many Ton of Wine?  
 Quarts in a Ton }  
 per Table 3. } 1008) 158256 (157 = Tons of Wine, Answer.

$$\begin{array}{r}
 \hline
 5745 \\
 \hline
 7056 \\
 \hline
 0
 \end{array}$$

These six Examples are the Reverse, and prove the Truth of the six first in the Use of Multiplication.

And as a seventh Example, I shall give the Use of Multiplication and Division, in shewing how to find all the aliquot Parts into which any Number is capable of being divided: and the Use thereof.

*Example*

*Example 1.]* To find how many even Parts into which 360 is divisible. See the whole Operation.

|        |                                    |                   |
|--------|------------------------------------|-------------------|
| 2) 360 | The Divisors and<br>last Quote } = | 2, 2, 2, 3, 3, 5, |
| 2) 180 |                                    | — 4, 8,           |
| 2) 90  |                                    | 6, 12, 24, 9,     |
| 3) 45  |                                    | 18, 36, 72,       |
| 3) 15  |                                    | —                 |
| 5      |                                    | 10, 20, 40, 15,   |
|        |                                    | 30, 60, 120, 45,  |
|        |                                    | 90, 180, 360.     |

1. I divide the given Number by 2, 3, 5, 7, or other that will divide without a Remainer, and then place the Divisors and last Quote as in the Example.

2. I multiply the 2 next the left hand by the next 2, which produceth 4, placed under the 2, and that Product by the first 2 gives 8.

3. I multiply the 3d (2) the 4, 8, and 2d (3) by the 1st 3, which produceth 6, 12, 24, and 9 placed under the respective Multiplcands, (as I do all the rest following.)

4. In like manner I multiply 6, 12, and 24, by the 3, (because that Digit is repeated, otherwise I should have multiplied the 2, 2, 2, 3, and the 4, 8, also by it) and the Products are 18, 36, and 72.

5. I multiply all that is before by 5, (except where the Products would be the same) as 2, 4, 8, and 2d (3) in the first and second Lines, which produceth 10, 20, 40, 15, in the fifth Line. Also by the same 5 (next the right hand) I multiply 6, 12, 24, and 9, which produceth 30, 60, 120, and 45 in the sixth Line; and 18, 36, and 72 in the fourth Line (of the Numbers above) by the same 5 produceth 90, 180, and 360.

And if there were any more different Digits in the first Line towards the right hand, I should multiply all the above 7 Lines thereby (where the Products would not be the same, for I omit repeating one and the same Product). But because the last Number is 360, and cannot therefore be properly said to be a part of the Number given, I therefore omit that, and put 1 always instead of the Number given.

And tho' you change the places in the Figure in the first Line, the Answer will be true; as in the subsequent Example will appear, as

by the above Rules the aliquot Parts in the last Example of 360 are, found 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, and 180.

*Example 2.*] To find all the aliquot Parts into which 10350 may be divided.

10350 divided by 2 = 5175; that by 3 = 1725; that by 3 = 575; that by 5 = 115; that by 5 = 23.

Which Divisors  
and the last  
Quote are }

3, 3, 2, 5, 5, 23 (the 2 being transposed)

|                        |                                                           |
|------------------------|-----------------------------------------------------------|
| 9, . . . . .           | = 3 by the 3.                                             |
| 6, 18, . . . . .       | = 3, and 9 by the 2 in the upper Line.                    |
| 15, 45, 10, . . .      | = 3, 9 and 2 by 5 in the upper Line.                      |
| 30, 90, . . . . .      | = 6 and 18 by 5.                                          |
| 75, 225, 50, 25,       | = also the 15, 45, and 10 by said 5; it being repeated.   |
| 150, 450,              | = 30 and 90 by 5, for the same reason.                    |
| 69, —, 46, 115,        | = 3, 2 and 5 in 23, the first in the upper Line.          |
| 207,                   | = 9 in 2d Line by said 23.                                |
| 138, 414,              | = 6 and 18 in the 3d Line by said 23.                     |
| 345, 1035, 230,        | = 15, 45, and 10 in the 4th Line by 23.                   |
| 690, 2070,             | = 30 and 90 in the 5th Line by the 23.                    |
| 1725, 5175, 1150, 575, | = the 6th line by 23, the first Number in the first Line. |
| 3450, 10350.           | = 150 and 450 in the 7th Line by said 23.                 |

So the even Parts of 10350 are 1, 2, 3, 5, 6, 9, 10, 15, 18, 23, 25, 30, 45, 46, 50, 69, 75, 90, 115, 138, 150, 207, 225, 230, 345, 414, 450, 575, 690, 1035, 1150, 1725, 2070, 3450, and 5175.

The



*The Use of the Rules for finding the Aliquot Parts of a Number.*

This will appear thus : Suppose I would find all the aliquot Parts of a Pound Sterling, I reduce it to its least Denomination, as 960 Farthings ; the aliquot Parts of which are found 2, 3, 4, 5, 6, 8, 10, 12, 15, 16, 20, 24, 30, 32, 40, 48, 60, 64, 80, 96, 120, 160, 192, 240, 320, 480, and 960.

Which Numbers being made Denominators, and 1 the Numerator, there will arise the Parts of Coin following ; which being supposed the Price of any Integer,

|     |     |      |      |      |      |     |    |
|-----|-----|------|------|------|------|-----|----|
| 2,  | 2,  | 2,   | 2,   | 2,   | 2,   | 3,  | 5. |
| —   | 4   | 8    | 16   | 32   | 64   |     |    |
| 6,  | 12, | 24,  | 48,  | 96,  | 192. |     |    |
| 10, | 20, | 40,  | 80,  | 160, | 320, | 15, |    |
| 30, | 60, | 120, | 240, | 480, | 960. |     |    |

the Value of any Number of them is found by dividing the Number of Integers by the respective Denominators at once.

| Parts of a Pound. | s. | d. | q. | Parts of a Pound. | s. | d. | q. | Parts of a Pound. | s. | d. | q. | Parts of a Pound. | d. | q. |
|-------------------|----|----|----|-------------------|----|----|----|-------------------|----|----|----|-------------------|----|----|
| 10                | :  | 0  | :  | 1                 | :  | 8  | :  | 0                 | 0  | :  | 6  | :                 | 0  | :  |
| 6                 | :  | 8  | :  | 1                 | :  | 4  | :  | 0                 | 0  | :  | 5  | :                 | 0  | :  |
| 5                 | :  | 0  | :  | 1                 | :  | 3  | :  | 0                 | 0  | :  | 4  | :                 | 0  | :  |
| 4                 | :  | 0  | :  | 1                 | :  | 0  | :  | 0                 | 0  | :  | 3  | :                 | 3  | :  |
| 3                 | :  | 4  | :  | 0                 | :  | 10 | :  | 0                 | 0  | :  | 3  | :                 | 0  | :  |
| 2                 | :  | 6  | :  | 0                 | :  | 8  | :  | 0                 | 0  | :  | 2  | :                 | 2  | :  |
| 2                 | :  | 0  | :  | 0                 | :  | 7  | :  | 2                 | 0  | :  | 2  | :                 | 0  | :  |

*Example 1.* What doth 7358 amount to, at 1 d. 1 q. each?

192) 7358 (38 l. 6 s. 5 d.  $\frac{1}{4}$  = Answer.

1598

62 rest so many 5 Farth. or 6 s. 5 d.  $\frac{1}{4}$ .

*Example 2.* What doth 31987 come to at 3 d. 3 q. each?

64) 31987 (499 l. 15 s. 11 d.  $\frac{1}{4}$  = Answer.

...

638

627

rests 51 so many 3 d. 3 q. or 15 s. 11 d.  $\frac{1}{4}$ .

*Note,* 1 d. 1 q. in the first is  $\frac{1}{192}$  ; and 3 d. 3 q. in the second Example, is  $\frac{1}{64}$  of a Pound.

Also by the same Rule, what follows are the even Parts of a Ton in Averdupois greater Weight.

| Parts of a Ton. | C. | qr. | lb. | Parts of a Ton. | C. | qr. | lb. | Parts of a Ton. | C. | qr. | lb. | Parts of a Ton. | C. | qr. | lb. |
|-----------------|----|-----|-----|-----------------|----|-----|-----|-----------------|----|-----|-----|-----------------|----|-----|-----|
| 10              | :  | 0   | :   | 1               | :  | 1   | :   | 0               | 0  | :   | 1   | :               | 7  | :   | 0   |
| 5               | :  | 0   | :   | 1               | :  | 0   | :   | 0               | 0  | :   | 1   | :               | 4  | :   | 0   |
| 4               | :  | 0   | :   | 0               | :  | 2   | :   | 24              | 0  | :   | 1   | :               | 0  | :   | 0   |
| 2               | :  | 3   | :   | 0               | :  | 2   | :   | 14              | 0  | :   | 0   | :               | 20 | :   | 0   |
| 2               | :  | 2   | :   | 0               | :  | 2   | :   | 8               | 0  | :   | 0   | :               | 16 | :   | 0   |
| 2               | :  | 0   | :   | 0               | :  | 2   | :   | 0               | 0  | :   | 0   | :               | 14 | :   | 0   |
| 1               | :  | 1   | :   | 20              | :  | 1   | :   | 12              | 0  | :   | 0   | :               | 10 | :   | 0   |

The

*The Proof of Multiplication.*

This can only be done by Division: as in any of the 6 Examples foregoing, the Products of those in the Use of Multiplication are proved to be true by dividing those Products by the Multiplier, the Quotient is the Multiplicand; or if you divide the Product by either of the Factors, the Quotient will be the other. But to pretend to prove Multiplication by casting out the Nines, is a Mistake, as I have elsewhere demonstrated; for why divide by 9 more than 2, or any other Digit, which would prove the Work as well? But the easiest way is to divide the Factors by 10, and the Product of the Remainders by 10, which will leave a Remainder equal to that of the Product divided by 10. But the mischief is, that if there be a Mistake in the Product of just your Divisor, or any Power thereof, this Way of proving will not shew it.

*The Proof of Division.*

This is either performed by Multiplication or Division, as appears in the Margin. For Example, I have divided 17002125 by 357, and find the Quotient 47625. And if 17002125 be divided by that Quotient, the later Quotient will be 357 = the former Divisor.

|               |                  |                                          |
|---------------|------------------|------------------------------------------|
| 357) 17002125 | (47625) 17002125 | (357 = Quotient<br>or former<br>Divisor. |
| .....         | ...              |                                          |
| 2722          | 271462           |                                          |
| 2231          | 333375           |                                          |
| 892           | 0                |                                          |
| 1785          |                  |                                          |
| 0             |                  |                                          |

This is proved by Multiplication of the Quotient 47625 by 357 the Divisor. See *Example 2. Prop. 2. of Sect. 4.*

SECT. VI. *Of Extracting the Roots of Numbers, called Evolution.*

**I**N this Section I shall shew,

I. The Extraction of the Square Root.

II. The Extraction of the Cube Root.

III. The Extraction of the Biquadrate Roots of Numbers.

I. The

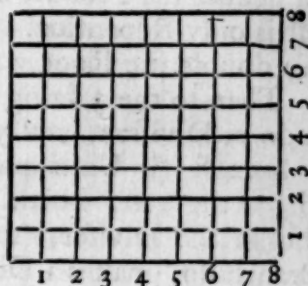
# SECT. VI. *Of Extracting the Roots of Numbers.* 55

I. The Square Root of a Number is such a one, as being multiplied in itself, produceth the Number given (for to square any Number, is to multiply it by itself) Thus the square Numbers, whose Roots are the 9 Digits, are as follows in the Margin.

This may be illustrated by any square Superficies, as Glass, Board, &c. for if the superficial Content of a Square thereof (whose 4 Sides are equal) be 81, then one of the 4 Sides is 9; if the square Superficies be 64, then 1 of the 4 equal Sides is 8, &c. as appears by the Table. And the square Figure whose Surface is 64 square Inches, Feet, or other thing; each Side is therefore 8, which is called the Root of that Superficies, as you see. So that when the square Root of any Number is demanded, it is as much as to require what the Side of a Geometrical Square is, whose Area (or superficial, or outside Content) is any Number given. Hence it follows, that by extracting the Square

| Squares. | Their<br>Roots. |
|----------|-----------------|
| 81       | 9               |
| 64       | 8               |
| 49       | 7               |
| 36       | 6               |
| 25       | 5               |
| 16       | 4               |
| 9        | 3               |
| 4        | 2               |
| 1        | 1               |

Root of any Superficies, you reduce it to a compleat (or Geometrical) Square. Thus the long Quadratick Figure being 16 in length, and 4 in breadth, the Superficies is upon the flat 64, (or 4 times 16) and the square Root of 64 by the Table above is 8. Therefore a Geometrical Square, one of whose four equal Sides is 8, (as that above) is equal to the long Square, or any other Figure whose flat Surface is 64.



Such Numbers as the above, when the Root may be extracted without



a Remainder, may be called compleat or perfect square Numbers; but there are abundantly more Numbers, whose Roots cannot be precisely extracted: and these are called imperfect Squares, or Surd Numbers, of which more in Decimals, Logarithms, and especially in Algebra, hereafter treated on.

*Question*



*Question 1.* What is the Square Root of 10274589?

Place the Number so that you may conveniently perform the Work, and point over the first and every second afterward; which Points shew how many places the Roots will consist of.

$$\begin{array}{r}
 \text{Root} \\
 10 \overline{) 27458916} \quad (32054 \\
 \underline{6) 127} \\
 640 \overline{) 34589} \\
 \underline{6410) 256416} \\
 \text{o remains.}
 \end{array}$$

Then consider what square Number in the foregoing little Table is next to and can be taken from the first branch towards the left hand of the square Number given, (as here 10;) 9 I find is the next, whose Root (3) I put down (like a Quotient) as you see, and subtracting the Square of 3 (or 9) from 10, there remains 1.

To which Remainder 1 I bring down the two next Figures (27) and divide that 127 (except the Figure next the right hand) by 6, which is double the Root (3) saying the 6's in 12 is 2; put that in the Root, makes it 32. Then square 2, and deduct from 7, (in the 127) and the rest is 3; then multiply 6, the double Root, by 2, and deduct the Product from 12 (in the 127) and there rests 0 the rest is only Repetition.

Then double the Root 32 makes 64, and that is your next Divisor. Then to the 3 bring down the next Branch 45, marking it as you do in Division, and you have 345 for a Dividend.

But because (if Units place, here 5, be excluded) you cannot have 64 in the rest, which is 34; therefore put (0) in the Root, and also in the Divisor, (to 64, as you see) and bring down the next Branch 89, makes a Dividend 34589 to divide by 640. And finding I can have 6 in 34, 5 times, I put 5 in the Root; and squaring 5 (as was taught for the other Figures of the Root) I say 25 from 29, (of the 34589) and there rests 4, and carry 2; 5 times 0 is 0, but 2 from 8 rests 6, and so 5 times 4, proceed as in Division, and the whole Remainder is 2564.

*Lastly,* To this bring down the last Branch (16) and divide the 256416 by double the Root 3205, viz. 6410, and the Quote being 4, I put it in the Root, and subtracting the square thereof from 16, (in the 256416) and also the Product of the Divisor thereby, as before shewn, there remains (0) and 32054 is the Answer.

*A second*

*A second Example.*

|                                   | Square Root. |
|-----------------------------------|--------------|
| 1   1473   25   96   43   (107113 |              |
| 20) 1473                          |              |
| 214) 2425                         |              |
| 2142) 28496                       |              |
| 21422) 707543                     |              |
|                                   | 64874 refts. |

*A third Example.*

|                                    | Square Root. |
|------------------------------------|--------------|
| 97   65   43   21   0123   (988202 |              |
| 18) 1665                           |              |
| 196) 16143                         |              |
| 1976) 39921                        |              |
| 197640) 3970123                    |              |
|                                    | 17319 refts. |

 II. *The Extraction of the Cube Root.*

The Cube Root of any Number is that whose Square multiplied by the Root, produceth the Cube Number given. Thus the Cube Numbers, whose Roots are the 9 Digits, are as follows.

A Cube is a solid Body bounded by six Geometrical square Superficies's, as a Dye, whose Length, Breadth, and Depth, (or Thickness) are all equal: And to extract the Cube Root of any Number, is to suppose that Number the Content of a Cube in Feet, Inches, &c. given, to find the Side of one of the six Squares that bounds it. So that as the Extraction of the Square Root is chiefly used in measuring and proportioning of Surfaces; so is the Cube Root in doing that of Solids. And as there are Surd Numbers, whose Square Roots cannot be extracted without a Remainder, so it is in Cube, and other Roots.

I shall give but one Example, but 'tis so plainly demonstrated, as may be sufficient to enable any one to extract the Root of any whole Number: And where there are Remainders, I shall shew how to proceed, when I shew the Use of Decimals in extracting the Cube Root.

What is the Cube Root of 32934168093464?

The whole Operation follows, with the Name of each Line or how it ariseth.

| Cube Numbers. | Squares. | Roots of the Squares and Cubes. |
|---------------|----------|---------------------------------|
| 1             | 1        | 1                               |
| 8             | 4        | 2                               |
| 27            | 9        | 3                               |
| 64            | 16       | 4                               |
| 125           | 25       | 5                               |
| 216           | 36       | 6                               |
| 343           | 49       | 7                               |
| 512           | 64       | 8                               |
| 729           | 81       | 9                               |

|            |             |     |     |     |                                             |
|------------|-------------|-----|-----|-----|---------------------------------------------|
| 32         | 934         | 168 | 093 | 464 | (32054 = Cube Root sought.                  |
| 27         |             |     |     |     | The Cube of 3 (1st in the Root) deduct.     |
| 279)       | 5934        |     |     |     | The first Resolvend or Dividend.            |
|            | 9           |     |     |     | The treble Root (3).                        |
|            | 27          |     |     |     | Treble the Square of that 3.                |
|            | 279         |     |     |     | Sum, or the first Divisor.                  |
|            | 8           |     |     |     | Cube of 2, the second put in the Root.      |
|            | 36          |     |     |     | Squ. of that 2, multip. in last treb. Root. |
|            | 54          |     |     |     | The treble Square of the Root in 2.         |
|            | 5768        |     |     |     | The Sum of the 3 last Lines = a Subtrah.    |
| 30816)     | 166168      |     |     |     | { The 2d Resolvend, being 166, the Rem.     |
|            | 96          |     |     |     | and 168 carried down to that.               |
|            | 3072        |     |     |     | Treble the Root 32.                         |
|            | 30816       |     |     |     | Treble the Square of that Root.             |
|            |             |     |     |     | Sum, or the 2d Divisor.                     |
| 3072960)   | 166168093   |     |     |     | The 3d Resolvend.                           |
|            | 960         |     |     |     | Treble the Root 320.                        |
|            | 307200      |     |     |     | Treble the Square of that Root.             |
|            | 3072960     |     |     |     | Sum, (or the 3d Divisor)                    |
|            | 125         |     |     |     | Cube of 5 (last put in the Root.)           |
|            | 24000       |     |     |     | Square of that 5, in the last treble Root.  |
|            | 1536000     |     |     |     | Last treble Square of the Root in that 5.   |
|            | 153840125   |     |     |     | The Sum, or a Subtrahend.                   |
| 308170365) | 12327968464 |     |     |     | The 4th Resolvend.                          |
|            | 9615        |     |     |     | Treble the Root 3205.                       |
|            | 30816075    |     |     |     | Treble the Square of that Root.             |
|            | 308170365   |     |     |     | Sum, (or the 4th Divisor.)                  |
|            | 64          |     |     |     | The Cube of 4, last put in the Root.        |
|            | 153840      |     |     |     | Square of that 4 in the last treble Root.   |
|            | 123264300   |     |     |     | Last Treble Square of the Root in that 4.   |
|            | 12327968464 |     |     |     | { The Sum (or Subtrahend) which de-         |
|            | (0) rests.  |     |     |     | ducted from the last Resolvend,             |

The Steps in the Performance of the Work of Extraction of the Cube Root are very evident in this last Example : For,

I. You



SECT. VI. *Of Extracting the Roots of Numbers.* 59

1. You point over the Figure in Units place, and then over every third Figure, which divides the whole Cube Number into 5 Parts; which shews that the Root will consist of 5 places.

2. I find that 3 being cubed, produceth 27, (as appears by the little Table foregoing) which is the next less Cube Number to 32, (the first part towards the left hand) I therefore put the Root 3, in the Cube Root sought as you see, and deducting the 27 from 32, the Remainder is 5.

3. To the Remainder we always bring down the next part, (as here 934) and that makes the Resolvend, which is always the Dividend, in order to find the next Figure in the Root.

4. And to find the Divisor whereby to divide that Dividend, you may observe that it is always composed of the treble Root, *i. e.* 9, and of the treble Square of the Root, *viz.* 27.

5. I find I can have 279 the Sum, in 593 (for the 4 in Units place of the Resolvend 5934, you are in this case to take no notice of) 2 times, which put in the Root makes it 32.

6. The next thing is to frame your Subtrahend, which is always composed, 1<sup>st</sup>, Of the Cube of the Figure last put in the Root: 2<sup>dly</sup>, The Square of that Figure multiplied in the last treble Root: And, 3<sup>dly</sup>, The last treble Square of the Root multiplied in the said Figure last put in the Root: The Sum of which three Numbers, as the Example plainly shews, is the Subtrahend.

7. The Subtrahend must always be deducted from the last Resolvend, as here 5768 from 5934, and the Remainder is 166; to which bring down (as before) the next three Figures (168) and you have a new Dividend (or Resolvend): all the rest is Repetition of the same Method of working; except here, that you cannot have 30816 in 16616, therefore you put a (0) in the Root, and from that 320 (the Root) you make a new Divisor, which is 3072960; and for the Dividend, you bring down the 3 next Figures (093) to the last Resolvend, and that makes your Dividend 166168093, &c. which is all very obvious in the Example.

III. *The Extraction of the Biquadrate Root.*

The Biquadrate Root is the Root of the 4<sup>th</sup> Power, as the Cube is that of the 3<sup>d</sup>, and the Square Root that of the 2<sup>d</sup> Power, according to the following Table.

Whence it appears, that the second Power of any Number is the Square of it, or Product of any Number by itself; the Cube

or 3d Power is produced by multiplying the Square of any Number by its Root; the Biquadrate or 4th Power is discovered by multiplying the Cube by the Root, as appears in these nine Examples.

But, of the Powers of Numbers to the 10th inclusive, and the proper Name of each, you have a full Account in the Extraction of Roots in Algebra; where the Reason of that abstruse Method of extracting the Square, Cube, Biquadrate, Surfolid, &c. Roots, is fully explained from the Algebraical Canons for each, both by Numbers and Symbols.

A Table of Powers and their Roots.

| The Biquadrates or 4th Powers. | The Cubes or 3d Powers. | The Squares or 2d Powers. | The Root or first Power, viz. of the 2d, 3d, and 4th Powers. |
|--------------------------------|-------------------------|---------------------------|--------------------------------------------------------------|
| 1                              | 1                       | 1                         | 1                                                            |
| 16                             | 8                       | 4                         | 2                                                            |
| 81                             | 27                      | 9                         | 3                                                            |
| 256                            | 64                      | 16                        | 4                                                            |
| 625                            | 125                     | 25                        | 5                                                            |
| 1296                           | 216                     | 36                        | 6                                                            |
| 2401                           | 343                     | 49                        | 7                                                            |
| 4096                           | 512                     | 64                        | 8                                                            |
| 6561                           | 729                     | 81                        | 9                                                            |

What is the Biquadrate Root of 2998219536?

The Example, with Rules how each Line is produced, follows.

1. You see that having pointed over Units place, and every 4th place afterwards, the Root will consist of 3 places, as there are 3 points.

2. You must consider what 4th Power in the little Table above is next to and less than the first Part or Branch 29: you see 16 is, whose Root you have there (2); then subtracting the Biquadrate 16 from 29, there rests 13.

3. To that 13 bring down the next branch or Part 9821, and you have 139821 for a Dividend, of which the Units place must not be considered, in asking how often the Divisor can be had therein.

4. To find the Divisor, you see it is composed, 1st, Of 4 times the Figures then in the Root (as here 2). 2dly, Of 6 times the Square of that: And, 3dly, 4 times the Cube thereof: the Sum of which being a Divisor,

5. There ariseth from the Division, 3 in the Root.

6. You must find a Subtrahend, by adding together as the Directions against the 4 Numbers 81, 216, 216, and 96, do express; and subtracting the Sum from the last Resolvend, there rests 19980: to which bringing down 9536, you have 199809536. The rest is only Repetition of the three last Steps, as to Method.

|                                                               |                                                                                                           |
|---------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------|
| $\begin{array}{r} 29 \overline{) 98219536} \\ 16 \end{array}$ | $(234 = \text{the Biquadrate Root sought.})$<br>The Biquadrate of 2, 1st put in the Root.                 |
| $3448 \overline{) 139821}$                                    | $\{ 13 \text{ (the Remainer) and } 9821 \text{ brought down to it, being the 1st Resolvend or Dividend.}$ |
| $\begin{array}{r} 8 \end{array}$                              | $4 \text{ times the } 2 \text{ put first in the Root.}$                                                   |
| $\begin{array}{r} 24 \end{array}$                             | $6 \text{ times the Square of that } 2.$                                                                  |
| $\begin{array}{r} 32 \end{array}$                             | $4 \text{ times the Cube of that } 2.$                                                                    |
| $\begin{array}{r} 3448 \end{array}$                           | $\text{The Sum of these } 3, \text{ a Divisor.}$                                                          |
| $\begin{array}{r} 81 \end{array}$                             | $\text{The Biquadrate of } 3, \text{ the last in the Root.}$                                              |
| $\begin{array}{r} 216 \end{array}$                            | $4 \text{ times the } 2 \text{ in the Cube of the } 3, \text{ in the Root.}$                              |
| $\begin{array}{r} 216 \end{array}$                            | $6 \text{ times the Square of } 2, \text{ in the Square of } 3.$                                          |
| $\begin{array}{r} 96 \end{array}$                             | $4 \text{ times the Cube of } 2, \text{ in } 3.$                                                          |
| $\begin{array}{r} 119841 \end{array}$                         | $\{ \text{The Sum, or a Subtrahend to take from the last Resolvend above.}$                               |
| $4898632 \overline{) 199809536}$                              | $\text{A 2d Resolvend, or Dividend.}$                                                                     |
| $\begin{array}{r} 92 \end{array}$                             | $4 \text{ times the } 23 \text{ in the Root.}$                                                            |
| $\begin{array}{r} 3174 \end{array}$                           | $6 \text{ times the Square of } 23.$                                                                      |
| $\begin{array}{r} 48668 \end{array}$                          | $4 \text{ times the Cube of that } 23.$                                                                   |
| $\begin{array}{r} 4898632 \end{array}$                        | $\text{The Sum of the } 3 \text{ Lines, a 2d Divisor.}$                                                   |
| $\begin{array}{r} 256 \end{array}$                            | $\text{The Biquadrate of } 4, \text{ the last in the Root.}$                                              |
| $\begin{array}{r} 5888 \end{array}$                           | $4 \text{ times } 23, \text{ in the Cube of } 4 \text{ in the Root.}$                                     |
| $\begin{array}{r} 50784 \end{array}$                          | $6 \text{ times the Sq. of that } 23, \text{ in the Sq. of the } 4.$                                      |
| $\begin{array}{r} 194672 \end{array}$                         | $4 \text{ times the Cube of that } 23, \text{ in the said } 4.$                                           |
| $\begin{array}{r} 199809536 \end{array}$                      | $\{ \text{Sum} = \text{a Subtrahend, which taken from the Resolvend last above,}$                         |
| $(0) \text{ remains.}$                                        |                                                                                                           |

I know that the Biquadrate Root is the Square Root of the Square Root, and consequently may be performed by extracting the Square Root twice. But this seems a more natural way to perform it, as the Extraction of the Cube, Surfolid Root, &c. may be done from Algebraic Canons, and I have inserted the Method here, because to me it is new; for I never saw, nor heard of its being done thus before: and I was pleased when I considered it from the 4th Power of  $a+b$  in Algebra, which wonderfully, tho' plainly, points out all these Rules above, which seem so intricate, as to be impossible first to discover. This Extraction of the Biquadrate Root is useful in some



some Computations of Compound Interest; finding 3 mean Proportionals between 2 Extremes, as in the 5th Head of Geometrical Progression, &c.

The Proof of the Square Root (as appears from the little Table to it) is to multiply the Root in itself; the Proof of the Cube is to multiply that Square in the Root; and of the Biquadrate to multiply the Cube in the Root; for they respectively produce the Number given to have the Root extracted, provided that nothing remain; and if any thing do, add such Remainder.



## CHAP. II.

*Contains the Application of the Fundamental Parts of Arithmetic, to Vulgar Fractions, Progression, the Rules of Proportion, Practice, Loss and Gain, Fellowship, Barter, Exchange, Equation of Payments, Alligation, and Rules of False Position, in eleven Sections.*

### SECT. I. Of Vulgar Fractions.

I. **N**OTATION and Numeration,] Teacheth what a Fraction is, and how to read or write down any one.

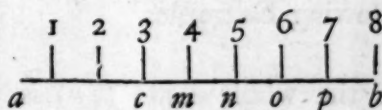
By a Fraction here is meant a broken Number, that is to say, one or more Parts of a Unit; for as there is Infinity or Units, so a Unit may be, or be supposed to be, divided into any Number of Parts.

A Fraction consists of two Parts, a *Denominator*, and a *Numerator*: The former shews how many Parts the Unit is divided into, which is wrote below the Line; the later shews how many of those Parts are contained in the Fraction, which is wrote above the Line; a

$\frac{3}{8}$  of

$\frac{3}{8}$  of a Pound Sterling, or 7 s. 6 d. here  $\frac{3}{8}$  is the Numerator.  
the Denominator.  
And these 2 Parts are called the Terms of a Fraction.

Or, to be more plain yet ;  
the Line ( $ab$ ) represents 1 or a  
Unit divided into 8 parts, and  
doth represent the said Deno-  
minator of the Fraction given ;



and the Line ( $ac$ ) is 3 of those 8 Parts ; ( $am$ ) = 4 Eights or  $\frac{4}{8}$  ;  
( $ao$ ) =  $\frac{6}{8}$  ; ( $ap$ ) =  $\frac{7}{8}$ , &c.

Having thus shewn  
what a Fraction is,  
you are next to know  
how to read it ; and  
that is, by first men-  
tioning the Numerator,  
and then the  
Denominator. As by  
this Example you see  
where 1/12th of the  
Unit (or Line  $mn$ )  
is wrote thus  $\frac{1}{12}$ , and  
read One Twelfth,  
&c.  $\frac{2}{12}$  = 2 Twelfths,  
&c. But the Line

A Table of Simple Fractions.

| $m$ |                                    | Read thus,       |
|-----|------------------------------------|------------------|
| 1   | — 12ths, wrote thus $\frac{1}{12}$ | One Twelfth.     |
| 2   | — 12ths, — $\frac{2}{12}$          | Two Twelfths.    |
| 3   | — 12ths, — $\frac{3}{12}$          | Three Twelfths.  |
| 4   | — 12ths, — $\frac{4}{12}$          | Four Twelfths.   |
| 5   | — 12ths, — $\frac{5}{12}$          | Five Twelfths.   |
| 6   | — 12ths, — $\frac{6}{12}$          | Six Twelfths.    |
| 7   | — 12ths, — $\frac{7}{12}$          | Seven Twelfths.  |
| 8   | — 12ths, — $\frac{8}{12}$          | Eight Twelfths.  |
| 9   | — 12ths, — $\frac{9}{12}$          | Nine Twelfths.   |
| 10  | — 12ths, — $\frac{10}{12}$         | Ten Twelfths.    |
| 11  | — 12ths, — $\frac{11}{12}$         | Eleven Twelfths. |
| 12  | — 12ths, — $\frac{12}{12}$ or 1    | Twelve Twelfths. |
| $n$ |                                    |                  |

might more commodiously be made downright, and then Fractions  
would stand as in the Examples next the left hand, whereby a Line  
in Printing might often be gained, if  $\frac{1}{12}$  were wrote 1/12 ;  $\frac{2}{12}$ ,  
2/12, &c. But let Custom have its own way ; and then Fractions  
are wrote and read as in the foregoing Tabulet, and by the same  
Rule  $\frac{17}{28}$  is seventeen 28th Parts ;  $\frac{365}{1728}$  is 365, seventeen hundred  
and 28 Parts ; that is, if a Unit were divided into 1728 Parts,  
this Fraction does contain 365 of those Parts.

But there are various kinds of Fractions, as Proper, Improper,  
Simple, and Compound.

A Proper Fraction is one whose Numerator is less than the  
Denominator, as those above.

An Improper Fraction is when the Numerator is greater or e-  
qual to the Denominator, as  $\frac{5}{3}$ ,  $\frac{7}{7}$ , &c.

A Simple or Single Fraction, as any of those foregoing is immediately the Fraction of a whole Unit. But

A Compound Fraction, is a Fraction of a Fraction, or Part of another Part of a Unit, as  $\frac{2}{3}$  of  $\frac{1}{4}$ , or  $\frac{1}{3}$  of  $\frac{1}{4}$ , &c. and is illustrated in the following Examples.

Where the whole Line (*ln*) is divided into 10 equal Parts, and each of those are subdivided into two Parts: so that supposing the Unit (*ln*) to be 1 *l.* each 10th is 2 *s.* and every half of a 10th is 1 *s.* so that  $\frac{1}{4}$  of  $\frac{1}{10}$  is a Fraction of a Fraction, whose Value is 13 *s.* But the Value here is not intended to be so observable as the Nature of the Compound Fraction; for here  $\frac{1}{10}$  is  $\frac{1}{10}$  of the Line *ln*, and  $\frac{1}{4}$  of  $\frac{1}{10}$  is 13 *s.* for  $\frac{1}{10}$  is 2 *s.* and consequently  $\frac{1}{10}$  is 14 *s.* and  $\frac{1}{4}$  of 14 *s.* must needs be 13 *s.* Sometimes you have a Fraction of a Fraction of a Fraction, &c. of a Unit, as 1 Farthing is  $\frac{1}{4}$  of  $\frac{1}{2}$  of  $\frac{1}{10}$  of a Pound, &c.

Having thus shewn what a Fraction is, and how to read the same, I proceed to

| A Table of Compound Fractions. |                                                   |   |           |
|--------------------------------|---------------------------------------------------|---|-----------|
|                                | <i>l.</i>                                         |   | <i>s.</i> |
| —                              | $\frac{1}{2}$ of $\frac{1}{10}$ of 1 <i>l.</i> or | — | 1         |
| 1—                             | $\frac{1}{10}$ or                                 | — | 2         |
| —                              | $\frac{1}{4}$ of $\frac{1}{10}$                   | — | 3         |
| 2—                             | $\frac{1}{10}$                                    | — | 4         |
| —                              | $\frac{1}{6}$ of $\frac{1}{10}$                   | — | 5         |
| 3—                             | $\frac{1}{10}$                                    | — | 6         |
| —                              | $\frac{1}{8}$ of $\frac{1}{10}$                   | — | 7         |
| 4—                             | $\frac{1}{10}$                                    | — | 8         |
| —                              | $\frac{1}{10}$ of $\frac{1}{10}$                  | — | 9         |
| 5—                             | $\frac{1}{10}$                                    | — | 10        |
| —                              | $\frac{1}{12}$ of $\frac{1}{10}$                  | — | 11        |
| 6—                             | $\frac{1}{10}$                                    | — | 12        |
| —                              | $\frac{1}{4}$ of $\frac{1}{10}$                   | — | 13        |
| 7—                             | $\frac{1}{10}$                                    | — | 14        |
| —                              | $\frac{1}{16}$ of $\frac{1}{10}$                  | — | 15        |
| 8—                             | $\frac{1}{10}$                                    | — | 16        |
| —                              | $\frac{1}{18}$ of $\frac{1}{10}$                  | — | 17        |
| 9—                             | $\frac{1}{10}$                                    | — | 18        |
| —                              | $\frac{1}{20}$ of $\frac{1}{10}$                  | — | 19        |
| 10—                            | $\frac{1}{10}$                                    | — | 20        |
|                                | <i>n</i>                                          |   |           |

## II. Reduction of Vulgar Fractions.

This Rule must necessarily be taught before Addition and Subtraction, because they cannot be performed till the Fractions given to be added or subtracted, are fitted by Reduction for that purpose.

Case 1. To reduce a mix'd Number to an Improper Fraction. As for Example  $12 \frac{3}{4}$ .

Rule.] Multiply the intire Part by the Denominator of the Fraction, and to the Product add the Numerator (3), and the Sum placed



placed over the Denominator (7) is the Answer, and will stand thus  $\frac{87}{7}$ .

Case 2. To reduce an improper Fraction to a whole or mix'd Number.  
Example, reduce  $\frac{87}{7}$ .

Rule.] Divide the Numerator (87) by (7) the Denominator, and the Quotient is the intire Number, and the Remainder (3) is the Numerator to place over the Denominator (7); so the Answer is  $12\frac{3}{7}$ , and proves the first Case true. And by the same rule  $\frac{14}{3}$  is  $= 4\frac{2}{3}$ ;  $\frac{144}{12}$  is  $= 12$ ;  $\frac{81}{9}$  is 9;  $\frac{44}{11}$  is 4, &c.

Case 3. To reduce Compound Fractions to Simple.

Example. Reduce  $\frac{1}{4}$  of  $\frac{2}{3}$  of  $\frac{3}{8}$  into one simple Fraction.

Rule.] Multiply the Numerators together for a new Numerator, and also the Denominators together for a new Denominator, and it stands thus  $\frac{1 \times 2 \times 3}{4 \times 3 \times 8}$ ; or this is  $\frac{1}{2}$  of  $\frac{2}{8}$  or  $\frac{1}{4}$ , by changing the Parts of the first and second Fractions, and omitting those which are the same; (as 3 in each) and  $\frac{1}{4}$  of  $\frac{1}{2}$  of  $\frac{1}{4}$  of  $\frac{1}{4}$  is  $\frac{1}{64}$ .

The Truth of this Rule is easily proved from any self-evident Instance, as  $\frac{1}{2}$  of  $\frac{2}{3}$  of a Pound Sterling is 6 s. 8 d. so by the Rule it is  $\frac{1}{3}$ . Now  $\frac{1}{2}$  being 3 s. 4 d.  $\frac{2}{3}$  must be 6 s. 8 d. or  $\frac{1}{3}$  of  $\frac{2}{3}$  is  $\frac{1}{3}$ : And if the Parts be changed, and the two of a sort be omitted, it proves the same; and the Reason is plain, for the Terms of a Fraction multiplied by the same Number, does not increase or alter the Value of such Fraction.

Case 4. To reduce a Fraction to its lowest Terms.

Example. Reduce  $\frac{45}{36}$  to its lowest Terms.

Rule 1.] Divide the Parts of the Fraction by any Number that will divide both without a Remainder: so this  $\frac{45}{36}$  is reduced to  $\frac{5}{4}$ , for Answer.

Rule 2.] Divide the greater by the lesser Part of the Fraction, and if any thing remain, divide the last Divisor by that; and if any thing yet remain, divide the last Divisor by that, till nothing remain; and then the last Divisor divideth both Parts of your Fraction, so as to reduce it to its lowest Term. See the Work in the Margin, where 9 the last Divisor reduces  $\frac{45}{36}$  to  $\frac{5}{4}$ ; so  $\frac{108}{72}$  is  $\frac{3}{2}$ ;  $\frac{36}{24}$  is  $\frac{3}{2}$ ; and  $\frac{18}{12}$  is  $\frac{3}{2}$ , and  $\frac{18}{12}$  is  $\frac{3}{2}$ , 35 dividing both Parts of the Fraction; found as per the 2d Rule.

for  $\frac{45}{36}$  divided by 3 is  $\frac{15}{12}$ , and  
 $\frac{15}{12}$  divided by 3 is  $\frac{5}{4}$

---


$$\begin{array}{r} 36 \overline{) 45} \quad (1 \\ \underline{36} \phantom{00} \\ 9 \phantom{00} \\ 9 \overline{) 36} \quad (4 \\ \underline{36} \phantom{00} \\ 0 \end{array}$$

*The Truth of this Rule:* That a Fraction reduced to its lowest Terms is of the same Value with that given, is thus proved,  $\frac{1}{2}$  of a Pound in its lowest Terms is  $\frac{1}{2}$  of a Pound, or 13 s. 4 d. And  $\frac{1}{2}$  of a Pound being 20 d. 8 times that is 13 s. 4 d.

Case 5. To find the Value of any Fraction of Weight, &c.

Example. What is the Value of  $\frac{1}{4}$  of a Pound Averdupois?

*Rule.]* Multiply the Numerator of the Fraction by such a Number of Units of the next less Denomination, as is equal to a Unit of that name which the Fraction is of, and divide the Product continually by the Denominator, and the Quotient or Quotients answer your Question. See the Work of the following Examples.

Example 2. What is the Value of  $\frac{1}{4}$  of a Pound Sterling?

$$\begin{array}{r}
 17 \text{ Numerator} \\
 20 \text{ Shill. in 1 L.} \} \text{Mult.} \\
 \hline
 26) 340 \text{ (13 s.} \\
 \quad \cdot \cdot \\
 \quad \cdot \cdot \\
 \hline
 80 \\
 \hline
 2 \text{ remains Sh.} \\
 12 \text{ Pence in 1 s.} \} \text{Mult.} \\
 \hline
 26) 240 \text{ d.} \\
 \quad 4 \text{ Farth. in 1 d. Mult.} \\
 \hline
 26) 96 \text{ (3} \frac{2}{3} \text{ Farthings.} \\
 \hline
 18
 \end{array}$$

So the Answer is

s. d. gr.  
13 : 0 : 3  $\frac{2}{3}$

Example 1. to Case 5.

$$\begin{array}{r}
 31 \text{ Pounds wt.} \\
 16 \text{ Ounces in 1 lb} \\
 \hline
 186 \\
 31 \\
 \hline
 42) 496 \text{ (11 Ounces.} \\
 \quad \cdot \cdot \\
 \quad \cdot \cdot \\
 \hline
 76 \\
 \hline
 34 \text{ remains.} \\
 16 \text{ Drams in 1 Ounce} \\
 \text{multiply.} \\
 \hline
 204 \\
 34 \\
 \hline
 42) 544 \text{ (12 Drams.} \\
 \quad \cdot \cdot \\
 \quad \cdot \cdot \\
 \hline
 124 \\
 40 \text{ remains to place} \\
 \text{over the 42.} \\
 \hline
 \text{So the Answer is} \\
 \text{Oun. Dram.} \\
 11 : 12 \frac{4}{12} \text{ or } \frac{30}{11}.
 \end{array}$$

And by the same Rule any other Fraction of Money, Weight, Measure, &c. hath its Value found.

The

The Truth of this Rule is manifest; for suppose  $\frac{2}{3}$  of a Pound Sterling we know is 17 s. 6 d. or 7 Half Crowns; and it will appear to be so by the Rule, if work'd as the two Examples above are.

Case 6. *To reduce Fractions of different Denominators to those of the same Value, which have a Common (or one and the same) Denominator.*

*Example.* Reduce  $\frac{2}{3}$ ,  $\frac{1}{4}$ , and  $\frac{3}{5}$  to a common Denominator.

*Rule.]* Multiply the Denominators one in another for the common Denominator, as 9 times 7 is 63, and 5 times 63 is 315 = the common Denominator.

Then for the 3 new Numerators, multiply every Numerator into all the Denominators except its own, and the Product is a Numerator answering to the Fraction whose Numerator you multiplied: as the Numerator 2 in 7 and 5 produceth 70, so is  $\frac{70}{315}$  equal to  $\frac{2}{3}$ . Then 3 multiplied in 9 and 5 gives 135, so is  $\frac{135}{315}$  equal to  $\frac{1}{4}$ . And lastly, 4 multiplied in 7 and 9 is = 252, so is  $\frac{252}{315}$  equal to  $\frac{3}{5}$ . So the 3 new Fractions have each the same Denominator, and are in value the same as those given. So also  $\frac{1}{2}$  and  $\frac{2}{3}$  are =  $\frac{3}{6}$  and  $\frac{4}{6}$ ;  $\frac{2}{3}$  and  $\frac{1}{4}$  are =  $\frac{2}{12}$ , and  $\frac{3}{4}$ , &c.

*The Truth of this Rule* will be evident by reducing any of the Fractions which have the same Denominator to its lowest Term, and that you'll find the primitive Fraction given: as  $\frac{70}{315}$  in its lowest Term is  $\frac{2}{9}$  = the Fraction given, and so of all the rest.

This Rule ought to be well minded, it being of principal Use in Addition and Subtraction of Fractions.

Case 7. *To reduce Fractions of a smaller Denomination to Fractions of a greater.*

*Example.* What Fraction of a Pound Sterling is  $\frac{1}{4}$  of a Farthing?

*Rule.]* Consider that  $\frac{1}{4}$  of a qr. is  $\frac{1}{4}$  of  $\frac{1}{4}$  of  $\frac{1}{4}$  of  $\frac{1}{4}$  of a Pound: then reduce this compound Fraction to a simple, by Case 3. and you'll find it  $\frac{3}{6720}$  of a Pound. And by the same Rule  $\frac{1}{12}$  of an Ounce is  $\frac{1}{304}$  of a Pound, &c.

*The Truth of this* appears by the next Case.

Case 8. *To reduce Fractions of a greater to a smaller Denomination.*

*Example.* What Fraction of a Farthing is  $\frac{3}{6720}$  of a Pound Sterling?

*Rule.]* Multiply the Numerator of the Fraction by such a Number of Units of the lesser, as make one of the greater Denomination: So here 960 Farthings making 1 l. I multiply the Numerator 3 in 960, and place the Product for a Numerator to the Denominator of the Fraction given, which makes the Answer  $\frac{2880}{6720}$  of a Farthing; which Fraction in its lowest Terms is  $\frac{1}{2}$ , and proves the last Case 7.



to be right. And  $\frac{15}{32}$  lb Averdupois is  $\frac{35}{64}$  of an Ounce, which in its lowest Term is  $\frac{15}{32}$ , as above. So that these two Cases 7 and 8, prove the Truth of each other.

### III. Addition of Vulgar Fractions.

*Example 1.* What is the Sum of  $\frac{1}{2}$  and  $\frac{1}{3}$ ?

*Rule.]* Reduce the Fractions to the same Denominator by Case 6. of Reduction, (which you'll find  $\frac{3}{6}$  and  $\frac{2}{6}$ .) Then add the Numerators 56 and 36 make 92, which placed over the common Denominator 96 is  $\frac{92}{96}$ , the Answer, or  $\frac{23}{24}$ .

*The Truth of this, &c. is thus proved:* Suppose the given Fractions be of a Pound Sterling; 7 12ths (or 7 times 20 d.) is 11 s. 8 d. and  $\frac{1}{3}$  of a Pound is 7 s. 6 d. the Sum of which is 19 s. 2 d. which you'll find to be the Value of the Answer  $\frac{92}{96}$ , by the fifth Case of Reduction of Fractions.

*Example 2.* What is the Sum of  $\frac{1}{2}$ , and  $\frac{2}{3}$  of  $\frac{1}{4}$  of  $\frac{1}{5}$ ?

*Rule.]* First reduce the compound Fraction  $\frac{2}{3}$  of  $\frac{1}{4}$  or  $\frac{1}{4}$  of  $\frac{1}{5}$  to a single Fraction, which you'll find  $\frac{1}{10}$  or  $\frac{1}{10}$ ; to which add the  $\frac{1}{2}$ , as per Example 1. and you'll find the Sum  $\frac{1}{2} + \frac{1}{10} = \frac{6}{10}$ , or  $\frac{3}{5}$ , (the Answer by Case 3. of Reduction.)

*Example 3.* What is the Sum of  $17\frac{1}{4}$  and  $\frac{1}{2}$  of  $\frac{1}{4}$ ?

*Rule.]* The compound Fraction in a simple is  $\frac{1}{8}$ , which added to the Fraction part of the mixt Number, maketh  $\frac{1}{8}$ , which by the second Case of Reduction is  $1\frac{1}{8}$  or  $1\frac{1}{4}$ ; which added to 17, gives the Sum  $18\frac{1}{4}$ . *The Truth of which is proved* by supposing  $17\frac{1}{4}$  to be 17 s. 9 d. and  $\frac{1}{2}$  of  $\frac{1}{4}$  to be 6 d. of a Shilling, which is 6 d. the Sum of which is 18 s. 3 d. or  $18\frac{1}{4}$ , as above.

### IV. Subtraction of Vulgar Fractions.

*Example 1.* From  $\frac{7}{8}$  take  $\frac{1}{8}$ .

*Rule.]* The Fractions in a common Denominator are  $\frac{7}{8}$  (equal to  $\frac{7}{8}$ ) and  $\frac{1}{8}$  (equal to  $\frac{1}{8}$ ) therefore 288 deducted from 736, the other Numerator, the Remainder is 448; so the Answer is  $\frac{448}{736}$  (which in its lowest Term is  $\frac{1}{2}$ , and proves the first Example in Addition of Vulgar Fractions). And this Remainder (as in Whole Numbers) add to the Subtrahend  $\frac{1}{8}$ , gives the Sum  $\frac{7}{8}$ , which proves (that way also) this and the first in Addition of Vulgar Fractions to be truly performed.

*Example 2.* From  $\frac{1}{2}$  take  $\frac{1}{3}$  of  $\frac{1}{4}$  of  $\frac{1}{5}$ .

*Rule.]*

*Rule.]* 1<sup>st</sup>, Reduce the compound Fractions to a single, which you'll find by the third Case in Reduction to be  $\frac{70}{4}$ .

2<sup>dly</sup>, Reduce  $\frac{18}{2}$  and  $\frac{70}{2}$  to one and the same Denominator by the sixth Case of Reduction, which are  $\frac{218448}{1270080}$  (equal to  $\frac{18}{2}$ ) and  $\frac{1270080}{1270080}$  (equal to  $\frac{70}{2}$ .)

3<sup>dly</sup>, Deduct the Numerator 176400 from the other Numerator 938448, and the Remainder is 762048: so the Answer is  $\frac{762048}{1270080}$ . And that in its lowest Terms (by the fourth Case of Reduction) you shall find  $\frac{1}{2}$ , by dividing each part of the Fraction by the common Measurer or Divisor 254016. *And this proves not only the Truth of the second Example in Addition, but also that this Example is rightly performed.*

*Example 3.* From  $18\frac{1}{2}$  take  $\frac{3}{4}$  of  $\frac{1}{4}$ .

*Rule.]* 1<sup>st</sup>, Reduce  $\frac{3}{4}$  of  $\frac{1}{4}$  to 1 Fraction as before, which is  $\frac{3}{16}$ .

2<sup>dly</sup>, Reduce  $\frac{18}{1}$  and  $\frac{3}{16}$  (the Fraction-part of the mixt Number given) to a common Denominator, which are  $\frac{288}{16}$  (equal to  $\frac{18}{1}$ ) and  $\frac{3}{16}$  (equal to  $\frac{3}{16}$ .)

3<sup>dly</sup>, Now you should take  $\frac{3}{16}$  from  $\frac{288}{16}$ , but you cannot, as being less; therefore borrow 1 or  $\frac{16}{16}$  from the 18, will leave 17. Then add  $\frac{16}{16}$  to the  $\frac{288}{16}$  makes  $\frac{304}{16}$ , from which take the  $\frac{3}{16}$ , and there remains  $\frac{301}{16}$ , which in its lowest Term is  $17\frac{1}{16}$ ; so the Remainder or Answer is  $17\frac{1}{16}$ , and *proves* the third Example in Addition. But this is done shorter by omitting the Numerators and Denominators which are the same Digits as  $\frac{3}{4}$  of  $\frac{1}{4}$  is  $\frac{3}{16}$  or  $\frac{1}{5}$ , &c. as under Case 3. of Reduction.

Which 3 Examples in Substraction, and those 3 in Addition, do mutually prove each other.

And thus I have given as many Examples as are necessary, in order to the perfect understanding of Addition and Substraction, which will prove very easy, (as they are so plainly expressed) especially to such as have a due knowledge of Reduction of Vulgar Fractions. I shall therefore proceed to

### V. Multiplication of Vulgar Fractions.

*Example 1.* Multiply  $\frac{7}{9}$  by  $\frac{9}{10}$ .

*Rule.]* Multiply the Numerators together for a new Numerator, as here 7 by 9 is 63; and the Denominators together for a new Denominator, as 15 by 10 is 150: so that  $\frac{63}{150}$  is the Answer.

*Example 2.* Multiply  $\frac{1}{2}$  of  $\frac{2}{3}$  by  $\frac{3}{4}$  of  $\frac{4}{5}$ .

*Rule.]*

*Rule.]* Work as before; for the Product of 5 in 2 in 7 and in 8, is 560 the Numerator; and 7, 3, 8, and 9 together, gives 1512: so is  $\frac{560}{1512}$  the Answer, or  $\frac{10}{27}$  in its lowest Terms.

*Example 3.* Multiply 25 by  $\frac{3}{4}$ .

*Rule.]* The whole Number wrote Fraction-ways is  $\frac{25}{1}$ , which multiply as by the Rule 1. and the Product is 175, (of the Numerators) and that of the Denominators is 12: so the Answer is  $\frac{175}{12}$ , or by the second Case of Reduction  $14\frac{7}{12}$ .

Thus you see in Multiplication of Fractions the Product is less than one of the Factors; and 'tis so much less, as the Numerator of the Multiplier is less than the Denominator, (in this Example) or so much as 7 is less than 12 its Denominator: for if the Numerator 7 were 12, it is plain that  $\frac{7}{12}$  (as is said before) is 1, and 1 time 25 would be 25, which would make the Product equal to the Multiplicand; and therefore as 7 is less than 12, so must the Product  $14\frac{7}{12}$  be less than 25.

*Example 4.* Multiply  $34\frac{7}{8}$  by  $13\frac{1}{2}$ .

*Rule.]* Reduce both the mixt Numbers to improper Fractions, and then proceed as by the first Rule in Multiplication of Fractions.

Thus  $34\frac{7}{8}$  is  $\frac{279}{8}$ , and  $13\frac{1}{2}$  is  $\frac{167}{2}$ , and the Product of the Numerators 279 and 167 is 46593, and of the Denominators (96) so the Answer is  $\frac{46593}{96}$ , or  $485\frac{3}{8}$ .

## VI. Division of Vulgar Fractions.

*Example 1.* Divide  $\frac{5}{3}$  by  $\frac{2}{3}$ .

*Rule.]* Place the Fractions as in the Margin;  
 then multiply the Numerator of the Divisor  $\frac{2}{3}$  by the Denominator of the Dividend, and the Product is 1050, the Denominator of the Quotient. And the Denominator of the Divisor in the Numerator of the Dividend produceth 945, the Numerator of the Quotient; which is therefore  $\frac{1050}{945}$ , in its least Terms  $\frac{2}{3}$ : And proves the Truth of the first Example in Multiplication of Vulgar Fractions.

*Example 2.* Divide  $14\frac{1}{2}$  by 25.

*Rule.]* Reduce the mixt Number to an improper Fraction, and put a Unit under the 25, then work as per the last Rule, and as in the Margin. Where you see the Quotient is  $\frac{1}{300}$ , or in its least Terms is  $\frac{1}{300}$ . And this proves the Truth of Example 3. in Multiplication, as that proves this true.

|               |                                   |                                    |
|---------------|-----------------------------------|------------------------------------|
| Divis.        | Dividend                          | Quot.                              |
| $\frac{2}{3}$ | $14\frac{1}{2}$ or $\frac{1}{25}$ | $\frac{1}{300}$ or $\frac{1}{300}$ |

*Example*



*Example 3.* Divide  $48\frac{5}{9}$  by  $13\frac{1}{2}$ .

*Rule.]* Reduce the mixt Numbers to Fractions, and you'll have  $\frac{465}{9}$  equal to  $48\frac{5}{9}$  = the Dividend,  $\frac{16}{2} = 13\frac{1}{2}$  = the Divisor, and  $\frac{1521}{16032}$  = the Quotient; which being reduced to a mixt Number, is  $34\frac{1}{8}$ , or  $34\frac{7}{8}$ , the Fraction being in its lowest Terms, (dividing each part by the common Measurer 2004).  $\frac{152}{12}$  ( $\frac{465}{9}$ ) ( $\frac{1521}{16032}$ ) or  $34\frac{7}{8}$  = the Answer.

*And this manifestly proves the Truth of the 4th Example in Multiplication of Fractions.*

I have in the *Merchant's Magazine* shewn the Reason of all the Rules abovementioned, for Reducing, Adding, &c. Vulgar Fractions; and have not room to repeat that here: but have fully shew'd how one Rule proves the Truth of another, which is a good Demonstration of the Genuineness of all.

## VII. To Extract the Roots of Vulgar Fractions.

*Rule 1.]* You must always reduce your Fraction into its least Terms: for if the Root can be justly extracted in any Terms, it can be so in its lowest. Tho' when you can see, as sometimes it happens, that the Roots may be immediately extracted of the Fraction in the Terms given, then you need not reduce it to its lowest. Thus for

*Example,* The Square Root of the Fraction  $\frac{16}{49}$  is  $= \frac{4}{7}$ , the Answer  $= \frac{4}{7}$ . And if the  $\frac{16}{49}$  had been reduced to its least Terms, they would have been  $= \frac{2}{7}$ , whose Square Roots are  $\frac{2}{7}$ , as before.

*Example 2.* When the Square Root of a mixt Number is required, as suppose of  $56\frac{1}{4}$ ; reduce the mixt Number to a Fraction, and extract the Root of both Terms as before: so  $56\frac{1}{4}$  is  $= \frac{225}{4}$ , whose Square Root is  $\frac{15}{2} = 7\frac{1}{2}$  the Answer, (for the  $\frac{225}{4}$  is given in its least Terms) and if you multiply  $7\frac{1}{2}$  by  $7\frac{1}{2}$ , the Product is  $56\frac{1}{4}$  for *Proof.* But

*Note,* That when a Fraction is given, whose Root cannot be extracted without Remainder, you must reduce it to a Decimal, and proceed to get the Root thereof, as *per* the Rules and Examples at the End of Decimals; or it may in most Cases be done near enough by Logarithms, as at the end of *Sect. 3. of Chap. VI.*

II. To extract the Cube Root of Vulgar Fractions. Reduce the Fraction to its least Terms, and then extract the Cube Root of each for those of the Root, unless you can see that the Roots may be immediately

immediately extracted of the Terms given. For Example, to get the Cube Root of  $\frac{27}{125}$ , this Fraction in its least Terms is  $\frac{3}{5}$ , whose Cube Root is  $\frac{3}{5}$ . Or if you had done it without reducing to its lowest Terms, the Roots of the Terms given would be 9 and 15, or  $\frac{3}{5}$ , which is equal to  $\frac{3}{5}$ , as before.

For the Root of a mixt Number, reduce it to an improper Fraction, and proceed as by the last Example. But when the Root cannot be accurately enough extracted without trouble, you may proceed by Decimals, or Logarithms, as is said above.

*Note*, That the Use of the Square and Cube Roots you have partly in the two next Sections.

### VIII. The Application or Use of Vulgar Fractions.

*Quest. 1.* Two Bags contain 470 $\frac{2}{3}$  Dollars, but the greater exceeds the lesser 32 $\frac{2}{3}$  Dollars: how much is in each Bag?

*Rule.*] Subtract the Difference from the Sum, and there rests 438 $\frac{1}{3}$  Dollars; half of which is 219 $\frac{2}{3}$ , the Content of the lesser Bag; to which add the Difference 32 $\frac{2}{3}$ , and the Sum is 251 $\frac{2}{3}$  Dollars in the greater Bag: *The Sum of which for Proof is 470 $\frac{2}{3}$ .*

*Quest. 2.* A Brick-Wall contains 1792 $\frac{1}{4}$  square Feet on the Surfacies, how many Rod is that of 272 $\frac{1}{4}$  Feet to the Rod?

*Rule.*] Divide 1792 $\frac{1}{4}$  by 272 $\frac{1}{4}$ , and the Quotient is  $\frac{108780}{77319}$ , or 6 $\frac{4300}{77319}$  Rod for Answer.

*Quest. 3.* In 846 $\frac{1}{8}$  Pole, how many Yards of 5 $\frac{1}{2}$  Yards to the Pole?

*Rule.*] Multiply the one by the other gives the Answer 4655 $\frac{1}{8}$  Yards.

*Quest. 4.* What is the Value of 87 C. — q. 13 lb at 1 l. 9 s. per C?

*Rule.*] The Weight is 87  $\frac{13}{16}$  C. or  $\frac{2237}{16}$  C. the Money is  $\frac{3}{8}$  l. multiply the Fractions together gives  $\frac{242201}{128}$  l. which by the fifth Case of Reduction of Fractions is = 126 l. 6 s. 4 d. 1 q.  $\frac{5}{8}$ .

*Quest. 5.* If I give l. 126 : 6 : 4 : 1  $\frac{5}{8}$  for 87 C. o q. 13 lb of Sugar, what is that per Hundred weight of 112 per C?

*Rule.*] Divide the Money (as 126 $\frac{37}{80}$ ) by the Weight 87 $\frac{13}{16}$  C. and the Quotient is exactly l. 1 : 9 : —, the Answer.

*And this proves the Truth of the last Question.*

*Quest. 6.* What is the Product of 5 s. 7 d. by 7 s. 5 d?

*Rule.*] Multiply 5 $\frac{7}{12}$  s. by 7 $\frac{5}{12}$  s. (the Pence being so many 12ths of a Shilling) and the Product is 41 $\frac{19}{144}$  Shillings, or l. 2 : 1 : 4 : 3 $\frac{7}{12}$ .

*Quest. 7.* Divide l. 2 : 1 : 4 : 3 $\frac{7}{12}$  by 7 $\frac{5}{12}$  Shillings, (or 7 s. 5 d.) The first, in a mixt Number is 41 $\frac{19}{144}$  s. which divided by  $\frac{85}{12}$ , (41 $\frac{19}{144}$  being

being first reduced to an improper Fraction) the Quotient is  $\frac{7111}{16}$  or  $5\frac{1}{2}$  s. This proves the last Question to be truly wrought.

Quest. 8. What is the Product of  $l. 5 : 7 : 10\frac{1}{2}$  multiplied in itself?

Rule.] Reduce the  $7$  s.  $10$  d.  $\frac{1}{2}$  into Farthings, and those Farthings into the Fraction of a Pound by the seventh Case of Reduction of Fractions: then you have  $l. 5\frac{127}{64}$  to square, (or multiply by itself) which produceth  $l. 29 : 1 : 7 : 2\frac{1123}{128}$ , the Answer.

I should not have inserted this and the sixth Question above, but that there is much noise (tho' little real Use) of them; some Persons being deceived by thinking to answer them (for example) by reducing in the 6th Question the  $5$  s.  $7$  d. into Pence, and multiplying them by the Pence in  $7$  s.  $5$  d. and then reducing the Pence of the Product into Shillings or Pounds by dividing by 12, &c. But this way gives 12 times the Answer; because, as appears, the Pence should be divided by 144, or 12 times 12 = the Product of the Denominators of the Fraction. And so in the last Example they will divide the Farthings in the Product of those in the Number given multiplied in itself by 960; whereas it ought to be divided by the Square of that, viz. 921600.

On the contrary, others think to perform the Work of the sixth Question by reducing the 67 and 89 Pence to the Fraction of a Pound, multiplying  $\frac{67}{16}$  by  $\frac{89}{16}$ . But here 'tis plain the Answer will be so much too little, as  $\frac{1261}{768}$  l. is less than  $\frac{1261}{64}$  l. i. e. it will be but  $\frac{1}{12}$  of the true Answer.

Hence it appears, that such Questions are only naturally and accurately resolved by Fractions, as in the said Examples; for which reason I have given those two above in this part of the Use of Fractions, but hope the ingenious Reader will excuse this Digression on so trivial an Instance.

## SECT. II. PROGRESSION.

THIS Part of Arithmetic, tho' neglected both by many Authors of Arithmetical Tracts, and Teachers of the Science, is however of excellent Use, as shewing in a great Variety of Instances the wonderful Power and Harmony of Numbers, and their Relation one to another.

This Relation of Numbers is either Arithmetical or Geometrical.

L

I. Arithmetical



I. *Arithmetical Progression*, or Relation of Numbers, is when a Series thereof differ by the Addition of some Number to the first, second, &c. So these Numbers,

|   |   |   |    |    |    |    |    |    |    |    |         |
|---|---|---|----|----|----|----|----|----|----|----|---------|
| 1 | 2 | 3 | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12, &c. |
| 2 | 4 | 6 | 8  | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24, &c. |
| 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 | 33 | 36, &c. |

Each of those in the first Series or Row differ by 1, each of those in the second Series by 2, and in the third by the common Addition of 3. And a Series of abundant Numbers are 30, 42, 54, 66, &c. which differ by 12. Now,

1. To find the Sum of any of the Series's, you must multiply half the Sum of the first and last Terms by the Number of Terms; or half the Number of Terms by the Sum of the first and last, and the Product gives the Sum required.

Thus 1 and 12 is 13 by 6 (half the Terms) gives 78 the Sum.

2 and 24 is 26 by 6, gives 156 = the Sum of the 2d Series.

3 and 36 is 39 by 6, gives 234 = the Sum of the 3d Series.

And the like is to be observed in summing up any other Rows of Numbers, tho' never so large, or however differing, if Arithmetically continued.

2. It may be observed, that the several Sums of the three Lines above, or any other of equal Numbers of Terms, beginning with successive Digits continued, do differ also in Arithmetical Proportion; the common Difference being the Sum of the first Series, as 78, 156, 234, &c.

*Question 1.* How many Strokes does the Hammer of a Clock strike in the 12 Hours? This is done as the first Example above, the Answer being 78, and would have been the same had you multiplied half the first and last, viz.  $6\frac{1}{2}$  by 12, the whole Number of Terms.

*Quest. 2.* Admit a Boy is to collect 100 Apples, which lie a Yard distant from each other, and to put each of them singly into a Basket placed one Yard from the first; how many Yards does he pass?

Here the first Term is 2 Yards, the last 200, (to the last Apple, and back to the Basket) Sum 202, which multiplied by 50, (half the Number of Terms) produceth the Sum of the Yards passed by the Boy, which is 10100, or upwards of five Miles.

*Quest.*

*Quest. 3.* How many Feet does an heavy Body fall in 11 Seconds of Time, supposing it to fall 16 Foot the first Second, 3 times that the next Second, 5 times that the 3d Second, &c. as in the Example, (or by continually adding 32 to the first Term) the 11th Term is 336 Feet, which the Body falls in the 11th Second, (so prodigious is the Increase of the Velocity) and if  $\frac{1}{2}$  the last and first together be multiplied by the 11, the Sum of all the Feet that it falls is 1936.

| <i>The Seconds<br/>a Body<br/>falls.</i> | <i>Multi-<br/>pliers.</i> | <i>Feet an heavy<br/>Body falls in<br/>each Second.</i> |
|------------------------------------------|---------------------------|---------------------------------------------------------|
| 1                                        | 1                         | 16                                                      |
| 2                                        | 3                         | 48                                                      |
| 3                                        | 5                         | 80                                                      |
| 4                                        | 7                         | 112                                                     |
| 5                                        | 9                         | 144                                                     |
| 6                                        | 11                        | 176                                                     |
| 7                                        | 13                        | 208                                                     |
| 8                                        | 15                        | 240                                                     |
| 9                                        | 17                        | 272                                                     |
| 10                                       | 19                        | 304                                                     |
| 11                                       | 21                        | 336                                                     |

Thus if you would take the Depth of a Well, or the like, suppose by a Watch that vibrates Quarter Seconds, I find a Stone 44 Quarters or 11 Seconds in falling as above, the Depth of such a Well, &c. is found 1936 Feet.

And in this Progression 'tis plain that any of the Terms are found without the intermediate, by multiplying 16 by double the number of Seconds less 1. Thus I find that it falls in the 8th Second, 240 Feet, by multiplying 16 by 15.

3. *To find a mean Arithmetical Proportional between any two Numbers.* Take half the Sum of the 2 Numbers or Extremes for Answer, as in the Examples above; or add half the Difference to the lesser.

A Mean between 9 and 11 in the first Line is 10.

20 and 24 in the 2d Series is 22.

27 and 33 in the 3d . . . . . 30, &c.

4. *The common Difference and Number of Terms given to find the last Term.* Thus in the 2d Series the Number of Terms (12) being multiplied by the common Difference 2, gives 24, the last Term, &c. or in any Series multiply the Number of Terms less 1 by the common Difference, and add the first Term.

5. *Any 2 Numbers standing together given to find a third, &c.* Take their Difference, and add to the greater, gives the 3d, &c. Or subtracted from the lesser, gives the lesser Terms, as in the third Series 21 and 24 are given to find the 3d Term; the Difference is 3, which added to 24 gives 27, the next Term higher; or subtracted from 21 gives 18, the next Term lower.

6. If any four Numbers are in *Arithmetical Proportion*, whether continued or interrupted, the Sum of the two middle Numbers are equal to the Sum of the two Extremes. Thus,

In the first Line 7, 8, 9, 10; 7 and 10 are equal to 8 and 9.

Also in 7, 8, 14, 15; 7 and 15 are equal to 14 and 8.

And in the 3d Line 12, 15, 18, 21; 12 and 21 are equal to 15 and 18.

7. If three Numbers are in *Arithmetical Proportion* continued, the Double of the Mean is equal to the Sum of the two Extremes, as 12, 15, 18; 2 times 15 is equal to 18 and 12.

8. If three Numbers are given, a fourth may be found by adding together the 2d and 3d, and from that Sum subtracting the first, as in 14, 15, 16, the Sum of 15 and 16 is 31, from which take 14, and the Remainder is 17, the 4th in *Arithmetical Proportion*; and this also holds, tho' the Progression be interrupted, as 14, 15, 25, 26.

9. The Total of the Progression, and the first and last Terms given, to find the Number of Places; divide the Total by half the Sum of the first and last Terms, and the Quotient is the Number of Terms or Places.

10. The last Number and common Difference given, to find the Number of Terms; divide the last Number by the Excess or common Difference,

11. The Sum of the Progression, and the first and last Terms given, to find the common Difference. Divide the Total of the Progression by half the Sum of the first and last Terms, and the Quotient is the Number of Terms. Then from the last Term take the first, and the Remainder divide by the Number of Terms less 1, and the Quotient is the Excess or Difference sought, as will appear by any of the three Series's above.

*The Reason of the Rule for summing up an Arithmetical Progression.* According to the 3d Proposition last above, half the Sum of the two Extremes is an *Arithmetical Mean*; and a Mean between the first and last Terms of a Progression, according to the same Rule, is found by the first Proposition; and since that is a Mean between the extreme Terms, therefore that being multiplied by the Number of Terms, must necessarily give the Total of them all.

Or more plainly: Take any odd Number of Terms in the three Series's above, and you'll find that Term standing in the middle to be the Mean, according to the said 3d Prop. Thus in the seven first Terms in the second Series, the Mean (or middle Number) is 8, there being three on each side; so that one with another, each

Term



Term is 8, as you'll find it: for 6 (the next towards the left hand) is 2 less than 8, but then 10 (the next towards the right hand) is 2 more than 8; and 4 (the 2d towards the left hand from the 8) is 4 less than 8, but then 12 (the 2d toward the right hand from 8) is 4 more than 8: so lastly, 2 (the 3d toward the left hand from the 8) is 6 less than 8; but then 14 (the 3d toward the right hand) is 6 more than 8. So that nothing is more plain, than that each Term (one with another) being 8, that multiplied by the Number of Terms must give the Sum of all the Terms (or Eights): so the Sum of 7 of the Numbers of the 2d Series aforesaid is 56 (or 7 times 8). And so of any other Series, grounded on the first *Prop.* above.

*Geometrical Progression.*

This is when a Series of Numbers are increased by a continual Multiplication of the first Term, and the Products arising, by some certain Number called the Ratio or common Factor: as

1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024.

3, 9, 27, 81, 243, 729, 2187, 6561, 19683, &c.

4, 16, 64, 256, 1024, 4096, 16384, 65536, &c.

*Prop. 1.* To sum up any Series of Numbers, whose Relation is in a Geometrical Progression.

*The first Way to find the Total of a Geometrical Progression.*

*Rule.]* Multiply the last Number by the common Multiplier.

2dly, From that Product deduct the first Number.

3dly, Divide the Remainder by the said Multiplier less one; and the Quotient is the Sum sought.

*The Example* shall be in the finding the Aggregate of the middle Series above, the last Number whereof is 19683; which multiplied by the common Multiplier is 59049, from which take the first Term of the Progression (3) refts 59046, which divide by the said Multiplier less (1) viz. by 2, and the Quotient is 29523 = the Sum required.

*The second and much briefer Way to find the Total of a Geometrical Progression, without many of the intermediate Terms, or the last Term being given.*

Consider that in the Series of the last Example or middle Series there are 9 Terms; therefore if you multiply the 5th (or 243) by itself,

self, it will at once produce the 59049 as before, (which would be the 10th Term, if the Progression had run so far) whence the Sum is found as above. For,

*Note 1.* That when the Number of Terms are odd, and the first is above a Unit, the Square of the middle Term gives that next above the last Term, as in the last Example.

*Note 2.* If the Number of Terms be odd, and the first Term be but a Unit, then the Square of the middle Term gives the last Term of the Progression, as in the first Series the Square of 32 is the last Term = 1024.

*Note 3.* If the Terms be an even Number, and the first Term be One or Unity; then the Square of the Sum in the place of half the Number of Terms gives the last Term save one: as in the 10 first Terms of the first Series, the Square of the fifth Term (16) gives  $256 =$  the last of the 10 Terms but one.

*Note 4.* If the Number of Terms be even, and the first Term be more than a Unit, then the Square of the Sum in the place of half the Number of Terms, gives the last Term of the Progression, as in the third Series the Square of 256 is the last Term 65536.

These Rules are more particular and useful in the summing up all kinds of Geometrical Progressions than I have any where observed to be exhibited, and therefore worth nothing, for they always hold, as in the two Series's above; and where the Nature of the Progression is as in the first Rank or Line; as is thus farther demonstrated:

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10,  
1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024,

The upper Series being Arithmetical, the Addition of any two, or Double of any one, shews respectively what is produced by multiplying or squaring those under them in the lower Line. Thus 4 doubled gives 8 in the upper Line, under which stands 256, or the Square of the Number under the 4, &c. So any 2 added in the upper Line, as 3 and 6, give 9; which stands over the Product of the 2 Numbers 64 and 8, which is 512.

*The Reason of the Rule for summing up a Geometrical Progression.*

It will be the more easily understood by a Series of a few Terms. Thus if I would know the Sum of 4 and 8, it is 4 (or the first Term) less than the last Term 8 multiplied by the common Factor

(2)

(2) which plainly shews why you deduct the first Term (here 4). So also in 2, 4, 8, the Double of 8 is 2 (or the first Term) more than the Sum 14, and therefore from the 16 must be deducted the first Term (2). Lastly, in 1, 2, 4, 8, twice 8 (the last Term) is 16, which is the first Term (1) more than the Sum (15). And this is sufficient to shew the Reason why we multiply the last Term by the common Factor, and why from the Product the first Term is deducted.

Now, as when the Ratio or common Factor is (2) the Sum of the Progression is  $\frac{1}{2}$  the last Number after the first is deducted from the Product; so when the common Factor is 3, the Answer is  $\frac{1}{2}$ ; if the common Factor be 4, the Answer is  $\frac{1}{2}$  of the last Number (or Term) having before Division the first Term deducted from the Product. Thus in 3 and 9, the Sum is 4 three's or 12, which is thrice the last Term (9) less the first Term (3) divided by 2. So also in 4 and 16, the Sum is 4 times 16, less the first Term (4) dividing the Remainder by 3; as might easily (from so few Terms) at first be discovered. But see more at the end of this Section.

*Quest. 1.* A King of much Virtue and Valour, admirably esteem'd by his Subjects, returning from the Wars with Victory and Peace, very much Emulation appear'd who should exceed in Demonstrations of Joy. Among the rest, the Governour of the Fortrefs intending to signalize himself, orders his Sub-Officers to make so many Discharges of Cannon; viz. 3 for the first Year of the King's Age, (which was 25 Years) 9 for the second Year, 27 for the third Year, &c. (not considering the Impossibility of the Performance) the Question is, how many Discharges were to be made, and how much Powder consumed, supposing the Guns to be each a Culverin, whose Requisite of Powder is 10 lb?

The



|               |          |         |        |          |
|---------------|----------|---------|--------|----------|
| The 13th Term | 1594323  | } Mult. | Years. | Dischar. |
| Ditto ..      | 1594323  |         | 1st —  | 3        |
|               | 4782969  |         | 2d —   | 9        |
|               | 3188646  |         | 3d —   | 27       |
|               | 4782969  |         | 4th —  | 81       |
|               | 6377292  |         | 5th —  | 243      |
|               | 14348907 |         | 6th —  | 729      |
|               | 7971615  |         |        | 729      |
|               | 1594323  |         |        | 6561     |
|               |          |         |        | 1458     |
|               |          |         |        | 5103     |

This is the 6th  
Term (or the next  
above the last) per.  
Note 1. foregoing.

2541865828329

The first Deduct . . . . . 3

12th Term }  
by Note 4. } 531441

3

Rests = 2541865828326

13th Term . . . 1594323

Half of which is = 1270932914163,  
the Sum of the Progression, or the  
Number of Discharges order'd.

Which, at 10 Pound of Powder at each Discharge, gives  
12709329141630 lb. at 100 lb. each Barrel, is 1270932914163  
Barrels; which would lade 14184965 Ships, at 400 Ton each Ship.

*Quest 2.* On the same Occasion  
all the Bells in the King's Metro-  
polis were directed to ring as many  
Changes as they were capable of.  
Now there being 9 Churches, 1  
of which had 2 Bells, another, 3,  
&c. the last 10 Bells; how many  
Changes could be rung on all the  
Bells at each Church, and what  
the Sum Total of all that Pro-  
gression?

The Number of Bells multipli-  
plied (any way) one in another,  
gives the last Term, or Changes  
that can be rung thereon. Then for the Sum, multiply the last  
Term by the correspondent Number of Bells, and divide the Pro-  
duct

| Churches.      | Bells. | Changes to<br>be rang<br>at each. |
|----------------|--------|-----------------------------------|
| St Peter's —   | 2 —    | 2                                 |
| St Andrew's —  | 3 —    | 6                                 |
| St James's —   | 4 —    | 24                                |
| St John's —    | 5 —    | 120                               |
| St Bartholomew | 6 —    | 720                               |
| St Thomas's —  | 7 —    | 5040                              |
| St Matthew's — | 8 —    | 40320                             |
| St Simon's —   | 9 —    | 362880                            |
| St Jude's —    | 10 —   | 3628800                           |

duct by the Number of Terms, and the Quote is the Answer for four Bells: For 5 Bells add 2 to the Rule, for 6 add 8, for 7 add 32, for 8 Bells add 152, for 9 add 872, and for 10 add 5912, gives the Sum of the Progressions, here 4037912.

*Note.* The 2 is the first Term, the 8 the Sum of the 2 first, the 32 = the 3 first, the 152 = the 4 first, the 872 = the 5 first, and the 5912 = the Sum of the 6 first Terms.

*Quest.* 3. Any Line or Number given, to divide the same into extreme and mean Proportionals. To the Square of the whole Line or Number given, add  $\frac{1}{4}$  of that Square; extract the Square Root of the Sum, from which Root deduct half the Line or Number given, and the Remainder is the greater part. And for a Proof, the Rectangle or Product of the whole Line by the lesser part, is equal to the Square of the greater part, by *Euclid* 11. 2.

4thly, If three Numbers be given in Geometrical Proportion continued, the Product of the two Extremes is equal to the Square of the Mean, as 5, 20, 80, equal to 400.

5thly, If two Numbers be given, it follows from the last, that a mean Proportional may be found by multiplying them together, and extracting the Square Root of the Product. Thus in the last Example 5 times 80 is 400; the Square Root of which is 20 = the Mean sought.

6thly, If you would find two Geometrical mean Proportionals between two Numbers given, as suppose between 5 and 320, you must divide the greater by the less; then extract the Cube Root of the Quotient; lastly, by that Root multiply the first Term, gives the first Mean, which multiply by the said Root gives the second, so is 5, 20, 80, 320, the Numbers in order, 20 and 80 being the Means required.

7thly, Or three mean Proportionals may be found thus: Suppose between 5 and 1280. Divide the greater Extreme by the lesser, extract the Biquadrate Root of the Quotient, and by that Root multiply the lesser Extreme continually 3 times; so will the Answer be found 5. 20. 80. 320. 1280.

8thly, If 4 Numbers be given in Geometrical Proportion, the Product of the two Extremes is equal to that of the two Means: so in the last Example 5 times 320 is equal to 20 times 80, viz. 1600.

9thly, If two Numbers be given, a third in Geometrical Proportion may be found by dividing the greater by the lesser, and multiplying such greater by the Quotient. As per the 5th, 80 is found

in proportion as 5 to 20; for as 5 to 20, so is 20 to 80, as may be proved by the 3d or 6th Cases above.

10thly, Three Numbers given in Geometrical Proportion, a fourth may be found by multiplying the second and third together, and dividing the Product by the first: as in the 5th 20 times 80 is 1600, which divided by 5, gives the 4th Proportional 320. This last is the Foundation of the Rule of Three Direct, which is the next thing to be taught after I have shewed

*The Process for finding out the Canon whereby to discover the Sum of a Geometrical Progression.*

But first note that

$\therefore$  { shews a Geometrical Pro-  
portion.  
— signifies less.  
= signifies equal to.  
 $\times$  signifies multiplied in.

And in the Literal Work,  
f the first Numb. of the Progression.  
r the Ratio or common Factor.  
u { the Sum of the Progression,  
which is unknown.  
l the last Term of the Progression.

The Sum of the Progression to be found is this Example,

1, 2, 4, 8, 16, 32.

Literally performed.

$$\begin{aligned} f. fr : : u - l. u - f \\ fu - ff = r fu - rf l. \\ u - f = ru - rl \\ ru - u = rl - f \\ u = \frac{rl - f}{r - 1} \end{aligned}$$

Numerally performed.

$$\begin{aligned} \text{that is } 1. 2 \times 1 : : u - 32. u - 1 \\ \text{that is } u - 1 \times 1 - 2 \times 1 \times u - 32 \times 2 \times 1 \\ \text{that is } u - 1 = 2 \times u - 2 \times 32 \\ \text{that is } 2u - u = 2 \times 32 - 1 \\ \text{that is } u = \frac{2 \times 32 - 1}{2 - 1} = \text{the Answer.} \end{aligned}$$

So that 'tis plain the Answer is (2) the Ratio multiplied in (32) the last Term, made less by (1) the first Term, and that Remainder divided by (2) the Ratio, less 1, the Quotient is (63) the Sum of the Progression (or u): And this Canon (or Rule) holds in all Geometrical Progressions continued. Which the Reader will more easily apprehend, when he comes to the Algebraic Part, and till then he may pass this, if he finds it difficult to understand; but I could not well omit it in this place, to which it most properly belongs.

SECT.



## SECT. III. Rules of Proportion.

**I**N this Section I shall shew the Operation, 1. Of the single Rule of Direct Proportion. 2. The single Rule of Reverse Proportion. 3. The double Rule of Direct Proportion. 4. The double Rule of Reverse Proportion. 5. The Rule of Proportion by 5 Numbers Direct. 6. The double Rule by 5 Numbers Reverse. 7. A third Variety thereof. 8. Duplicate Proportion Direct. 9. Duplicate Proportion Reverse. 10. Triplicate Proportion. 11. Harmonical Proportion. And, 12. Sefquiplicate Proportion.

## I. The Single Rule of Direct Proportion.

This Rule has its Foundation from the 8th and 10th Propositions last foregoing, and is demonstrated by *Euclid* in the 16. 6.

The main difficulty is to state the 3 Numbers given right; for there are always 3 given to find a 4th, and hence some call this *the Rule of 3*; others, because of its great Use, have termed it *the Golden Rule*: and 'tis called the Rule of Proportion, because the Number sought bears such Proportion to the 3d, as the 2d does to the first of the Numbers given.

*For the right disposing or placing the 3 Numbers given.*

*Rule.]* There being always 2 of one Denomination, and 1 of another, put the last mentioned down first: 2dly, Put that towards the left hand thereof which has dependance on it: and, 3dly, Put the other next the right hand. Thus if the Interest of *l. 750* were sought for a Year at the Rate of *5 per Cent.* it appears by the State in the Margin, that only 1 of the 3 given Numbers is Interest, therefore I place that in the middle. And because 100 has dependance on it (as being its Principal) therefore I put 100 towards the left hand, and the 3d toward the right hand (in course) Then

|                  |                  |                                        |
|------------------|------------------|----------------------------------------|
| <i>l. Princ.</i> | <i>l. Inter.</i> | <i>l. Princ.</i>                       |
| 100.             | 5                | : : 750                                |
| —                | —                | —                                      |
|                  |                  | 3750                                   |
|                  |                  | —                                      |
|                  |                  | 37 <sup>1</sup> / <sub>2</sub> Answer. |

*To find the 4th Proportional, or Answer to the Question.*

*Rule.]* Multiply the 2d and 3d (or those towards the right hand) together, and divide the Product by the first (or that next the left hand) and the Quotient is the Answer, or the 4th Proportional sought for. So in the

Example above, the 4th Proportional is  $37\frac{1}{2}$  for 100.  $5 :: 750. 37\frac{1}{2}$ , that is, as 100 is in proportion to 5, so is 750 to  $37\frac{1}{2}$ . So that you may prove this Rule by comparing the Product of the 1st and 4th with the Product of the 2d and 3d, as by the sixth last mention'd Prop. in Geometrical Progression.

Case 2.] When the first Number is a Unit only, the Work is done by Multiplication alone.

Thus if 1 Piece of Broad-Cloth cost 17 l. what will 71 cost? Here the 4th Number is found 1207 l. which is the 4th Proportional sought.

$$\begin{array}{r} \text{Pc.} \quad l. \quad \text{Pc.} \quad l. \\ 17 : 71 :: 1207 \\ \hline 17 \\ 497 \\ 71 \\ \hline 1207 \end{array}$$

2dly, If I give 3 s. 9 d. for an Ell of Holland, what will 137 Ells cost at that rate?

By the Rule above the Question is stated as in the Margin, and you may multiply  $137\frac{1}{2}$  by 3 s.  $\frac{9}{2}$ , as in the 4th Example of Multiplication of Fractions, and the Answer is  $493\frac{6}{96}$  Shillings; which, by the 5th Case of Reduction of Vulg. Fractions, is l. 257 : 2 : 7 :  $3\frac{1}{2}$ , as per Margin. But these Questions are sooner solved by Practice, as per the next Section.

$$\begin{array}{r} \text{Ell} \quad s. \quad d. \quad \text{Ells} \\ 1 \quad 3 : 9 :: 137\frac{1}{2} \\ 96) 493695 (5142 s. \end{array}$$

$$\begin{array}{r} 136 \\ \hline 409 \end{array}$$

Case 3.] When the 3d Number is only a Unit, the Question is answered by Division. Thus if I give l. 257 : 2 : 7 :  $3\frac{1}{2}$  for 137 Ells of Holland, what is that per Ell?

The Question is thus stated: and being divided (as per

$$\begin{array}{r} \text{Ells} \quad \text{Shill.} \quad \text{Ell} \\ 1097\frac{1}{2} \quad 4936\frac{9}{96} :: 1 \end{array}$$

the first Example of Division of Vulgar Fractions) the 2d by the 1st, the Quotient is  $1\frac{242}{3276}$  s. which by the said 5th Case of Reduction of Vulgar Fractions is 3 s. 9 d. and proves the Truth of the last Question.

$$\begin{array}{r} 255 \\ \hline 63 s. \text{ Rem.} \\ 12 \text{ Mult.} \end{array}$$

$$\begin{array}{r} 96) 756 (7 d. \\ \hline 84 d. \text{ Rem.} \\ 4 \text{ Mult.} \end{array}$$

$$\begin{array}{r} 96) 336 (3 q. \frac{1}{2} \\ \hline 48 \end{array}$$

Note, That where the first, or third Number, or both, are of diverse Denominations, you must reduce both such 1st and 3d into the least Name mentioned in either, and then

then multiply by the second Number in the lowest Denomination that is in it when given. And then having divided by the first,

*Note, 2dly,* That your first Quotient will always be of the same Denomination with the lowest mention'd and given in your second Number. To illustrate these three Cases,

*Case 4.]* If I give  $l. 7$  for the Interest of  $l. 100 : 10 : -$  : what must I give for the Interest of  $l. 30$  for the same time; See the Work in the Margin, where the 1st and 3d be reduced to Shillings (that being the least Term in the 1st Number) yet the Answer retains the same Denomination with the 2d Number, and is  $l. 2 \frac{1}{2} \frac{8}{10}$ ; or by the 5th Case of Reduction of Vulgar Fractions  $l. 2 : 1 : 9 \frac{1}{2}$  *prope.*

|       |                            |                            |      |      |       |       |
|-------|----------------------------|----------------------------|------|------|-------|-------|
| Prin. | $l.$                       | $s.$                       | $l.$ | Int. | $l.$  | Prin. |
|       | 100                        | 10                         | 7    | ::   | 30    |       |
|       | 20                         |                            |      |      | 20    |       |
|       | <hr/>                      |                            |      |      | <hr/> |       |
|       | 2010                       | Shil.                      |      |      | 600   | Shil. |
|       |                            |                            |      |      | 7     |       |
|       |                            |                            |      |      | <hr/> |       |
|       | 2010                       | 4200                       | (2   | $l.$ | Int.  |       |
|       | $\underline{\hspace{1cm}}$ | $\underline{\hspace{1cm}}$ |      |      |       |       |
|       |                            |                            |      |      | 18    |       |

*Case 5.]* If I give  $7 \text{ } l.$  for 30 Ounces of Silver, what will 7 Ounces and 10 Penny-Weight cost at that rate?

|                            |                            |                        |               |
|----------------------------|----------------------------|------------------------|---------------|
| $\text{ } \frac{3}{4}$     | $l.$                       | $\text{ } \frac{3}{4}$ | dw.           |
| 30                         | 7 ::                       | 7                      | 10            |
| 20                         |                            | 20                     |               |
| <hr/>                      |                            | <hr/>                  |               |
| 600                        | dw.                        | 150                    | Penny-Weight. |
|                            |                            | 7                      |               |
|                            | <hr/>                      |                        |               |
| 600                        | 1050                       | (1                     | $l.$          |
| $\underline{\hspace{1cm}}$ | $\underline{\hspace{1cm}}$ |                        |               |
|                            | 450                        |                        |               |
|                            | 20                         |                        |               |
|                            | <hr/>                      |                        |               |
| 600                        | 9000                       | (15                    | $s.$          |
| $\underline{\hspace{1cm}}$ | $\underline{\hspace{1cm}}$ |                        |               |
|                            | 30                         |                        |               |
|                            | <hr/>                      |                        |               |
|                            | 0                          |                        |               |

Here it is plain that because the  $7 \text{ } \frac{3}{4} 10 \text{ dw.}$  are reduced into Penny-Weight, the  $30 \text{ } \frac{3}{4}$  must be so too: Also that the Quote is of the same Denomination with the 2d Number, *viz.*  $l. 1 \frac{1}{2}$ ; or, being reduced as aforesaid,  $l. 1 : 15$ .

*Case 6.]* If 25 Ounces of Ambergreace cost  $l. 52 : 6 : 8$ , what will 2 Ounces cost?

See



See the Operation, where the 2d Number in its least Denomination being 12560 Pence ; therefore the Quote 1004 is Pence.

|     |    |    |    |   |
|-----|----|----|----|---|
| 3   | l. | s. | d. | 3 |
| 25. | 52 | :  | 6  | : |
|     | 20 |    |    | 2 |

1046 s.

12

12560 Pence } Mult.  
2 Ounces }

25) 25120 (1004  $\frac{3}{4}$  Pence.

120 or l. 4 : 3 : 8 $\frac{1}{2}$  the Answer.

20

## II. The Single Rule of Proportion Reverse.

This teaches, upon a due stating of the 3 Numbers, to find a 4th ; which shall bear such a Ratio to the 2d, as the 1st does to the 3d Number.

*Quest.* 1. How much Matting of 2 Foot broad will line a Gallery which is 7 Foot broad and 40 Foot long ?

To state a Question in this Rule,

1. Put down the Number which has the same Denomination as that required.

2. Towards the left hand put down that Number which is joined in the Sense of the Question to the last.

3. Put down the other which has the same Denomination as that towards the left hand.

Then to find the Answer by this,

*Rule.* Multiply the 2 next the left hand one in the other, and divide the Product by the 3d, or that next the right hand, and the Quotient is the Answer.

Hence the Reason is evident why this Rule is said to be Reverse because the lesser the 3d Number is, the greater is the 4th ; but in Direct

broad. long. broad long.

7. 40. 2. 140.

7

2) 280 (140 = the Answer.

And as 140. 40 :: 7. 2.

Direct Proportion the greater the 3d is, the greater is the 4th Number.

So that by this Rule it is easy to know when a Question is to be solved as Direct, and when it is Reverse: For when the 3d Number is less than the first, and yet requires more than it does, or being more, requires less than the first requires; in these Cases your Proportion is Reverse, and so you are to work by the last Rule.

*Example 2.]* How much Ground in length, being 12 Perches broad, will make an Acre?

|        |       |                      |
|--------|-------|----------------------|
| broad. | long. | broad.               |
| 4.     | 40.   | 12.                  |
|        | 4     |                      |
|        | 12)   | 160(13 $\frac{1}{3}$ |
|        |       | Perch. long,         |
|        |       | for Answer.          |

To do this, consider that 4 Perches in breadth requires 40 in length to make an Acre; therefore state and work as *per* Margin.

But altho' Writers have prescribed the foregoing Method for stating a Question, as being very natural, and thereby have made it necessary to make this a different kind of Operation and Proportion; yet I shall shew

*How to reduce Questions commonly said to be in Reverse Proportion into Direct.*

And this is only to consider (to instance in the last Example) That as (12) the Breadth on which the Answer depends, Is to (4) the Breadth belonging to the Length given :: So is (40) the Length given, To (13 $\frac{1}{3}$ ) the Length required.

Here the 4 Numbers are in Direct Proportion, because the Product of the 2 Means is equal to that of the 2 Extremes; *which, by the by, serves for Proof of this Direct Proportion.* But in the Reverse the Product of the 1st and 2d is equal to that of the 3d and 4th Numbers; *which proves the Reverse Proportion.*

### III. The Double Rule of Direct Proportion.

This is when a Question requires to be twice stated, and has two such Operations, as under the single Rule. For Example,

If I give 5 *l.* for the Interest of *l.* 100 for one Year, what Interest will *l.* 700 gain at that rate in seven Months?

1<sup>st</sup> say, If  $l. 100$  require  $l. 5$ , what will  $l. 700$  require in the same time? You find as *per* the Margin that it will gain  $l. 35$ .

$l. \quad l. \quad l.$   
 $100. \quad 5 :: 700$

$5$

$l. 3500$

$l.$

2<sup>dly</sup> say, If 12 Months require  $l. 35$ , what will 7 Months require? Work and you find the Answer  $l. 20\frac{1}{2}$ , or  $l. 20 : 8 : 4$ .

Mon.  $l.$  Mon.

$12. \quad 35 :: 7$

$7$

$l.$

$12) 245 (20\frac{1}{2}$

$5$

And by the same Rule you may find the Interest of any Sum for any Number of Days; as suppose  $l. 700$  for 119 Days.

Days.  $l.$  Days.

$365. \quad 35 :: 119$

For a Year, by the first Operation, the Interest is  $l. 35$ : then in the second Working say, if 365 Days requires  $l. 35$ , what will 119 Days require? You'll find the answer to be  $l. 11 : 8 : 2 : 2\frac{1}{2}$ .

#### IV. The Double Rule of Reverse Proportion.

This is also performed at two Operations, the first Direct, the later Reverse. For instance,

What Principal will gain  $l. 20\frac{1}{2}$  in 7 Months, as the Rate of 5 per Cent?

The 1<sup>st</sup> Answer is, that  $241\frac{1}{2}\%$   $l.$  Principal will raise the  $20\frac{1}{2}$  in a Year.

$l. \text{ Int.} \quad l. \text{ Princ.} \quad l. \text{ Int.} \quad \text{Princ.}$   
 1<sup>st</sup> say  $5. \quad 100 :: 20\frac{1}{2} \quad 241\frac{1}{2}\%$   $l.$

Mon.  $l.$  Mon.  $l.$   
 2<sup>dly</sup> say  $12. \quad 241\frac{1}{2}\% : 7. \quad 700$

The 2<sup>d</sup>, or Answer to the Question, is, That  $l. 700$  is the Principal that will raise the Interest  $l. 20\frac{1}{2}$  in 7 Months: by multiplying the 2<sup>d</sup> Number by 12, and dividing by 7. Note, This proves the first Question in the third Head above.

#### V. The Rule of Proportion by 5 Numbers Direct.

This Rule hath 5 Numbers given to find a 6th. It is the same with the double Rule of Proportion Direct; only whereas that was done



done by twice stating the Question, this is done by one stating :  
*the Rule for which is to place the 5 Numbers as followeth.* Where 'tis  
 obvious, That the 3d must be of the same Denomination with  
 that sought ; the 1st and 2d are the 2 on which the 3d depends: so  
 that the 4th must be of the same Denomina-  
 tion with the 1st, and the 5th the same with the 2d.

|          |       |         |             |       |
|----------|-------|---------|-------------|-------|
| l. Prin. | Time. | l. Int. | l. Prin.    | Time. |
| 100.     | 12.   | 5.      | 700.        | 7     |
| 12       |       |         | 7 multiply. |       |

*Example.]* What is the Interest of  $\text{£} 700$  for 7 Months at the Rate of 5 per Cent. per Ann. ? By the Work in the Margin, you see

1200 = the Di-  
 visor.

4900 } multiply.  
 5

1200) 24500 (20½ = the  
 Answer

*The Rule for the Operation is,* Multiply the 3 Numbers next the right in each other for a Dividend, and the 2 next the left hand for a Divisor, and the Quotient is the 6th Proportional sought ; as appears in the Method, by double stating under Head III.

*And to prove this ;* The Product of the 1st, 2d, and 6th, is equal to that of the 3d, 4th, and 5th.

*Quest. 2.]* If 1000 Men in 24 Hours can dig a Trench 18 foot broad, 9 deep, and 500 long; what length of the like Trench can 9800 Men dig in 10 Hours? The Answer as per Margin, is 2041½ Feet.

|       |        |       |              |        |
|-------|--------|-------|--------------|--------|
| Men.  | Hours. | Feet. | Men.         | Hours. |
| 1000. | 24.    | 500.  | 9800.        | 10.    |
| 24    |        |       | 10 multiply. |        |

98000 } multiply.  
 500

24000) 49000000 (2041½ foot  
 Answer.

Whence it appears, That as the Product of the 1st and 2d Numbers is to the 3d : so is the Product of the 4th and 5th to the 6th Number. Which also serves to prove the Rule ;

For as 24000. 500 :: 98000. 2041½.

And 24000 multiplied in 2041½ is equal to 98000 multiplied by 500, viz. 49000000, as per Prop. 6. of Geom. Progression.

## VI. The second Variety of the Rule of Proportion by 5 Numbers.

This is commonly called *the Rule by 5 Numbers Reverse*.

*Example.*] How many Men can dig 2041 $\frac{1}{2}$  feet in 10 Hours, at the rate of 1000 digging 500 feet in 24 Hours. *State your Question thus:*

| Men.   | Hours. | Feet. | Feet.              | Hours. |
|--------|--------|-------|--------------------|--------|
| 1000 : | 24 :   | 500.  | 2041 $\frac{1}{2}$ | 10.    |

*i. e.* If 1000 Men in 24 Hours dig a Trench of 500 Feet in length, how many Men can dig a Trench of 2041 $\frac{1}{2}$  feet long in 10 Hours?

To answer this, multiply the 1st, 2d, and 4th Numbers together for a Dividend, which divide by 5000, the Product of the 3d and 5th Numbers, and the Quotient is 9800 Men for Answer.

## VII. A third Variety of the Rule of Proportion by 5 Numbers.

This (altho' omitted by Authors) is as likely to be used as the last: for (to put the same Question again, that the varying may the better appear)

If 1000 Men in 24 Hours can dig 500 foot of a Trench, (as mention'd under the 5th Head above,) in how many Hours can 9800 Men dig 2041 $\frac{1}{2}$  feet (of the like Trench)?

*Rule.*] Multiply the 1st, 2d, and 5th together, produceth 49000000 for a Dividend: Then multiply the 3d and 4th together for a Divisor, which is 4900000. So the Quote is 10 Hours, as in the two Cases above.

|        |   |                                                                                 |   |                          |
|--------|---|---------------------------------------------------------------------------------|---|--------------------------|
| In the | { | 1st Case the Feet to be digged was sought,                                      | } | 2041 $\frac{1}{2}$ Feet. |
|        |   | and found                                                                       |   | 9800 Men.                |
|        |   | 2d — the Number of Men to dig the larger Trench in the lesser Time, and found   |   | 10 Hours.                |
|        | { | 3d — the Time in which the larger Number could dig the larger Trench, and found | } |                          |

For Proof of the 2d Variety or Kind of Proportion by 5 Numbers: The Product made of the 1st, 2d, and 4th, is equal to that of the 3d, 5th, and 6th.

And for Proof of the 3d Variety: The Product under the 3d, 4th, and 6th, is equal to that of the 1st, 2d, and 5th.

## VIII. Of Duplicate Proportion Direct.

This differs from the single Rule of Proportion Direct, in this: 2 of the 3 (the 1st and 3d) requiring to be squared, when that sought (or the 4th Number) is not a Square, and 1, viz. the middle or 2d Number, when that sought is a square Number (or one whose Root must be extracted to give the specifick Answer).

Or else it is when the Ratio is as the Square of one thing is to that of another, &c.

*Examples.*

The Diameter of a Circle being 2, the Area is  $3\frac{1}{4}$ : therefore

What is the Area or superficial Content of a Circle, whose Diameter is 8?

|                       |                |                       |                 |
|-----------------------|----------------|-----------------------|-----------------|
| The Sqr. of the Diam. | The Area.      | The Sqr. of the Diam. | The Area.       |
| 4                     | $3\frac{1}{4}$ | 64                    | $50\frac{1}{4}$ |

*The Rule for stating the Question* is the same as for the single Rule Direct: And the Diameters being squared as you see (4 being the Square of 2, and 64 of 8) *the Rule for performing the Operation* is the same as for single Direct Proportion; for  $3\frac{1}{4}$  multiplied by 64, and that Product divided by 4, the Quotient is  $50\frac{1}{4}$  for Answer. For

*Euclid* has demonstrated, that Circles are in proportion as the Squares of their Diameters; and accordingly, by this Proportion above we find the Area of a Circle  $50\frac{1}{4}$ , whose Diameter is 8.

*Example 2.*] What is the Diameter of a Circle, whose Area is  $50\frac{1}{4}$ ? supposing the Diameter of Circle, whose Area is  $3\frac{1}{4}$ , be 2, (as it really is.)

This is stated as per Margin, and the Operation done as before. For

|                |                       |                 |                       |
|----------------|-----------------------|-----------------|-----------------------|
| The Area.      | The Sqr. of the Diam. | The Area.       | The Sqr. of the Diam. |
| $3\frac{1}{4}$ | 4                     | $50\frac{1}{4}$ | 64                    |

As  $3\frac{1}{4}$  (the Area of a Circle)

Is to 4 (the Square of that Circle's Diameter:)

So is  $50\frac{1}{4}$  (or the Area of any other Circle)

To (64) the Square of the Diameter of that Circle.

Now extract the Square Root of 64, and you have the Answer, viz. 8.

*This and the last Example prove each other.*



## IX. Duplicate Proportion Reverse.

*Example.]* Admit the Pendulum of our common Clocks is 39 Inches; (as it is very little more) we know that such a Clock vibrateth Seconds (or 60 times in a Minute). Now what is the Length of a Pendulum that vibrateth Half-Seconds (or 120 times in a Minute)?

Questions under this Rule are most naturally stated as in single Reverse Proportion above; and being so stated, must be so wrought after the Vibrations are squared.

By this State it appears

|             |         |         |                |
|-------------|---------|---------|----------------|
| Vibrations. | Inches. | Vibrat. | Inches.        |
| 60.         | 39.     | 120.    | $9\frac{1}{2}$ |

that the Proportion is Reverse: for if 60 Vibrations

require a Pendulum 39 Inches long, 'tis plain that a Clock, whose Vibrations are 120 in a Minute, must have its Pendulum shorter (for the longer the Pendulum, the fewer the Vibrations in a Minute; and the contrary): therefore more requiring less, shews the greater Extreme (or 120) must be your Divisor, as appears by what is said under the first Example of single Reverse Proportion. So that multiplying the Square of 60 = 3600 by 39, and dividing the Product by the Square of 120 = 14400, the Quotient or Answer is  $9\frac{1}{2}$  Inches.

*Example 2.]* A Pendulum of 39 Inches vibrating 60 times in a Minute; how many times does that Pendulum vibrate in a Minute, whose Length is  $9\frac{1}{2}$ ?

Here, by multiplying the two first towards the left hand together, viz.

|         |                         |                |                         |
|---------|-------------------------|----------------|-------------------------|
| Inches. | The Sqr. of the Vibrat. | Inches.        | The Sqr. of the Vibrat. |
| 39.     | 3600.                   | $9\frac{1}{2}$ | 14400                   |

39 and 3600 = (the Square of 60) and dividing the Product by  $9\frac{1}{2}$  (as taught in Division of Vulgar Fractions) the Quotient is 14400, the Square of the Answer: therefore extract the Square Root of 14400, and you'll find it 120 = the Number of Vibrations in a Minute of a Pendulum whose Length is  $9\frac{1}{2}$  Inches.

*Example 3.]* Admit a Body on the Surface of the Earth (or 4000 Miles distant from the Center thereof) weigh 20 Hundred weight, what will that Body weigh if it were 12000 miles above the Surface, or 16000 from the Center of it? State your Question thus;

|        |        |        |
|--------|--------|--------|
| Miles. | C. wt. | Miles. |
| 4000   | 20     | 16000  |

Multiply

Multiply the Square of the 1st by the 2d, and divide the Product by the Square of the 3d, and the Quote is  $1\frac{1}{4}$  C. Which shews, that a Ton weight, if 12000 miles high, will weigh but  $1\frac{1}{4}$  C. and if 189314 miles high, 'twill weigh but a Pound, by the same Rule. So likewise the Velocity of Swiftneſs of Deſcent is alſo in proportion to a heavy Body's diſtance from the Center of the Earth in a duplicate Ratio: for the Celerity of the Fall on the Earth (or 4000 miles from the Center) being 16 foot in a Second; if the ſame Body be 16000 miles from the Center, it will fall but 1 foot in a Second, as you may eaſily prove by this preſent Rule, as aboveſaid.

I have given an Example under the ſingle Rule of Proportion Reverse, how theſe laſt, &c. may be performed as in Direct Proportion, by ſaying (in the 2d Example)

As  $9\frac{1}{4}$  to 39 :: ſo 3600. to 14400, whoſe Root is 120.

But then this neither agrees, as to ſtating the Queſtion, with the Rules given for that purpoſe under the ſingle Rules Direct nor Reverse, and conſequently theſe Queſtions are moſt properly and naturally to be performed as directed above, as agreeing with the Rules given for ſtating and working ſuch Queſtions.

#### X. Triplicate Proportion.

As in Duplicate Proportion the Ratio is as the Square of one Number is to another Number, or the contrary; ſo this kind, the Ratio is as one Number is to the Cube of another, or as the Cube of one Number is to ſome other Number.

*Example 1.]* If a Sphere (or Ball) which is 8 Inches Diameter, weighs 48 Pounds; what will another Sphere of the ſame Specie of Matter weigh, whoſe Diameter is 4 Inches?

This is a ſingle direct Proportion, (working with the Cubes of the Diameters) where the 4th Proportional is found lb 6. For

|                             |       |                             |
|-----------------------------|-------|-----------------------------|
| The Cube of<br>the Diam. 8. | It is | The Cube of<br>the Diam. 4. |
| 512.                        | 48 :: | 64.                         |

As 512 = (the Cube of the Diameter 8) is to 48 Pounds weight, So is 64 (the Cube of the Diameter 4) to 6 Pounds, the Weight required.

*Example 2.]* If a Sphere weighing 48 lb. be 8 Inches Diameter, what is the Diameter of another Sphere of the ſame Matter, whoſe Weight is 6 lb?

Here

Here by multiplying the 2d  $\frac{1}{2}$  the Weight. The Cube of  $\frac{1}{2}$  the and 3d Numbers together, and the Diam. 8. Weight. dividing the Product by the 1st; 48. 512 :: 6. the 4th Proportional is 64, the Cube Root whereof is 4 = the Answer. For

As 48 Pounds weight is to 512 (the Cube of the Diameter 8)

So is 6 Pounds weight to 64 (the Cube of the Diameter sought) whose Root is 4.

And as the Weight, so the Solidity of Spheres is found by the same Ratio.

*Example 3.]* Ad- The Cube of The Soli- The Cube of The Soli-  
mit the solid Con- the Diam. 2. dity. the Diam. 4. dity.  
tent of a Sphere = 8  $4\frac{1}{1000} :: 64$   $33\frac{1}{1000}$   
be  $4\frac{1}{1000}$ , whose

Diameter is 2; what is the Solidity of another Sphere, whose Diameter is 4?

Here you see that As the Cube of the Diameter 2, viz. 8.

Is to the Solidity of that Sphere  $4\frac{1}{1000}$ :

So is the Cube of the Diameter 4, viz. 64.

To the solid Content of that Sphere, viz.  $33\frac{1}{1000}$ .

*Example 4.]* The Diameter of a Sphere being 2, whose Solidity is  $4\frac{1}{1000}$ ; what is the Diameter of another Sphere, whose Solidity is  $33\frac{1}{1000}$ ? Answer 4; the Cube Root of 64: for as  $4\frac{1}{1000}$ . 8 ::  $33\frac{1}{1000}$ . 64.

### XI. Harmonical Proportion.

In this kind of Proportion there are 3 Numbers given to find a 4th, which shall bear such direct Proportion to the first, as the Difference between the 3d and 4th bears to the Difference between the 1st and 2d.

It is called Harmonical (or Musical Proportion) probably by reason of the double Ratio of the Numbers among themselves.

The Rule to find the 4th Proportional, is, Multiply the 1st and 3d together, and divide the Product by double the 1st, less the 2d; and the Quotient is the 4th Number required.

*Example.]* What is the 4th Proportional Harmonical to these 3 Numbers 7 : 10 : 12? Answer 21.



See the Work in the Margin.

For Proof of this, as abovesaid :

$$21. 7 :: 9. 3$$

That is, as 21, the 4th Number (or that required in the Question) is to 7 (the 1st Number) so is the Difference between 12 and 21, viz. 9. to the Difference between the 1st and 2d, or 7 and 10, viz. 3.

Hence as a Corollary it will appear, that double the 1st Number must exceed the 2d Number, otherways you'll want a Divisor, and cannot find the requisite Number without the Arithmetic of Negatives.

XII. Sefquiplicate Proportion.

In this kind of Ratio the Square of one Number is in proportion to the Cube of another, and the contrary.

It is used in finding the Distance of the Planets in the Solar System from the Sun their Center, about which they revolve: For

As the Square of the Time in which any Planet finisheth its Periodical Revolution,

Is to the Cube of its Distance from the Sun ::

So is the Square of any other Planet's Time

To the Cube of its Distance. Thus

|                        | Days.               | And its Distance  | Miles.    |
|------------------------|---------------------|-------------------|-----------|
| The Earth finishes its | } 365 $\frac{1}{4}$ | } from the Sun is | 86000000  |
| Period in — —          |                     |                   |           |
| Mars — — —             | 625                 | — — — about       | 130000000 |
| Jupiter — — —          | 4333                | — — — — —         | 440000000 |
| &c.                    |                     |                   |           |

SECT. IV. Rules of Practice.

**T**HIS is a briefer Method of casting up any Quantity of Merchandize than the Rule (commonly used) of Proportion or Multiplication. And 'tis done by considering what even Parts of a Pound the given Price is, or is reducible to, and then framing a Rule accordingly.

It

It is called the Rule of Practice because of its excellent Use in the Practice of Merchandize, for dispatching many Computations with much ease and in a short time.

I shall not trouble the Reader with shewing him a Table of the even Parts of a Shilling or a Pound, by reason I have given an intire Table, with the Fabrick thereof, at the end of Division, and have reduced the whole Business of Practice into Rules arising gradually in the Value of the Integer in so regular and copious a Method, that any one may easily find his Rate or Value of the Yard, Ell, Pound, Ounce, &c. and right against it a Rule how to perform the Operation after the best and most concise manner. And he will generally know which are the even Parts of a Pound by the Shortness of the Rule, being only one Division required to bring the given Number into Pounds respectively. I have marked the Aliquot Parts of a Pound with *A. P. L.* in the Table: And I think I may affirm, that this most copious, regular, and short Way of Practice, was first (and only) shewn by myself, where I take no notice of the Parts of a Shilling, but of a Pound only.

The main thing that makes the Rules of Practice preferable, is the performing the Operation mentally, without putting down any Figures but the Quotients or Answers for the most part: And this is done by chusing proper Divisors, few or none of which exceed 12; to which Number inclusive my Multiplication-Table foregoing extends, and is supposed to be perfectly in the Reader's memory.

But before I proceed to the Tables, it will be necessary to give these previous Cases.

*Case 1.] When the Value of the Integer is 2 s. you have the Answer by only cutting off the Units place (or dividing by 10) 2 s. being the 10th part of a Pound; and the Figure in Units place is so many two Shillings. Thus 34765 Ells at 2 s. each is l. 3476 : 10; those*

*to the left hand Units place being so many Pounds. So also 9734 at 2 s. is l. 973 : 8 : —; and 875 is l. 87 : 10 : —*

*Case 2.] From the last Case it will follow, that if the Price is any even Number of Shillings for the Unit, the whole Parcel will be half so many Shillings as the Value of the Unit is, multiplied in what such Parcel would amount to at 2 s. Thus 7876 at 18 s. is l. 7876 at*

*2 s, therefore it is 9 times that at 18 s. So multiply 7876 by 9 thus,*

9 times

9 times 6 (cut off) is 54 two Shillings, put down 8 s. and carry 1.5; then 9 times 7 l. is 63, and 5 carried is 68; and so proceed to multiply the 78, and you will find the Answer to be l. 7088 : 8 : 0. And by the same Rule 19468 at 16 s. is l. 15574 : 8 : 0. I say 8 times 8 is 64, twice the 4 is 8 s. and carry 1.6, &c. Likewise 97357 at 14 s. is l. 68149 : 18 : 00, &c.

Case 3.] When the Value of the Integer or Unit is 1 s. what is the Value of 135694?

135694

1

You must take a 20th part, cutting off Units place, and taking half what rests to the left hand, as half 13 is 6,  $\frac{1}{2}$  of 15 is 7,  $\frac{1}{2}$  of 16 is 8, and  $\frac{1}{2}$  of 9 is 4, and the 10 s. remaining, and the 4 cut off is 14 s. because the Value at 1 s. per Unit is half as much as at 2 s.

Answer l. 6784 : 14 : 00

Case 4.] When the Price of the Unit is 6 d. take a 4th of the given Number, which suppose 17372, cut off the 2 from 17372, and take a 4th of the rest: So the Answer at one Work is l. 434 : 6 s. for 12 Sixpences remains. For 4 in 17 is 4, 4 in 13 is 3, and 4 in 17 is 4, the 434 l.

Case 4.] When the Price of the Unit is 4 d. and you are directed to take a 60th part of the given Number, which suppose — — — — —

5567

Here the Units place being cut off, I take a 6th of what is to the left hand the dash, thus 6 in 55 is 9 and rests 1; 6 in 16 is 2, and there rests 4, which with the 7 is 47 Groats, or 15 s. 8 d. the Answer.

 $\frac{1}{60}$  is l. 92 : 15 : 8

Case 6.] When the Value of the Unit is 3 d. and you are directed in the subsequent Rules to take  $\frac{1}{60}$  of the given Number, which suppose 7539; cut off the Units place, and take  $\frac{1}{8}$  of the remaining Figures, as 8 in 75 is 9, 8 in 33 is 4, and 19 Three-pences (or 4 s. 9 d.) over.

7539

1

 $\frac{1}{60}$  is l. 94 : 4 : 9

Case 7.] When the Price of the Unit is 2 d. and you are directed to take  $\frac{1}{120}$  of the given Number, which suppose 19739; cut off the 9 in Units place, and take  $\frac{1}{2}$  of the rest: so 12 in 19 is 1, 12 in 77 is 6, 12 in 53 is 4, and 59 Two-pences rests, or 9 s. 10 d. And that the Remainders in these

19739

1

 $\frac{1}{120} =$  l. 164 : 9 : 10



four last Cases may not, when large, seem difficult to cast up, I shall subjoin this little *Table of Remainers* in Shillings and Pence.

| Remainers. | Two-pences. | Three-pences. | Four-pences. | Six-pences. |
|------------|-------------|---------------|--------------|-------------|
| 10         | 1 s. 8 d.   | 2 s. 6 d.     | 3 s. 4 d.    | 5 s. 0 d.   |
| 20         | 3 4         | 5 0           | 6 8          | 10 0        |
| 30         | 5 0         | 7 6           | 10 0         | 15 0        |
| 40         | 6 8         | 10 0          | 13 4         |             |
| 50         | 8 4         | 12 6          | 16 8         |             |
| 60         | 10 0        | 15 0          |              |             |
| 70         | 11 8        | 17 6          |              |             |
| 80         | 13 4        |               |              |             |
| 90         | 15 0        |               |              |             |
| 100        | 16 8        |               |              |             |
| 110        | 18 4        |               |              |             |

Thus by this Table the 47 Groats which remained in the 5th Case above, is 15 s. 8 d. for 40 under Four-pences is 13 s. 4 d. and 7 Groats (or 2 s. 4 d.) is 15 s. 8 d.

*Note 1.* That the Remainers are of the same Denomination with the Dividends: As in the Example to the 5th Rule I suppose in the first Operation the Dividend Two-pences, and therefore dividing by 120, I find 47 Two-pences remaining, or 7 s. 10 d. And what remains after in taking any part of the Quotients, are Pounds, Shillings, or Pence, as those Quotients are. The Cases above, and the Rules and Examples following, make this very evident.

The

# SECT. IV.

# Rules of Practice.

99

The Value of Here followeth 149 Rules (with Examples for such the Pound, as seem the most difficult) for the brief working Number Ell, Yard, by Practice to give the Answer in Pounds, &c. of &c. the like not extant by any other Hand. Rules.

|          |        |                                                                                                    |    |
|----------|--------|----------------------------------------------------------------------------------------------------|----|
| A. P. L. | 1      | Take $\frac{1}{80}$ of $\frac{1}{12}$ of the given Number (or $\frac{1}{960}$ thereof.)            | 1  |
| A. P. L. | 2      | Take an 80th of a 6th of the given Number — — —                                                    | 2  |
| A. P. L. | 3      | Take $\frac{1}{80}$ of $\frac{1}{4}$ of the given Number (or $\frac{1}{320}$ thereof) — — —        | 3  |
| A. P. L. | 00:1:0 | Take a 20th of a 12th of the given Number, or take an 120th and deduct $\frac{1}{2}$ thereof — — — | 4  |
| A. P. L. | 1:1    | To $\frac{1}{2}$ of 120 of the given Number add a 4th of that $\frac{1}{2}$ — — —                  | 5  |
| A. P. L. | 1:2    | Take a 20th of an 8th of the given Number, or $\frac{1}{160}$ — — —                                | 6  |
|          | 1:3    | From 120th of the given Number take an 8th of that 120th — — —                                     | 7  |
| A. P. L. | 2:0    | Take an 120th of the given Number — — —                                                            | 8  |
|          | 2:1    | To an 120th of the given Number add an 8th of that 120th — — —                                     | 9  |
| A. P. L. | 2:2    | To an 120th of the given Number add $\frac{1}{4}$ of that 120th, or take $\frac{1}{90}$ — — —      | 10 |
|          | 2:3    | From an 80th of the given Number take a 12th of that 80th — — —                                    | 11 |
| A. P. L. | 3:0    | Take an 80th of the given Number, as per Case 6. — — —                                             | 12 |
|          | 3:1    | To an 80th of the given Number add a 12th of that 80 — — —                                         | 13 |
|          | 3:2    | To an 80th of the given Number add a 6th of that 80 — — —                                          | 14 |
| A. P. L. | 3:3    | To an 80th of the given Number add a 4th of that 80th, (or take a 64th) — — —                      | 15 |
| A. P. L. | 4:0    | Take a 60th of the given Number, as per previous Case 5. — — —                                     | 16 |
|          | 4:1    | To an 120th of the given Number twice put down, add an 8th of 120th — — —                          | 17 |
|          | 4:2    | To a 60th of the given Number add an 8th of that 60 — — —                                          | 18 |
|          | 4:3    | To a 60th of the given Number add an 8th of that 60th and $\frac{1}{2}$ that 8th — — —             | 19 |
| A. P. L. | 5:0    | Take an 8th of a 6th of the given Number or take $\frac{1}{48}$ — — —                              | 20 |
|          | 5:1    | From a 40th of the given Numb. take an 8th of that 40 — — —                                        | 21 |
|          | 5:2    | From a 40th of the given Number take a 12th of that 40th — — —                                     | 22 |
|          | 5:3    | From an 80th of the given Number twice put down, take a 12th of an 80th — — —                      | 23 |
| A. P. L. | 6:0    | Take a 40th of the given Number, as per previous Case 4. — — —                                     | 24 |

| Example to Rule 5.<br>19727 at 1d. 1 q.          | Example to Rule 17.<br>9376 at 4d. 1 q.      | Example to Rule 23.<br>8935 at 5d. 3 q.                  |
|--------------------------------------------------|----------------------------------------------|----------------------------------------------------------|
| $\frac{1}{120} = L. 164: 7: 10$                  | $\frac{1}{120} = L. 78: 2: 8$                | $\frac{1}{80} = L. 111: 13: 9$                           |
| $\frac{1}{2}$ of that $L. 82: 3: 11$             | ditto $L. 78: 2: 8$                          | ditto $L. 111: 13: 9$                                    |
| $\frac{1}{4}$ of that $L. 20: 10: 11\frac{1}{4}$ | $\frac{1}{8}$ of $\frac{1}{120} L. 9: 15: 4$ | Sum, $L. 223: 7: 6$                                      |
| Sum, $L. 102: 14: 10\frac{1}{4}$                 | Sum, $166: 00: 8$ Ans.                       | $\frac{1}{2}$ of $\frac{1}{80} = 9: 6: 1\frac{1}{4}$ ded |
| the Answer.                                      |                                              | Refts = $214: 1: 4\frac{1}{2}$ Ans.                      |

The Value  
of the  
Pound, Ell,  
Yard, &c.  
d. gr.

The Continuation of the Table of Rules for the Number  
Short Work of Practice, &c. as before. of  
Rules.

|        |        |                                                                                   |    |
|--------|--------|-----------------------------------------------------------------------------------|----|
|        | 6 : 1  | To an 80th of the given Number twice put down, add<br>a 12th of an 80th           | 25 |
|        | 6 : 2  | To a 40th of the given Number add a 12th of that 40th                             | 26 |
|        | 6 : 3  | To a 40th of the given Number add an 8th of that 40th                             | 27 |
|        | 7 : 0  | To a 40th of the given Number add a 6th of that 40th                              | 28 |
|        | 7 : 1  | To a 40th of the given Number add a 6th of that 40th,<br>and a 4th of that 6th    | 29 |
| A.P.L. | 7 : 2  | To a 40th of the given Number, add a 4th of that 40th<br>(or take $\frac{1}{3}$ ) | 30 |
|        | 7 : 3  | To a 40th of the given Number add a 4th of that 40th,<br>and a 6th of that 4th    | 31 |
| A.P.L. | 8 : 0  | Take a 30th of the given Number                                                   | 32 |
|        | 8 : 1  | From an 80th of the given Number put down three<br>times, take the 4th of an 80th | 33 |
|        | 8 : 2  | To a 60th of the given Number, put down twice, add<br>an 8th of a 60th            | 34 |
|        | 8 : 3  | From an 80th of the given Number thrice put down,<br>take a 12th of an 80th       | 35 |
|        | 9 : 0  | To a 40th of the given Number add half of that 40th                               | 36 |
|        | 9 : 1  | To an 8th of the given Number thrice put down, add a<br>12th of an 80th           | 37 |
|        | 9 : 2  | To an 80th of the given Number thrice put down,<br>add a 6th of an 80th           | 38 |
|        | 9 : 3  | To an 80th of the given Number thrice put down, add<br>a 4th of an 80th           | 39 |
| A.P.L. | 10 : 0 | Take an 8th of a 3d of the given Number, (or a 24th<br>part.)                     | 40 |
|        | 10 : 1 | To a 30th of the given Number add a 4th of that 30,<br>and an 8th of that 4th     | 41 |
|        | 10 : 2 | From a 20th of the given Number take an 8th of that 20                            | 42 |
|        | 10 : 3 | To a 40th and a 60th of the given Number add an 8th<br>of that 40th               | 43 |
|        | 11 : 0 | From a 20th of the given Number take a 12th of the 20                             | 44 |
|        | 11 : 1 | From a 40th of the given Number twice put down,<br>take an 8th of a 40th          | 45 |
|        | 11 : 2 | From a 40th of the given Number twice put down,<br>take a 12th of a 40th          | 46 |
|        | 13 : 3 | To a 30th of the given Number add an 80th, and a<br>4th of that 80th.             | 47 |

| Example to Rule 31.                                 | Example to Rule 35.                                     | Example to Rule 45.                                          |
|-----------------------------------------------------|---------------------------------------------------------|--------------------------------------------------------------|
| 7238 at 7 d. 3 q.                                   | 3929 at 9 d. 1 q.                                       | 1895 at 11 d. 1 q.                                           |
| $\frac{1}{40} = 1.180 : 19 : 0$                     | an 80th = 1.49 : 2 : 3                                  | a 40th = 1.47 : 7 : 6                                        |
| $\frac{1}{4}$ of the 40th 1.45 : 4 : 9              | ditto — 49 : 2 : 3                                      | ditto = 47 : 7 : 6                                           |
| $\frac{1}{6}$ of the 4th 1.7 : 10 : 9 $\frac{1}{2}$ | ditto — 49 : 2 : 3                                      | 1.94 : 15 : 0 Sum.                                           |
| Answer, 1.233 : 14 : 6 $\frac{1}{2}$                | $\frac{1}{2}$ of $\frac{1}{80} = 4 : 1 : 10\frac{1}{2}$ | $\frac{1}{8}$ of $\frac{1}{40} = 5 : 18 : 5\frac{1}{2}$ ded. |
|                                                     | Sum, Anf. 151 : 8 : 7 $\frac{1}{2}$                     | rests 1.88 : 16 : 6 $\frac{1}{2}$ Anf.                       |



The Value of  
the Pound,  
Ell, Yard,  
&c.

A Continuation of the Table of Rules for the  
Short Work of Practice.

Number  
of  
Rules.

|                 |       |                                                                                                                 |    |
|-----------------|-------|-----------------------------------------------------------------------------------------------------------------|----|
| <i>A. P. L.</i> | 1:0:0 | Take a 20th of the given Number, (as per previous Case 3.)                                                      | 48 |
|                 | 1:0:1 | To a 40th of the given Number, and a half of that 40th twice put down, add a 12th of one of the $\frac{1}{2}$ . | 49 |
|                 | 1:0:2 | To a 40th of the given Number twice put down, add a 12th of a 40th                                              | 50 |
|                 | 1:0:3 | To a 40th of the given Number twice put down, add an 8th of a 40th                                              | 51 |
|                 | 1:1:0 | To a 20th of the given Number add a 12th of that 20th                                                           | 52 |
|                 | 1:1:1 | To a 20th of the given Number add a 12th of that 20th, and a 4th of the 12th                                    | 53 |
|                 | 1:1:2 | To a 20th of the given Number add an 8th of that 20th                                                           | 54 |
|                 | 1:1:3 | To a 20th of the given Number add an 8th of that 20th, and a 6th of the 8th                                     | 55 |
|                 | 1:2:0 | To a 20th of the given Number add a 6th of that 20th                                                            | 56 |
|                 | 1:2:1 | To a 20th of the given Number add a 6th of that 20th and an 8th of that 6th                                     | 57 |
|                 | 1:2:2 | To a 20th of the given Number add a 6th of that 20th and a 4th of that 6th                                      | 58 |
|                 | 1:2:3 | To a 30th of the given Number add a 40th of it, and an 8th of the 40th                                          | 59 |
| <i>A. P. L.</i> | 1:3:0 | Take half of an 8th of the given Number, (or a 16th)                                                            | 60 |
|                 | 1:3:1 | To a 20th of the given Number add a 4th of the 20th and a 12th of the 4th                                       | 61 |
|                 | 1:3:2 | To a 20th of the given Number add a 4th of that 20th, and a 6th of the 4th                                      | 62 |
|                 | 1:3:3 | To a 20th of the given Number add a 4th of that 20th, and a 4th of that 4th                                     | 63 |
| <i>A. P. L.</i> | 1:4:0 | Take a 30th of the given Number, and double it (or put down twice) or take a 15th of the given Number           | 64 |
|                 | 1:4:1 | To $\frac{1}{2}$ of an 8th of the given Number add a 12th of that $\frac{1}{2}$                                 | 65 |
|                 | 1:4:2 | To a 20th of the given Number add a 3d of that 20th, and an 8th of the 3d                                       | 66 |
|                 | 1:4:3 | To a 40th of the given Number twice put down, add a 60th of the given Number and an 8th of a 40th               | 67 |
|                 | 1:5:0 | From a 12th take an 8th of the given Number                                                                     | 68 |
|                 | 1:5:1 | From a 20th of the given Number added to $\frac{1}{2}$ the 20th, take an 8th of that half                       | 69 |
|                 | 1:5:2 | From a 20th of the given Number added to half that 20, take a 12th of that half                                 | 70 |
|                 | 1:5:3 | From a 20th of the given Number added to a 4th of that 20 twice put down, take a 12th of the said 4th           | 71 |

| Example to Rule 65.                                       | Example to Rule 67.                          | Example to Rule 71.               |
|-----------------------------------------------------------|----------------------------------------------|-----------------------------------|
| 893 at 16 d. $\frac{1}{4}$                                | 1239 at 16 d. $\frac{1}{4}$                  | 952 at 17 d. $\frac{1}{4}$        |
| an 8th = 1.111:12:6                                       | a 40th 1.30:19:6                             | a 20th 1.47:12:00                 |
| $\frac{1}{2}$ an 8th, 1.55:16:3                           | ditto 30:19:6                                | $\frac{1}{4}$ of a 20, 1.11:18:00 |
| $\frac{1}{2}$ of the $\frac{1}{2}$ = 4:13:0 $\frac{1}{4}$ | a 60th = 20:13:0                             | ditto 1.11:18:00                  |
| Answer, 1.60:9:3 $\frac{1}{4}$                            | $\frac{1}{8}$ of a 40th 3:17:5 $\frac{1}{4}$ | Sum, 71:8:00                      |
|                                                           | Sum, 1.86:9:5 $\frac{1}{4}$                  | a 12 of 24 = 19:10 ded.           |
|                                                           |                                              | rests 1.70:8:2 Anf.               |

The Value of  
the Pound,  
Ell, Yard,  
&c.

*A Continuation of the Table of Rules for the  
Short Work of Practice.*

Number  
of  
Rules.

| s.       | d. |                                                     |                                                       |    |
|----------|----|-----------------------------------------------------|-------------------------------------------------------|----|
| 1        | 6  | To a 20th add half that 20th                        | 72                                                    |    |
| 1        | 7  | From a 12th of the given Number take a 20th of that | 73                                                    |    |
|          |    | 12th                                                |                                                       |    |
| A. P. L. | 1  | 8                                                   | Take a 12th of the given Number                       | 74 |
|          | 1  | 9                                                   | To a 12th of the given Number add a 20th of that 12th | 75 |
|          | 1  | 10                                                  | To a 12th of the given Number add a 10th of that      | 76 |
|          |    |                                                     | 12th                                                  |    |
|          | 1  | 11                                                  | Add an 8th to a 12th of the given Number              | 77 |
| A. P. L. | 2  | 00                                                  | Take a 10th of the given Number, as per previous      | 78 |
|          |    |                                                     | Case 1.                                               |    |
|          | 2  | 1                                                   | To a 12th of the given Number add a 4th of that 12th  | 79 |
|          | 2  | 2                                                   | To a 10th of the given Number add a 12th of that 10th | 80 |
|          | 2  | 3                                                   | To a 10th of the given Number add an 8th of that      | 81 |
|          |    |                                                     | 10th                                                  |    |
|          | 2  | 4                                                   | To a 10th of the given Number add a 6th of that       | 82 |
|          |    |                                                     | 10th                                                  |    |
|          | 2  | 5                                                   | To a 12th of the given Number add a 4th of that 12th  | 83 |
| A. P. L. | 2  | 6                                                   | Take an 8th of the given Number                       | 84 |
|          | 2  | 8                                                   | To a 10th of the given Number add a 3d of that 10th   | 85 |
|          | 2  | 9                                                   | To an 8th of the given Number add a 10th of that      | 86 |
|          |    |                                                     | 8th                                                   |    |
|          | 3  | 0                                                   | To 10th of the given Number, add half that 10th       | 87 |
|          | 3  | 3                                                   | From a 6th of the given Number take a 40th of that    | 88 |
|          |    |                                                     | 6th                                                   |    |
| A. P. L. | 3  | 4                                                   | Take a 6th of the given Number                        | 89 |
|          | 3  | 6                                                   | To a 6th of the given Number add a 20th of that       | 90 |
|          |    |                                                     | 6th                                                   |    |
|          | 3  | 8                                                   | To a 6th of the given Number add a 10th of that 6th   | 91 |
|          | 3  | 9                                                   | To a 6th of the given Number add an 8th of that 6th   | 92 |
| A. P. L. | 4  | 0                                                   | Take a 5th of the given Number, (or shorter) take 2   | 93 |
|          |    |                                                     | Tenths of the given Number                            |    |
|          | 4  | 3                                                   | To a 10th of the given Number twice put down, add     | 94 |
|          |    |                                                     | an 8th of a 10th                                      |    |
|          | 4  | 4                                                   | To a 10th twice put down, add a 6th of a 10th         | 95 |

| Example to Rule 73.     | Example to Rule 88.                       | Example to Rule 94.                |
|-------------------------|-------------------------------------------|------------------------------------|
| 1047 at 1 s. 7 d.       | 895 at 3 s. 3 d.                          | 739 at 4 s. 3 d.                   |
| a 12th = 1.87 : 5 : 0   | a 6th = 1.49 : 3 : 4                      | a 10th = 1.73 : 18 : 00            |
| a 20 of a 12, 4 : 7 : 3 | a 40th of $\frac{1}{6}$ 1.3 : 14 : 7 ded. | ditto 1.73 : 18 : 00               |
| Resteth 1.82 : 17 : 9   | Resteth 1.145 : 8 : 9                     | $\frac{1}{8}$ of a 10th, 9 : 4 : 9 |
| The Answer.             | the Answer.                               | Sum = 157 : 00 : 9 Anf.            |

|          | The Value<br>of the<br>Pound, Ell,<br>Yard, &c. | s. | d. | A Continuation of Rules for the short Work<br>of Practice.                                                 | Number<br>of<br>Rules. |
|----------|-------------------------------------------------|----|----|------------------------------------------------------------------------------------------------------------|------------------------|
|          |                                                 |    |    |                                                                                                            |                        |
|          |                                                 | 4  | 6  | To a 10th add an 8th of the given Number                                                                   | 96                     |
|          |                                                 | 4  | 8  | To a 10th of the given Number twice put down, add<br>a 3d of a 10th                                        | 97                     |
|          |                                                 | 4  | 9  | From a 4th of the given Number take a 20th of that 4th                                                     | 98                     |
| A. P. L. |                                                 | 5  | 0  | Take a 4th of the given Number                                                                             | 99                     |
|          |                                                 | 5  | 6  | To a 4th of the given Number add a 10th of that 4th                                                        | 100                    |
|          |                                                 | 6  | 0  | Take 3 Tenths of the given Number, as <i>per</i> previous<br>Case 2.                                       | 101                    |
| A. P. L. |                                                 | 6  | 8  | Take a 3d of the given Number                                                                              | 102                    |
|          |                                                 | 7  | 0  | Work for 6s. and 1s. as <i>per</i> previous Case 2, and 3                                                  | 103                    |
|          |                                                 | 7  | 6  | Take an 8th of the given Number, and put it down<br>thrice, or multiply by 3.                              | 104                    |
|          |                                                 | 8  | 0  | Take 4 Tenths of the given Number, as <i>per</i> previous<br>Case 2.                                       | 105                    |
|          |                                                 | 8  | 6  | To 4 Tenths add a 40th of the given Number                                                                 | 106                    |
|          |                                                 | 9  | 0  | To 4 Tenths of the given Number add a 20th thereof                                                         | 107                    |
|          |                                                 | 9  | 6  | From half the given Number take a 20th of that half                                                        | 108                    |
| A. P. L. |                                                 | 10 | 0  | Take half the given Number                                                                                 | 109                    |
|          |                                                 | 10 | 6  | To half the given Number add a 20th of that half                                                           | 110                    |
|          |                                                 | 11 | 0  | To half the given Number add a 10th of that half                                                           | 111                    |
|          |                                                 | 11 | 6  | From 6 Tenths take a 40th of the given Number                                                              | 112                    |
|          |                                                 | 12 | 0  | Take 6 10ths of the given Number, as <i>per</i> previous<br>Case 2.                                        | 113                    |
|          |                                                 | 12 | 6  | To half the given Number add a 4th of that half                                                            | 114                    |
|          |                                                 | 13 | 0  | Take 6 Tenths of the given Number, and add to a 20th<br>thereof                                            | 115                    |
|          |                                                 | 13 | 4  | To half add a 6th of the given Number                                                                      | 116                    |
|          |                                                 | 14 | 0  | Take 7 Tenths of the given Number, as <i>per</i> previous<br>Case 2.                                       | 117                    |
|          |                                                 | 14 | 6  | To 7 Tenths add a 40th of the given Number                                                                 | 118                    |
|          |                                                 | 15 | 0  | To $\frac{1}{2}$ the given Number add half of that half, (or from<br>the given Number take $\frac{1}{4}$ ) | 119                    |

| Example to Rule 104.                 | Example to Rule 112.          | Example to Rule 115.      |
|--------------------------------------|-------------------------------|---------------------------|
| 559 at 7s. 6d.                       | 365 at 11s. 6d.               | 169 at 13s.               |
| an 8th = £.69 : 17 : 6               | 6,10ths = 219 : as p. Caf. 2. | 6,10ths 101 : 8 : 00      |
| Multiply by - - - 3                  | a 40th = £.9 : 2 : 6 ded.     | a 20th = £.8 : 9 : 00     |
| Prod. = £.209 : 12 : 6<br>or Answer. | rests = £.209 : 17 : 6 Anf.   | Sum, £.109 : 17 : 00 Anf. |

Note, That A. P. L. signifies the Aliquot Parts of a Pound.



The Value  
of the  
Pound, Ell,  
Yard, &c.

*A Continuation of Rules for the short Work of  
Practice.*

Number  
of  
Rules.

| s. | d. |                                                                         |     |  |
|----|----|-------------------------------------------------------------------------|-----|--|
| 15 | 6  | From 8 Tenths (as per previous Case 2.) take a 40th of the given Number | 120 |  |
| 16 | 0  | Take 8 Tenths of the given Number                                       | 121 |  |
| 16 | 6  | Take 8 Tenths add a 40th of the given Number                            | 122 |  |
| 17 | 0  | To 8 Tenths add a 20th of the given Number                              | 123 |  |
| 17 | 6  | From the given Number take an 8th thereof                               | 124 |  |
| 18 | 0  | Take 9 Tenths of the given Number                                       | 125 |  |
| 18 | 6  | To 9 Tenths add a 40th of the given Number                              | 126 |  |
| 19 | 0  | From the given Number take a 20th thereof                               | 127 |  |
| 19 | 6  | From the given Number take a 40th thereof                               | 128 |  |

*Note.* That where Pounds, Shillings, &c. are the Price of a Unit; you must multiply the Units given by the Pounds, and work for the Shillings, &c. as before directed. See the 2d Case of the next Section.

per Cent. or 100. The following shew how Interest, Commission to Factors, Customs, &c. is cast up by the short Rules of Practice.

|                 |                                                                                 |     |
|-----------------|---------------------------------------------------------------------------------|-----|
| 1 per Cent.     | Take the 100th part of the given Number, (as per Rule 3. Prop. 3. in Division.) | 129 |
| 2               | Take a 50th of the given Number                                                 | 130 |
| 2 $\frac{1}{2}$ | Take a 40th of the given Number                                                 | 131 |
| 3               | To a 50th of the given Number add half that 50th                                | 132 |
| 4               | Put down a 50th of the given Number twice                                       | 133 |
| 4 $\frac{1}{2}$ | To a 50th add a 40th of the given Number                                        | 134 |
| 5               | Take a 20th of the given Number                                                 | 135 |
| 6               | Take a 50th of the given Number, and put it down thrice (or multiply it by 3.)  | 136 |
| 7               | Add a 20th to a 50th of the given Number                                        | 137 |
| 8               | Multiply a 50th of the given Number by 4.                                       | 138 |
| 9               | From a 10th of the given Number, take a 10th of the 10th                        | 139 |
| 10 per Cent.    | Take a 10th of the given Number                                                 | 140 |

*Eight Examples to the Rules for finding Interest, &c. by Practice, viz.*

|                                                         |                                                              |                                                |
|---------------------------------------------------------|--------------------------------------------------------------|------------------------------------------------|
| What is £. 1341 : 16 : 3 at 1 per C.                    | What £. 2132 : 12 : 6 $\frac{1}{2}$ at 2 $\frac{1}{2}$ p. C. | What 3235 : 17 : 11 $\frac{1}{4}$ at 5 per C.  |
| 100 part is £. 13 : 8 : 4 $\frac{1}{4}$ Answ.           | a 40th is £. 53 : 6 : 3 $\frac{3}{4}$ Answer.                | a 20th is £. 161 : 15 : 10 $\frac{1}{4}$ Answ. |
| What is £. 879 : 10 : 7 at 6 per C.                     | What £. 933 : 8 : 10 at 7 per C.                             | What 671 : 00 : 2 $\frac{1}{2}$ at 8 per Cent. |
| a 50th = £. 17 : 11 : 3                                 | a 20th £. 46 : 13 : 5 $\frac{1}{4}$ } Add.                   | a 50th £. 13 : 8 : 4 $\frac{3}{8}$             |
| Multiply by - - - - 3                                   | a 50th £. 18 : 13 : 4 $\frac{1}{2}$ }                        | which multiply by - 4                          |
| Produceth 52 : 15 : 5 $\frac{1}{4}$ Answer.             | Sum, £. 65 : 6 : 9 $\frac{1}{2}$ Answer.                     | Prod. £. 53 : 13 : 7 $\frac{1}{4}$ Answer.     |
| What is £. 472 : 14 : 5 $\frac{1}{2}$ at 9 per Cent.    | What £. 1976 : 19 : 11 $\frac{1}{4}$ at 10 per Cent.         |                                                |
| a 10th is = £. 47 : 5 : 5 $\frac{1}{4}$                 | a 10th is £. 197 : 13 : 10 $\frac{1}{4}$ Answer.             |                                                |
| a 10th of that 10th £. 4 : 14 : 6 $\frac{1}{2}$ deduct. |                                                              |                                                |
| resteth = £. 42 : 10 : 10 $\frac{1}{2}$ the Answer.     |                                                              |                                                |

See more of computing Interest, in the Use of Decimals, Chap. III.

I shall next very briefly shew the Use of Rules of Practice in making Allowance for Tare; which is an Allowance for the Bag, or whatsoever a Commodity is pack'd up in, for some things more, some less. The Weight of the Goods, and what contains them, is together called the Gross Weight, and the Weight of the Goods alone is the nett Weight.

| Goods.                                                                    | Allowance per the 112 lb. | No. of Rules. | Rules for the short Working.                                                                                                                |
|---------------------------------------------------------------------------|---------------------------|---------------|---------------------------------------------------------------------------------------------------------------------------------------------|
| Curran — — — — lb                                                         | 16                        | 141           | Take a 7th of the gross Weight.                                                                                                             |
| Almonds, Steel, Hemp —                                                    | 14                        | 142           | Take an 8th of the gross Wt.                                                                                                                |
| Allum, Salt-petre, Tallow, —                                              | 12                        | 143           | From an 8th take a 7th of that 8th.                                                                                                         |
| Brimstone, Copperas, }<br>Copper, — — — — }                               | 8                         | 144           | Take half of a 7th of the gross Weight.                                                                                                     |
| Iron Wire, Latten Wire —                                                  | 6                         | 145           | Take half of what it comes to at 12, as above.                                                                                              |
| Cotton - Wool, Lambs-<br>Wool, and Polish-Wool, }<br>Feathers, Hops — — } | 4                         | 146           | Take a 4th of a 7th of the gross Weight.                                                                                                    |
| Cotton-Yarn 5 lb at 100 weight.                                           |                           | 147           | Take a 20th, or half a 10th of the gross Weight.                                                                                            |
|                                                                           |                           | 148           | A General Rule (especially for such Rates of Tare as are most remote from being Aliquot Parts of 112 lb; such as 3, 5, 9, 11, &c.) is this. |

Example to Rule 143.

|                           |                                                      |
|---------------------------|------------------------------------------------------|
| C. gr. lb.                |                                                      |
| Gross                     | 29 : 3 : 24 at 12 per C.                             |
| $\frac{1}{8}$ is ...      | 3 : 2 : 27 $\frac{1}{2}$                             |
| a 7 <sup>th</sup> of that | 8,0 : 2 : 3 $\frac{1}{4}$ ded. from the 8th.         |
| Resteth Tare              | 3 : 0 : 23 $\frac{1}{4}$ = Answ.                     |
| or the                    |                                                      |
| Nett Wt.                  | 26 : 3 : 0 $\frac{1}{4}$ ; the Gross, less the Tare. |

Multiply the gross Hundreds by the Pounds to be allowed for 1 C. and for the Quarters of Hundreds and Pounds take a Part proportionable of the Allowance per Hundred, and the Sum is an easy way of giving the Answer.

P

Example

|                                             |                                       |                                                                          |                                     |
|---------------------------------------------|---------------------------------------|--------------------------------------------------------------------------|-------------------------------------|
| <i>Example to Rule 145.</i>                 |                                       | <i>The Example to Rule 143. done according to this General Rule 148.</i> |                                     |
| C. q. lb.                                   |                                       |                                                                          |                                     |
| Gross                                       | 37 : 2 : 12 at 12 lb per C.           |                                                                          |                                     |
| $\frac{1}{8}$ is . . . .                    | 4 : 2 : 22                            | Multip. {                                                                | 29 : 3 : 24 at 12 per C. or         |
| a 7 <sup>th</sup> of that 8 <sup>th</sup> , | 0 : 2 : 19 deduct from                |                                                                          | 112 lb.                             |
|                                             | the 8th.                              | Prod. =                                                                  | 348 lb.                             |
| Refts =                                     | 4 : 0 : 3 = the Tare at               | for the 3 q. 9                                                           |                                     |
|                                             | 12 per C.                             | the lb 24, 2 $\frac{1}{2}$                                               |                                     |
| $\frac{1}{2}$ of that is                    | 2 : 0 : 1 $\frac{1}{2}$ the Tare at 6 |                                                                          |                                     |
|                                             | per C. = Ans.                         | lb 359 $\frac{1}{2}$ Sum, or 3 C.                                        |                                     |
| And if ded. } there refts }                 | 35 : 2 : 10 $\frac{1}{2}$ Nett Weight |                                                                          | —qr. 23 $\frac{1}{2}$ lb as before. |

For Oil imported there is allowed 18 *per Cent.* (or 112 lb.) Tare: Now suppose I import 180 Hund. wt. how many Nett Tons and Gallons is there?

*The Proof, viz.*

|                         |              |                              |  |
|-------------------------|--------------|------------------------------|--|
| C. q. lb.               |              |                              |  |
| 180 : 0 : 0             | Gross.       | Multiply { 180 Hundr. Gross. |  |
| 1                       |              | 94 lb per Nett Hun.          |  |
| Ton                     | 9 : 00 Gall. |                              |  |
| Deduct                  | 12           |                              |  |
| Ton } 8 : 240 Gal. Anf. |              |                              |  |
| Nett }                  |              |                              |  |

|                                 |           |                |
|---------------------------------|-----------|----------------|
|                                 | 72        |                |
|                                 | 162       |                |
|                                 | _____     | Gall.          |
| lb per Gal. = 7 $\frac{1}{2}$ ) | 16920 lb. | (2256 (8 Ton.  |
|                                 | 252)      | _____          |
|                                 |           | 240 Gal. rest. |

*Rule 149.]* Take a 20th of the Gross Hundreds, (is here 9 Ton.) For every Ton deduct 1 $\frac{1}{2}$  Gallon; so 9 and  $\frac{1}{2}$  of 9 is 12, and the Remainder is the Answer, as is proved above.

Now 18 *per Cent.* is 12 $\frac{1}{2}$  Gallon *per* Nett C. therefore what Hundreds are in Units place must be multiplied by 12 $\frac{1}{2}$ ; and for the Quarters of C. and Pounds, take a proportionable part of 12 $\frac{1}{2}$ : from the Sum of all which, take the 1 $\frac{1}{2}$  Gallon *per* Ton.

*Another Example* makes this plain; which I am minded to do, because it has not been before.

What Nett Tons and Gallons are in 189 C. 3 q. 21 lb. Gross? See the Operation.

And



|                                  |
|----------------------------------|
| C. 189 : 3 : 21                  |
| 1                                |
| <hr/>                            |
| Ton. Gall.                       |
| 9 : 112½ for the 189 C. } Gross. |
| 9½ for 3 qrs. }                  |
| 2½ for the 21 lb. }              |
| <hr/>                            |
| 9 : 124 Sum, Gross.              |
| 12 = 9 and ⅓ of 9 deduct.        |
| <hr/>                            |
| 9 : 112 refts Nett Answer.       |

And for Proof of this Work by the longer way, do thus:

C. 189 : 3 : 21  
 lb in a Nett C. 94 multiply.

|                      |                           |
|----------------------|---------------------------|
| 756                  |                           |
| 1701                 |                           |
| <hr/>                |                           |
| 17766 lb in 189 C.   |                           |
| 88 lb ad.            | And in 3 qrs. 21 lb.      |
|                      | or 105 lb. gross,         |
|                      | there are Nett            |
| lb 7½ per) 17854 Sum | 88 lb. thus               |
| Gall.                | Gall. 112. 94 :: 105. 88. |
|                      | 252)2380(9 Ton.           |
|                      | Anf.                      |

and 112 lb rest,  
 as in the shorter way.

*The Reason of the short Rule.*

There are two things that seem to some as Difficulties in the shorter Way, viz.

1. Why the Hundreds in Units place, the Quarters and Pounds, are cast up at the Rate of 12½ per Cent.

2. Why 1⅓ is deducted for every Ton.

As to the 1st, you see in the Margin that in a Nett Hundred there are 12½ Gallons, and somewhat more. And

As to the 2d, you may observe from the Work in the Example above, that the Units place of C. the Quarters and Pounds given, are work'd

according to the Nett Measure of 12½ Gall. per Nett C. But in taking a 20th of the Hundreds Gross, (or half the Figures to the left hand the Units place of Hundreds) that half makes more than Nett Tons by 1⅓ Gallon in every Ton, as per Margin above; and that is the reason you deduct 1⅓ in each Ton.

*Note that Tret is an Allowance in Merchandize of l. 4 at 104 for the Dust or Refuse of the Commodity ; and this is deducted out of the Suttle (which is the Remainder when the Tare is deducted).*

SECT. V. Concerning GAIN and LOSS.

**T**HIS is a Rule whereby Merchants or other Traders know (when they have bought any Commodity by wholesale) how they may retale the same out, to make any certain Gain by the whole Parcel, or at any Rate *per Cent.* &c.

Or when any Goods are damaged, they know hereby what they shall lose *per Cent.* or by the whole, in selling the Pound, Ell, Yard, Ounce, &c. at any Rate less than it cost.

I shall place this and the following Sections relating to the Business of Trade, &c. in that Order which I judge most proper for them to be learnt ; not regarding what Method others have observed in treating thereof.

*Case 1.]* When a Merchant buyeth Goods for a certain Sum, to find how the same may be retaled to gain a required Sum by the whole.

*Examp.* A Merchant buys  $\text{th}$  3780 Nett of Cotton-Wool, for 126 l. what may he sell the same for *per th.* to gain l. 15 : 15 : by the Sale ?

*Rule.* Add l. 15 : 15 : — to the Cost, and the Sum is l. 141 : 15 —. Then say, If  $\text{th}$  3780  $\text{th.}$  l. s.  $\text{th.}$  cost me l. 141 : 15 : —, what will 1  $\text{th}$  cost?  $3780 : 142 : 15 :: 1$  Divide the Pence in 141 l. 15 s. by 3780, and the Quotient is 9 d. which it must be sold for *per Pound*, to gain l. 15 : 15 : in all.

*Case 2.]* Admit I bought 15 Bales of Linnen at l. 17 : 15 : *per Bale*, how much must I sell it for *per Bale*, to gain 10 *per Cent.* by the Sale ?

Add  $\left\{ \begin{array}{l} \text{l. } 266 : 5 : 0 \\ \text{l. } 26 : 12 : 6 \end{array} \right.$  Bales 15 at 17 l. 15 s. each.

The Value of the  
Linnen, and 10 *per*  
Cent. — — — } l. 292 : 17 : 6

$\begin{array}{r} 105 \\ 15 \\ \hline \text{l. } 255 \text{ at } 17 \text{ l.} \\ 7 : 10 \text{ at } 10. \\ 3 : 15 \text{ at } 5. \end{array}$

You see, by Practice, the Value of the Bales is l. 266 : 5 : —

Then as 100 to 10 : : so l. 266 : 5 to l. 26 : 12 : 6 ; which is the Advance at 10 *per Cent.* which added as above, makes l. 292 : 17 : 6.

$\begin{array}{r} \text{l. } 266 : 5 \text{ Val. of the Bales.} \\ \text{l. } \text{l. } \text{l. } \text{s. } \text{l.} \\ 100. 10. :: 266 : 5. \text{ 26 } \frac{10}{100} \\ \text{Bales, l. } \text{s. } \text{d. Bale. l. } \text{s. } \text{d.} \\ 15. 292 : 17 : 6 :: 1. 19 : 10 : 6 \\ \text{Then} \end{array}$

Then as 15 Bales to  $l. 292 : 17 : 6 ::$  so is 1 Bale to  $l. 19 : 10 : 6$ ; which is the Answer: for 15 Bales at  $l. 19 : 10 : 6$ , is  $l. 292 : 17 : 6$ , the Cost and Interest proposed.

*Case 3.]* A Merchant bought Broad-Cloth at  $l. 20$  per Piece ready Money, and sold it again for  $l. 25 : 10 : —$  per Piece, to be paid at the end of six Months: What does he gain per Cent. in 12 Months? Answer,  $l. 55$  per Cent.

First say, As 20 the Cost to  $l. 5 : 10 : —$  the Gain ::

So is  $l. 100$  to  $l. 27 : 10 : —$  the Gain in six Months.

Secondly say, As six Months to  $l. 27 : 10 : —$

So is 12 Months to  $l. 55$  the Gain per Cent. per Ann.

*Case 4.]* In Case of Loss you must deduct instead of adding. Thus a Merchant bought 20 Bags, containing 6020  $\text{lb}$  Nett of Cotton for  $l. 301$ ; but being damaged by Sea, he is willing to lose 10 per Cent. What must he sell it for per Pound, to lose that Sum?

1<sup>st</sup>, You see by the first Proportion, that the Loss by the whole will be  $l. 30 : 2 : —$ ; which deducted from the Cost, there remains  $l. 270 : 18$ . therefore as 6020  $\text{lb}$  Cotton is to  $l. 270 : 18 ::$  so is 1  $\text{lb}$  of Cotton to 10  $d. 3\frac{1}{3}$ , the Answer.

|      |        |      |            |       |
|------|--------|------|------------|-------|
| $l.$ | $l.$   | $l.$ | $l.$       | Loss. |
| 100. | 10. :: | 301. | 30 : 2 : — |       |
|      | Deduct | 30   | : 2 : —    |       |

Remains  $l. 270 : 18 : —$

therefore as 6020  $\text{lb}$  Cotton is to  $l. 270 : 18 ::$  so is 1  $\text{lb}$  of Cotton to 10  $d. 3\frac{1}{3}$ , the Answer.

This is sufficient to shew the Arithmetical Work of Loss and Gain: How the Account thereof (or of Profit and Loss) is kept in Books of Accounts, is shewn in my *Merchant's Magazine*. But this Book is not intended to be a Treatise of Merchandize, but to teach all kind of Arithmetic.

SECT. VI. *FELLOWSHIP.*

**F**ELLOWSHIP is the Application of the Rule of Proportion, shewing what share of Gain or Loss every Merchant of those who trade in company shall have, or bear in proportion to his share of the general Stock; and consequently by this Rule also those who have underwritten Policies know how to settle their Averages, in case part of what is insured be lost, proportionably to what each has subscribed.

So



So that this may be called the Rule of Distribution, or (in case of Loss) of Contribution, (of the Profits obtained, or of Loss sustained.)

*Quest. 1.]* Two Men let to freight a Ship, wherein the first hath  $\frac{3}{4}$ , the other 7; and when the Voyage was performed, and all Charges deducted, the Nett Profits are found  $\text{£} 480$ : What must each have?

*Rule.]* In this Case divide the whole Gain by the Number of Shares, and multiply the Quotient by each Man's particular Share, and the Products are the Answers, thus:

$$\begin{array}{rcl} \text{Or thus,} & & \\ 16.480 :: 9.270 \} & \text{2d way.} & 16) 480 (30 \text{ by } 9 \text{ is } = \text{£} 270 = 1\text{st.} \\ 16.480 :: 7.210 \} & & \quad \quad \quad \text{by } 7 \text{ is } = \text{£} 210 \text{ the 2d.} \\ \hline & & 0 \end{array}$$

480 Sum. Proof  $\text{£} 480$  Sum.

*Quest. 2.]* Five Merchants trade in Company; *A.* put in 1173  $\text{£}$ . *B.* 800, *C.* 977, *D.* 562, and *E.* 1000  $\text{£}$ . and having each continued his Share first put in Stock till the Profits were computed and a Dividend made, they find they had gained by Trading 25312  $\text{£}$ . What must each have?

|  | Sum or<br>whole<br>Stock. | The<br>whole<br>Gain. | Each<br>Man's<br>Stock. | Each Man's<br>Share of the<br>Gain. |
|--|---------------------------|-----------------------|-------------------------|-------------------------------------|
|  | 4512.                     | 25312 ::              | 1173.                   | <i>A.</i> 6580 $\frac{20}{112}$     |
|  | 4512.                     | 25312 ::              | 800.                    | <i>B.</i> 4487 $\frac{11}{112}$     |
|  | 4512.                     | 25312 ::              | 977.                    | <i>C.</i> 5480 $\frac{11}{112}$     |
|  | 4512.                     | 25312 ::              | 562.                    | <i>D.</i> 3152 $\frac{11}{112}$     |
|  | 4512.                     | 25312 ::              | 1000.                   | <i>E.</i> 5609 $\frac{11}{112}$     |

The Sum of the Fractions is 4, as *per* Addition of Vulgar Fractions, *Chap. II. Sect. 2. Head. 3.*

The Sum for Proof, 25312

And the Aggregate of each Man's Profit makes the whole Gain; which proves the Truth of the Rule and Operations.

*Quest. 3.]* Three Men underwrite a Policy of Insurance, viz.

*A.* is contented with the Insurance to pay in case of Loss,  $\text{£} 1000$

*B.* is contented with the Insurance for  $\text{£} 750$

*C.* for  $\text{£} 550$

Now it happen'd, that being closely pursued by Pirates, they were obliged to lighten the Ship, by Sum  $\text{£} 2300$   
throwing

throwing over-board Goods to the Value of  $l. 450$ ; what must each Subscriber bear of that Loss, in proportion to what he has insured?

The Proportion is as

| the whole Sum insured    | $l.$  | $l.$                    | Loss. $l.$                           |
|--------------------------|-------|-------------------------|--------------------------------------|
| is to the whole Loss     | 2300. | 450 :: 1000.            | $195\frac{1}{3} = A's \text{ Loss.}$ |
| $l. 450$ : so is the Sum | 2300. | 450 :: 750.             | $146\frac{2}{3} = B's \text{ Loss.}$ |
| each Insurer subscribed  | 2300. | 450 :: 550.             | $107\frac{1}{3} = C's \text{ Loss.}$ |
| to his Proportion of the |       |                         |                                      |
| Loss, as appears by the  |       |                         |                                      |
| Operation in the Margin. |       | Sum for Proof, $l. 450$ |                                      |

Or if you would know what the Loss is *per Cent.* to make the Average; this is only done by one single direct Proportion thus,  $2300. 450 :: 100. 19\frac{1}{3}$ , which is the Rate *per Cent.* to be paid, as is proved.

by multi-  $A's$  10 Hund. multipl. by the Rate  $19\frac{1}{3}$  gives  $l. 195\frac{1}{3}$   
 plying the  $B's$  7 $\frac{1}{2}$  Hund. by the Rate  $19\frac{1}{3}$  gives  $l. 146\frac{2}{3}$   
 100  $l.$  that  $C's$  4 $\frac{1}{2}$  by the Rate  $19\frac{1}{3}$  produceth  $l. 107\frac{1}{3}$   
 each insu-  
 red by the Rate *per Cent.* as in the Margin.

*Quest. 4.]* Three Men bought 120 Acres of Land for  $l. 2400$ ; for which  $A.$  was to pay  $\frac{1}{2}$ ,  $B.$  a 3d, and  $C.$  a 4th of the Charge, and were to have the like Proportion of the Land: how much must each pay, and have of the Land, according to the true Intent and Meaning of their Contract? For to think that  $\frac{1}{2}$ ,  $\frac{1}{3}$ , and  $\frac{1}{4}$ , will make but one, is absurd, because a 3d is more than a 4th, and  $\frac{1}{2}$  and  $\frac{1}{4}$  makes 1. The Answer is found as follows.

$\frac{1}{2}$  of Acres 120 is 60; and of  $l. 2400 = 1200$   
 $\frac{1}{3}$  is \_\_\_\_\_ 40; and \_\_\_\_\_  $l. 800$   
 $\frac{1}{4}$  is \_\_\_\_\_ 30; and \_\_\_\_\_  $l. 600$

Acres 130

Sums  $l. 2600$

Too much by 10

and —  $l. 200$

Therefore I say, If 130 abate 10, and  $l. 2600$  must abate  $l. 200$ ; what must each abate, to answer the intended Contract? Thus;

| Acres.                        | Acres.                         | $l.$                               |
|-------------------------------|--------------------------------|------------------------------------|
| 130. 10 :: 60. $4\frac{1}{3}$ | and if $l. 2600. 200 :: 1200.$ | $92\frac{2}{3} = A's \text{ Ded.}$ |
| 130. 10 :: 40. $3\frac{1}{3}$ | — $l. 2600. 200 :: 800.$       | $61\frac{1}{3} = B's$              |
| 130. 10 :: 30. $2\frac{1}{3}$ | — $l. 2600. 200 :: 600.$       | $46\frac{2}{3} = C's$              |
| Acres 10                      | Sums $l. 200$                  |                                    |

So

So that the Deductions of the respective Acres and Pounds being made (as *per* the Rules in Subtraction of Fractions) from their apparent Shares, above,

The Answer is,  $\left\{ \begin{array}{l} A. \text{ must have Acres } 55\frac{1}{3}, \text{ and pay } l. \ 1107\frac{1}{6}. \\ B. \text{ must have Acres } 36\frac{1}{3}, \text{ and pay } l. \ 738\frac{1}{6}. \\ C. \text{ must have Acres } 27\frac{1}{3}, \text{ and pay } l. \ 553\frac{1}{6}. \end{array} \right.$

Acres 120

Sums  $l. \ 2400$  Proof.

Or the Analogies (or Proportions) above, may be done from the Parts each is to have, by reducing the Fractions  $\frac{1}{3}$ ,  $\frac{2}{3}$ , and  $\frac{1}{2}$ , into a common Denominator; as  $\frac{2}{6}$ ,  $\frac{4}{6}$ , and  $\frac{3}{6}$ : For

As the Sum of the Numerators 13, is to 10, and to 200:

So is each Numerator 6, 4, and 3, to the Proportion that each is to deduct.

|             | Acres.       |                 | $l.$                 |
|-------------|--------------|-----------------|----------------------|
| Thus as 13. | 10 :: 6.     | 4 $\frac{1}{3}$ | And as 13. 200 :: 6. |
|             |              |                 | 92 $\frac{1}{3}$ .   |
|             | 13. 10 :: 4. | 3 $\frac{1}{3}$ | And as 13. 200 :: 4. |
|             |              |                 | 61 $\frac{1}{3}$ .   |
|             | 13. 10 :: 3. | 2 $\frac{1}{3}$ | And as 13. 200 :: 3. |
|             |              |                 | 46 $\frac{1}{3}$ .   |

When Time is considered in Partnership, You must multiply the Money in Stock by the Time it continued therein, and work with the Products as by the Rules above, and as in the Example following.

*Quest. 5.] A. and B. trade in Partnership for one Year. A. put into Stock  $l. \ 200$ , and at 4 Months end he withdrew  $l. \ 40$ , and at 9 Months end put in  $l. \ 170$ . B. put in at first  $l. \ 150$ , and at 7 Months end  $l. \ 200$  more, and at the end of 10 Months he takes out  $l. \ 160$ : they gain 700, what must each have of it?*

|                                         |                                                                                        |      |
|-----------------------------------------|----------------------------------------------------------------------------------------|------|
| <i>A's</i> $l. \ 200 : 4$ Months is 800 | $\left\{ \begin{array}{l} \text{Sum of these} \\ \text{Products} \end{array} \right\}$ | 2590 |
| $l. \ 160 : 5$ Months = 800             |                                                                                        |      |
| $l. \ 330 : 3$ Months = 990             |                                                                                        |      |
| <i>B's</i> $l. \ 150 : 7$ Months = 1050 | $\left\{ \begin{array}{l} \text{Sum of these} \\ \text{Products} \end{array} \right\}$ | 2480 |
| $l. \ 350 : 3$ Months = 1050            |                                                                                        |      |
| $l. \ 190 : 2$ Months = 380             |                                                                                        |      |
| Sum Total =                             |                                                                                        | 5070 |

Then 5070. 700 :: 2590.  $357\frac{1}{3}\frac{1}{3}$  = *A's* Profit.

5070. 700 :: 2480.  $342\frac{1}{3}\frac{1}{3}$  *B's*. Sum  $l. \ 700$  = the Proof.

*Quest. 6.] A Ship's Company take a Prize worth 6000  $l.$  Now the Captor having on board the following Officers, Midshipmen, and*



and Sailors, who are each to participate of the Prize-Money according to his Pay and Time entred on board; which was thus.

8 Officers entered on board 10 Mon. at 50 s. per Mo. } What must  
10 Midshipmen — — — 9 Mon. at 35 s. per Mo. } each have of  
150 Sailors — — — 8 Mon. at 25 s. per Mo. } the Prize?

*Mon.*

8 mult. by 10, and that Prod. by  $l. 2 : 10$ , produceth  $l. 200 : 00 : 00$   
10 mult. by 9, and that — — by  $1 : 15$ , gives — —  $157 : 10 : 00$   
150 — — by 8, and that — — by  $1 : 5$ , is — —  $1500 : 00 : 00$

Sum —  $1857 : 10 : 00$

Then I say,

As  $1857\frac{1}{2}. 6000 :: 200. \quad l. 646\frac{11}{13}$  { To be divided equally  
among the Officers.  
 $1857\frac{1}{2}. 6000 :: 157 : 10. \quad 508\frac{37}{13}$ , Among the Midshipmen.  
 $1857\frac{1}{2}. 6000 :: 1500. \quad 4845\frac{22}{13}$ , Among the Sailors.

Sum for Proof =  $l. 6000$

SECT. VII. *Equation of Payments.*

**B**Y this Rule or Part of the Application of Arithmetic, Men discover, when several Payments are due at different Times, how to fix upon one certain Time when the whole may be paid at once, without Loss to the Debtor or Creditor.

*The General and true Rule is,*

- As the Total of the Sums payable : is to a Unit (or 1 Month)
- So is the Sum of the Products made of each Sum in its respective Time,
- To the true Time when the whole ought to be paid.

There is a certain Gentleman, who makes some Figure in these kind of Sciences, and he denies the Truth of this Rule; which I shall prove under my second Question to be just, notwithstanding his Pretence of its being erroneous.

*Quest. 1.] A. is indebted to B.  $l. 100$ , to be paid at the end of three Months, also  $l. 200$  to be paid at the end of 4 Months, and  $l. 300$  to be paid at the end of 5 Months : Now to prevent the trouble of many Meetings, they agree to have but one Payment of the three Sums at one time; the Question is, when that must be, without loss to either A. or B.*

Q

According

According to the foregoing Rule,

|                                              |                                |
|----------------------------------------------|--------------------------------|
| l. 100 multiplied by 3 Months, produceth 300 | } Sum 2600 of<br>the Products. |
| 200 ————— by 4 — — — — — 800                 |                                |
| 300 ————— by 5 — — — — — 1500                |                                |

l. 600 Sum                      l. Mon.                      l. Mon.

Then I say, 600. 1 :: 2600. 4 $\frac{1}{3}$ .

The Time sought is 4 $\frac{1}{3}$  Months; at the end of which time, if the l. 600 be paid, neither Party will sustain Loss, as I shall prove by and by.

*Quest. 2.] A. oweth B. 5 Sums of Money to be paid at 5 Payments, viz.*

|                                                                      |                                                                                                          |
|----------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------|
| l. 100 ready Money                                                   | } At what time must<br>the whole be paid<br>without Loss to A.<br>or B?<br>Answ. 3 $\frac{1}{3}$ Months. |
| 200 at the end of $\frac{1}{2}$ a Month, or $\frac{1}{4}$ of a Year, |                                                                                                          |
| 300 at 1 Month's end,                                                |                                                                                                          |
| 400 at 3 Months end,                                                 |                                                                                                          |
| 500 at 6 Months end,                                                 |                                                                                                          |

l. 100 ready Money.

200 multiplied by  $\frac{1}{2}$  a Month = l. 100

300 by 1 Month — — — = 300

400 by 3 Months — — — = 1200

500 by 6 Months — — — = 3000

1500 = Sum.

4600

Then as 1500. 1 :: 4600. 3 $\frac{1}{3}$  = the Answer.

Now that this is the true equated Time, will appear thus: For if the Interest of the respective Sums for their Times be equal to that of the whole for the equated Time, I think the Truth of the Answer cannot be denied.

Now you'll find l. 100 ready Money Interest to be l. 0 : 0 : 0

Interest of l. 200 for  $\frac{1}{2}$  a Month at 5 per Cent. is 00 : 8 : 4

300 for 1 Month is — — — 1 : 5 : 00

400 for 3 Months is — — — 5 : 00 : 00

500 for 6 Mon. (all at 5 per Cent.) is 12 : 10 : 00

the Sum = 19 : 3 : 4.

*Note, The In-  
ter. for Months  
I take from  
that of a Year  
proportionably.*

And whoever will take the small pains to examine, will find, That the Interest of l. 1500 for 3 $\frac{1}{3}$  Months is the very same Sum. But no wonder he denies it, who denies the common Way of computing

putting simple Interest, tho' always practised by his Betters; and he may as well pretend that it ought to be computed by way of Compound Interest for every 2d in the Year, as object what he has advanced against the general Ways and Rules given by every body but himself, for working Questions both in Interest and Equation of Payments, which I have in the last example sufficiently proved to agree, tho' I never did see the Truth of Equation proved this way before; which I hope will be a Satisfaction to the Ingenious, as well also the Novelty of the last Question, &c.

And as a farther Proof to shew the Agreement of other Rules to the first, (or common one above) and that it is universally true for any Sum; I will find the equated time under the first Question, without taking notice of the Sums of Money, only the Parts of any Sum, and of the Time when payable. As,

| Parts of<br>any Sum | Payable at<br>these Mon. | Products of<br>these two. | Being<br>Months |                    |
|---------------------|--------------------------|---------------------------|-----------------|--------------------|
| $\frac{1}{2}$ at    | 3                        | $\frac{3}{2}$             | $\frac{1}{2}$   | } These three add. |
| $\frac{1}{3}$ at    | 4                        | $\frac{4}{3}$             | $1\frac{1}{3}$  |                    |
| $\frac{1}{5}$ at    | 5                        | $\frac{5}{5}$             | $2\frac{1}{5}$  |                    |

Sum = the equated Time above =  $4\frac{1}{5}$  as before.

And thus might the Equation of Time be reduced into Tables.

|                                        |           |                  |                  |
|----------------------------------------|-----------|------------------|------------------|
|                                        | <i>l.</i> | <i>Mon.</i>      | <i>l.</i>        |
| <i>Quest. 3.]</i> I lent my Friend     | If 500.   | 5.               | 750              |
| <i>l.</i> 500 for 5 Months; for what   | 5         |                  |                  |
| time must he lend me <i>l.</i> 750, to |           |                  |                  |
| recompence my Kindness to              | 75) 2500  | (3 $\frac{1}{5}$ | Months for Answ. |
| him? This is done as in the            | 1         |                  |                  |
| Margin.                                |           |                  |                  |

And for Proof of this,

25

Interest at 5 per Cent. of *l.* 500 : 5 Months is *l.* 10 : 8 : 4  
of *l.* 750 : 3 $\frac{1}{5}$  Months *l.* 10 : 8 : 4

## SECT. VIII. BARTER.

**B**arter or Commutation is a Rule among Traders, whereby they do, by considering the Price of their Goods, whether as for ready Money, or advanced in Barter, so proportion the Rates and Quantities, as to know how much of one Specie may be exchanged for any quantity of another kind of Commodity. And all this is discovered



discovered by that Golden Rule of Single Direct Proportion, as appears by the following Examples.

*Quest. 1.] A. hath 1752 Ells of Linnen at 2 s. 9 d. per Ell ; B. has Cheese at 31 s. per C. How much Cheese must B. give A. for his Linnen? See the Operation.*

In this Case it will be necessary to know the Value of A's Linnen only in Shillings thus,

|                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |               |    |     |              |     |                                    |  |     |  |      |  |    |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------|----|-----|--------------|-----|------------------------------------|--|-----|--|------|--|----|
| $\begin{array}{r} \text{1st, } 1752 \\ \quad \quad 2 \end{array} \left. \vphantom{\begin{array}{r} 1752 \\ 2 \end{array}} \right\} \text{Mult.}$ <hr style="width: 100px; margin-left: 0;"/> $\begin{array}{l} 3504 \text{ at } 2 \text{ s.} \\ \frac{1}{4} \text{ that} = 876 \text{ at } 6 \text{ d.} \\ \frac{1}{2} \text{ that} = 438 \text{ at } 3 \text{ d.} \end{array}$ <hr style="width: 100px; margin-left: 0;"/> $\text{Shill.} = 4818 \text{ Sum.}$ | <p style="text-align: center;">2dly, Then say, If 31 s. buy 1 C. of Cheese, what will 4818 s. buy? Answer, 155<math>\frac{1}{4}</math>.</p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="text-align: right;">s. C. Cheese.</td> <td style="text-align: left;">s.</td> </tr> <tr> <td style="text-align: right;">31.</td> <td style="text-align: left;">1 :: 4818 C.</td> </tr> <tr> <td style="text-align: right;">31)</td> <td style="text-align: left;"><math>\frac{4818}{31} = 155\frac{1}{4}</math></td> </tr> <tr> <td></td> <td style="text-align: left;">171</td> </tr> </table> <p style="text-align: center;">So that B. must give A.<br/>155<math>\frac{1}{4}</math> C. of Cheese<br/>for his Linnen.</p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%;"></td> <td style="width: 50%; text-align: right;">168.</td> </tr> <tr> <td></td> <td style="text-align: right;">13</td> </tr> </table> | s. C. Cheese. | s. | 31. | 1 :: 4818 C. | 31) | $\frac{4818}{31} = 155\frac{1}{4}$ |  | 171 |  | 168. |  | 13 |
| s. C. Cheese.                                                                                                                                                                                                                                                                                                                                                                                                                                                   | s.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |               |    |     |              |     |                                    |  |     |  |      |  |    |
| 31.                                                                                                                                                                                                                                                                                                                                                                                                                                                             | 1 :: 4818 C.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |               |    |     |              |     |                                    |  |     |  |      |  |    |
| 31)                                                                                                                                                                                                                                                                                                                                                                                                                                                             | $\frac{4818}{31} = 155\frac{1}{4}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |               |    |     |              |     |                                    |  |     |  |      |  |    |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | 171                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |               |    |     |              |     |                                    |  |     |  |      |  |    |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | 168.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |               |    |     |              |     |                                    |  |     |  |      |  |    |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | 13                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |               |    |     |              |     |                                    |  |     |  |      |  |    |

*Quest. 2.] A. has 52 Dozen of Hats (or 624) which he values at 2 s. 6 d. ready Money, But in Barter expects 2 s. 9 d. per Hat. B. has Cotton at 10 d. per Pound ready Money. How much Cotton must he give for the Hats, at a Price advanced in Barter proportionably to A's Advance in Barter?*

1st say, If A's 2 s. 6 d. advances 3 d. what does B's 10 d. advance?  
d. d. d. d.

30. 3 :: 10. 1 = B's advance on a Pound of Cotton ; so it is 11 d. per Pound in Barter.

2dly, The Value of 624 Hats at 2 s. 9 d. is d. 20592.

d.  $\frac{1}{2}$  Cott. d.  $\frac{1}{2}$  Cott.

3dly say, If 11. 1 :: 20592. 1872 for Answer.

And for Proof of this, 624 Hats at 2 s. 9 d. each is l. 85 : 16 : 00  
and so is 1872  $\frac{1}{2}$  of Cotton at 11 d. per  $\frac{1}{2}$  l. 85 : 16 : 00

*Quest. 3] Two Merchants have various kinds of Goods to Barter: A. has Indian Silk 735 Yards at 8 s. 6 d. but in Barter will have 10 s.*

Canes 532 at 3 s. but will in Barter have 3 s. 4 d.

Muslin 16 Pieces at l. 4, but will have in Barter 4 l. 10 s.

B. has Scarlet Cloath at l. 1. per Yard — —

Glass Manufacture at l. 0 : 1 : 8 per Pound, } Ready Money.

Ditto finer at ————— 2 : 4 per Pound, }

How

How many Yards of Cloth, and Pounds of each kind of Glass, (of each a like Number) must *B.* give *A.* advancing his Goods proportionably also in Barter? To answer this Question,

1st, See what *A.*'s Goods amount to at his bartering Price thus :

735 Yards at 10 *s.* each, is *l.* 367 : 10 : 00 }  
 532 Canes at 3 *s.* 4 *d.* — *l.* 88 : 13 : 4 } Sum *l.* 528 : 3 : 4  
 Mullin 16 Pieces at *l.* 4 : 10 : 00 *l.* 72 : 00 : 00 }

2dly, The Difference between the Sums of *A.*'s ready Money and bartering Price, is 11 *s.* 10 *d.* therefore what *B.* must advance in proportion is thus :

As the Sum of the Ready-Money Rate of one of each of *A.*'s Goods, *l.* 4 : 11 : 6

Is to 11 *s.* 10 *d.* the Sum advanced upon one of each in Barter :

So is *B.*'s *l.* 1 : 4 : 00 the Sum of one of each of his Goods,

To 3 *s.* 1 *d.*  $\frac{27}{98}$  *B.*'s Advance in Barter; which must therefore be 27 *s.* 1 *d.*  $\frac{27}{98}$  for the Price of 1 of each of his Goods.

3dly say, If 27 *s.* 1 *d.*  $\frac{27}{98}$  buy one of each of *B.*'s Commodities, what will *l.* 528 : 3 : 4, the Value of *A.*'s Goods, buy of each of *B.*? It will stand thus ;

*l.* *s.* *l.* *s.* *d.*  
 27 : 1  $\frac{27}{98}$  . 1 : : 528 : 3 : 4. 389  $\frac{33712}{100000}$

So that the Price of *A.*'s Goods will buy 389  $\frac{33712}{100000}$  Yards of *B.*'s Cloth, and as many Pounds, of each sort of his Glass.

And for Proof of this you'll find that 389  $\frac{33712}{100000}$  at 27 *s.* 1  $\frac{27}{98}$  *d.* each, (being a Set of *B.*'s Goods, or 1 of each) will amount to *l.* 528 : 3 : 4, the Value of *A.*'s Goods, according to his advanced Price in Barter.

Or you may find the Price in Barter that *B.* ought to rate each sort of his Goods : for,

*B.*'s Advance in Barter.  
*l.* *s.* *d.* *s.* *d.* *s.* *s.* *d.*  
 As *A.*'s 4 : 11 : 6. 11 : 10 : : 20. to 2 : 7  $\frac{27}{98}$  on Cloth.  
 4 : 11 : 6. 11 : 10 : : 1 : 8. to 0 : 2  $\frac{27}{98}$  on Glass.  
 4 : 11 : 6. 11 : 10 : : 2 : 4. to 0 : 3  $\frac{27}{98}$  on ditto.

And by these particular Prices of *B.*'s Goods, you may not only prove the Truth of the foregoing Operation, by seeing what

*s.* *d.*  
 389  $\frac{33712}{100000}$  will { 22 : 7  $\frac{27}{98}$  per Yard of Cloth, } Whose 3 Products  
 amount to at { 1 : 10  $\frac{27}{98}$  per lb of Glass, } added together, will  
 { 2 : 7  $\frac{27}{98}$  per lb ditto fine, } make *l.* 528 : 3 : 4.

But if you were not to deliver an equal Number of each, but according to any Proportion, you may now easily do the same.

And

And thus I have given the Reader a farther Account of the Usefulness and Extent of this Rule of Barter, &c. than has been done by any Author before, to make it agreeable to the other parts of the Book foregoing; all which I doubt not but will be gratefully accepted by the Ingenious Reader.

## SECT. IX. EXCHANGE.

AS the former or last Section exhibits Rules for the Bartering of one Commodity for another; so this shews how to exchange Money for Money, or, in the way of Negotiation, Money for Bills, &c. And therefore this is the proper place where Rules for Exchange ought to be inserted, as being another kind of Barter.

I shall not here trouble the Reader with an unnecessary and uncertain Account of the Value of foreign Coin of all Countries, (about which most differ) because they who have Money to remit, must be governed by the Course of Exchange, and it is notorious that does rise and fall according as foreign Trade is influenced by several Circumstances relating to this or that Country; only I shall give the Weight and Value of the four several sorts of Pieces of Eight, because they are current in most places.

|                                  | Weight. | True Value, |
|----------------------------------|---------|-------------|
|                                  | dw. gr. | s. d.       |
| The Piece of 8 <i>Pillar</i> ——— | 17 : 12 | 4 : 6½      |
| Piece of 8 <i>Seville</i> ———    | 17 : 12 | 4 : 6       |
| Piece of 8 <i>Mexico</i> ———     | 17 : 12 | 4 : 6       |
| Piece of 8 <i>Peru</i> ———       | 17 : 12 | 4 : 5       |

And for the other Denomination of foreign Coin at Places which have Exchange with *England*, they are,

1. The Pound *Flemish* 33 s. 4 d. in Value 1 Pound *Sterling*; in which Denomination *London* exchanges with *Amsterdam*, *Rotterdam*, *Antwerp*, and *Hamburg*.
2. In Pieces of 8 *Mexico* for *English* Pence, *London* exchanges with *Madrid*, *Cadiz*, *Genoa*, *Leghorn*.
3. For Ducats (one being in real Value 4 s. 4. d. ½ *Sterling*) *London* exchanges with *Venice*.
4. For the *French* Crown (in Value 4 s. 6 d. *Sterling*.) *London* exchanges with *Paris*.
5. For the Mill-rea of 6 s. 8 d. ½ *Sterling*, *London* exchanges with *Oporto* in *Spain*, and *Lisbon* in *Portugal*.

But





So that in *l.* 1272 : 13 : 4 *Flemish*, there are *l.* 763 : 12 : 00 *Sterl.* And thus *Dutch* Money may be reduced to *English* at any Rate per Pound *Sterling*; but in the above, of 33 *s.* 4 *d.* *Flemish* per Pound, the Work is performed much shorter, as above toward the Right-Hand Margin.

8. And if a Bill is drawn from *Lisbon* of Mill-Reas 1432 at 6 *s.* 10 *d.*  $\frac{1}{8}$  *Sterling* per Mill-Rea; how much *English* Money is that Bill?

|                  |           |                    |                   |                                                |
|------------------|-----------|--------------------|-------------------|------------------------------------------------|
| <i>Mill-rea.</i> | <i>s.</i> | <i>d.</i>          | <i>Mill-reas.</i> |                                                |
| 1.               | 6         | : 10 $\frac{1}{8}$ | :: 1432           | 1432 at 6 <i>s.</i> 10 <i>d.</i> $\frac{1}{8}$ |

By Practice,  $\frac{1}{8} = 477 : 6 : 8$   
 and  $\frac{1}{10} = 11 : 18 : 8$

And 1432 multiplied by 5, and divided by 8 = 3 : 14 : 7

Answer, *Sterling* = *l.* 492 : 19 : 11

9. Now suppose the *Dutch* Bill above be endorsed and sent to *Leghorn*, at 56 *d.* *Sterling* per Piece of Eight; and if it be again endorsed at *Leghorn*, and remitted to *Amsterdam*, the Exchange at 93 $\frac{1}{2}$  Pence *Flemish* per Piece of Eight: How many Pieces of Eight must be paid for the Bill at *Leghorn*, and how many Pounds *Flemish* at *Amsterdam*, according to this Course of Exchange?

1st, In *l.* 763 : 12 in the Bill, there are 183264 Pence *Sterling*; which divided by 56, gives 3272 $\frac{2}{3}$  Pieces of Eight.

2dly, 3272 $\frac{2}{3}$  being multiplied by 93 $\frac{1}{2}$  Pence *Flemish* for each Piece of Eight, or  $2290\frac{2}{3}$  by  $28\frac{2}{3}$  produceth  $64142\frac{2}{3}$  Pence *Flemish*; and being divided by 12, and then by 20, gives Pounds *Flemish* *l.* 1272 : 13 : 4 as above, which proves the Truth of the whole Work.

10. But these Questions will be so easy to those who have proceeded gradually to learn thus far, that I need not enlarge much farther on this Rule of Exchange; what has been observed being sufficient to shew how either the Coins, or Weights and Measures of one Country, are reduced to those of any other.

For in 32755 *Flemish* Ells there are 19653 *English*; 32755  
 for a *Flemish* Ell is  $\frac{1}{3}$  of an *English*, or  $\frac{1}{3}$  rather, be- 6  
 cause it saves the Work of Division

11. A *Dutchman* sells 29380 *Flemish* Ells of *Holland* 196530  
 Duck, to an *Englishman*, a *Spaniard*, an *Italian*, and |  
 a *Portuguese*, who are to have each a like quantity in their own  
 Country Measure; how much must each have therein?

The

The Dutch in English Ells are 17628, and

17628 Ells divided by 4, gives 4407 Ells English each, and  
Ells. Answer.

4407 is for the Briton — — — — — 4407 Ells.

4407 for the Spaniard at  $\frac{3}{4}$  Canes per Ell English 3084  $\frac{3}{4}$  Canes.

4407 for the Italian at 2 Braces per Ell English. 8814 Braces.

4407 for the Portuguese at 1 Vare per Ell is — — 4407 Vares.

This may be proved several ways, which I leave to the Reader's Judgment.

## SECT. X. ALLIGATION.

**A**LLIGATION may be called *The Rule of Mixture*, or of *compounding Ingredients*, because it teaches how to mix several Species of Simples according to any Intent or Design proposed. It is either Medial or Alternate.

*Alligation Medial* shews what the mean Price of a Pound, Ounce, &c. is worth, when several Quantities of several Values are mix'd together, &c. as per the Cases following.

*Alligation Alternate* shews how much of various kinds of Simples may be taken to make up any assigned Quantity of a Compound, which shall be worth a Price proposed.

## Of Medial Alligation.

*Case 1.]* A Goldsmith hath Gold 12  $\frac{3}{4}$  at 4*l.* per  $\frac{3}{4}$ ; 8  $\frac{3}{4}$  at *l.* 4 : 5 ; 3  $\frac{3}{4}$  at *l.* 4 : 6 : 8 ; and 9  $\frac{3}{4}$  at *l.* 4 : 13 : 4 per Ounce : what is an Ounce worth, suppose these be all melted down together? Answer, *l.* 4 : 7 : 5  $\frac{1}{2}$ .

*Rule.]* Multiply each Quantity given by the Price ; then by direct Proportion,

As the Sum of the Quantities given

Is to the Sum of the said Products ;

So is one Ounce of the Mixture

To its Value. See the Work following.

Ounces Gold - The Price of 1  $\frac{3}{4}$ .

12 multiplied by *l.* 4 : 00 : 0 the Product is *l.* 48

8 ——— by 4 : 5 : 0 produceth ——— 34

3 ——— by 4 : 6 : 8 produceth ——— 13

9 ——— by 4 : 13 : 4 produceth ——— 42

32 Sum.

R

137 Sum.

Then



3. l.      3. l.

Then say, 32. 137 :: 1. 4 $\frac{1}{2}$ , or to l. 4 : 5 : 7 $\frac{1}{2}$ .

And by the same Rule the Value of any other Quantity of that Composition is found: as suppose 7 in the last Example is worth 29 $\frac{1}{2}$ , for as

3. l.      3. l.  
32. 137 :: 7. 29 $\frac{1}{2}$ .

*Case 2.]* To increase or diminish a Compound proportionably, by knowing the several Quantities of the Simples in the Composition.

*Rule.* As the Sum of the particular Quantities of the Compound given

Is to the whole Quantity proposed to be augmented or lessened;

So is each particular Quantity in the given Compound

To the due proportion required of that Specie, Fineness, &c.

*Example.* I would augment the Compound in the last Case to 48 3, that is, I would add 16 to the 32; how much must I take of each simple Ingredient? See the Operation.

|                                       |                              |
|---------------------------------------|------------------------------|
| 12                                    | <i>Answer.</i>               |
| 8                                     | Then as 32. 16 :: 12. 3 6    |
| 3                                     | 32. 16 :: 8. 4               |
| 9                                     | 32. 16 :: 3. 1 $\frac{1}{2}$ |
| <hr style="width: 50px; margin: 0;"/> | 32. 16 :: 9. 4 $\frac{1}{2}$ |
| Sum = 32                              |                              |

Sum = 16 Proof to add.

So that I must have 18 3 of l. 4 per 3.

12 3 of l. 4 : 5 : —

4 $\frac{1}{2}$  of l. 4 : 6 : 8, and

13 $\frac{1}{2}$  of l. 4 : 13 : 4.

---

48 Sum for Proof, in the whole.

*Case 3.]* Having the Simples of any Compound given, to find how much of each kind of simple Ingredient is in any part of that Composition.

*Rule.* As the Total of the Composition

Is to the Quantity of any Simple in that Composition:

So is the Total Quantity proposed to be proportionably compounded,

To the Quantity of each Simple to be in that proposed Quantity.

*Example*

*Example.* I would know how much of each Ingredient (or Price of Gold mentioned in the first Case) is in a Pound or 12  $\frac{3}{4}$  of the 32, being the Compound given? See the Operation in the Margin.

*Answer.*

32. 12 :: 12  $\frac{3}{4}$  of l. 4. per  $\frac{3}{4}$ .  
 32. 12 :: 8. 3 of 4:5:—  
 32. 12 :: 3.  $1\frac{1}{8}$  of 4:6:8  
 32. 12 :: 9.  $3\frac{1}{8}$  of 4:13:4

$\frac{3}{4}$  12 Sum Proof.

*Case 4.]* The Total of the Compound of two Simples, with the Total Value of that Composition, and the Value of a Unit of each simple being given; to find the Quantity of each simple Ingredient in the Composition.

*Rule.* Multiply the Total Quantity of the Composition, (here 20) by the lesser Price of the Unit (here 4), then deduct the Product from

Gold at l. 4 per  $\frac{3}{4}$ .  
 Ditto at l. 4:5  
 Total of the } = 20  $\frac{3}{4}$ . Total Value l. 82  
 Composition }  
 4  
 80  
 l.  $\frac{1}{4}$  2 (8

the Total Value of the Composition (here 82), and divide the Remainder by the Difference in Value of a Unit of the two Simples given (as here 5 s. or  $\frac{1}{4}$  of a Pound) and the Quotient is the Quantity of the higher-priced Simple (here 8) whose Complement to 20 is 12: so that the Answer is 12  $\frac{3}{4}$  of l. 4 per Ounce, and 8  $\frac{3}{4}$  of l. 4:5 per Ounce. This Canon I discover'd by Algebra, as appears in the Solution of Questions by various Positions.

*Case 5.]* To find the Quantities of each simple Ingredient (when those Simples are more than 2 in Number) contained in a Composition, by having the Totals of the Quantity compounded, and of the Value; and also the Value of a Unit of each simple Ingredient given, as

$\frac{3}{4}$  of Gold at l. 4 per  $\frac{3}{4}$  =  
 $\frac{3}{4}$  ditto at l.  $4\frac{1}{4}$  =  
 $\frac{3}{4}$  ditto at l.  $4\frac{1}{2}$  =  
 $\frac{3}{4}$  ditto at l.  $4\frac{3}{4}$  =

Total of the Composition = 32  $\frac{3}{4}$ . Total Value l. 137

*Rule.* To these kind of Questions, as in those of Alligation Alternata, various Answers may be given, and yet all true. You may best

best do them by 2 at a time, as in the last Case. I suppose the 2 first 15 of the total Mixture, and 63 of the total Value, and so I find 3 at  $l. 4$ , and 12 at  $4\frac{1}{4}$ . Then the rest of the total Compound is 17, and of the Value 74; which, according to the 2 later Prices, gives 16 at  $4\frac{1}{2}$ , and 1 at  $4\frac{3}{4}$ .

But *note*, That you must so discreetly divide the total Quantity, and Value, that when the Product of the 1st in 1 of the 2 Prices is taken from the later, the Remainder may not be so

much as (when divided by the Difference of the Prices) will give a Quotient so great as that part of the total Quantity of the Ingredient which you fix'd upon, or supposed. See the Operation above.

|                           |                                                                        |
|---------------------------|------------------------------------------------------------------------|
| 1st 2 = 15                | and = 63                                                               |
| 4                         | less — 60 deduct.                                                      |
| Product = 60              | $\frac{1}{4}$ ) 3 (12 at $4\frac{1}{4}$ .                              |
| 2dly 15                   | 63 deduct.                                                             |
| $4\frac{1}{4}$            | $63\frac{1}{4}$                                                        |
| Product = $63\frac{1}{4}$ | $\frac{1}{4}$ ) $\frac{1}{4}$ ( $\frac{1}{4} = 3$ at 4.                |
| the 2d 2 = 17             | and 74                                                                 |
| $4\frac{1}{2}$            | $73\frac{1}{2}$ deduct.                                                |
| Product = $73\frac{1}{2}$ | $\frac{1}{2}$ ) $\frac{1}{2}$ (1 at $4\frac{1}{2}$ .                   |
| lastly — 17               | 74 deduct.                                                             |
| $4\frac{3}{4}$            | $79\frac{1}{4}$                                                        |
| $79\frac{1}{4}$           | $\frac{1}{4}$ ) $5\frac{3}{4}$ ( $\frac{1}{4}$ or 16 at $4\frac{3}{4}$ |

## II. Alligation Alternate.

*Quest. 1.* A Farmer hath 4 sorts of Wheat, viz. 5 s. 6 s. 7 s. and 7 s. 6 d. per Bushel; and he is minded to mix so much of each sort, as will make 64 Bushels worth 6 s. 6 d. per Bushel: how much of each sort must he take?

Having placed the Prices as you see, and the mean Price; take the Difference between the mean Price 6 s. 6 d. and 5 s. (the 1st Price) which is 1 s. 6 d. this you must put down

(in the 1st Way) against 7 s. 6 d. (because bigger than the mean Price.)

| Prices.          |   | Differences. |          |
|------------------|---|--------------|----------|
| s.               |   | s.           | d.       |
| 5                |   | 1            | 0        |
| s. d. 6          |   | 0            | 6        |
| Mean Price 6 : 6 | — | 0            | 6        |
| 7                |   | 1            | 6        |
| 7 : 6            |   | 3            | 6 = Sum. |

the 1st Way.



Price.) Then go on, and put the Difference between 6 s. 6 d. the mean Price, and 6 s. (the 2d Price) which is 6 d. against the Price (7 s.) because bigger than the mean Price.

| A Second Way. |    |    |
|---------------|----|----|
| s.            | s. | d. |
| 5             | 0  | 6  |
| s. d. 6       | 1  | 0  |
| 6 : 6         |    |    |
| 7             | 1  | 6  |
| 7 : 6         | 0  | 6  |
| Sum = 3 : 6   |    |    |

| Third Way.        |    |    | Sums. |       |
|-------------------|----|----|-------|-------|
| s.                | s. | d. | s.    | d.    |
| 5                 | 0  | 6  | 1 s.  | 1 : 6 |
| s. d. 6           | 0  | 6  | 1     | 1 : 6 |
| 6 : 6             |    |    |       |       |
| 7                 | 1  | 6  | 2     | 2 : 0 |
| 7 : 6             | 1  | 6  | 2     | 2 : 0 |
| Sum Total = 7 : 0 |    |    |       |       |

Then put the Difference between 6 s. 6 d. and 7 s. (the 3d Price) against 6 s. because that is less than the mean Price. Lastly, the Difference between 6 s. 6 d. and 7 s. 6 d. is 1 s. which put against 5, the first Price. And thus having put the Differences between the mean Price and those less than it against the Prices bigger than the mean Price ; and the Differences between the mean Price and those greater than it, right against those that are lesser alternately ;

2dly, Sum up the Differences, which you see is 3 s. 6 d.

3dly, Say by the single Rule of Proportion Direct,

| The Sum of the Differences. | Bushels the whole Mixture. | The Differences. | Bushels required.                              |
|-----------------------------|----------------------------|------------------|------------------------------------------------|
| s. d.                       |                            | d.               |                                                |
| As 3 : 6.                   | 64 ::                      | 12.              | $18\frac{1}{2}$ of that of 5 s. per Bushel.    |
| 3 : 6.                      | 64 ::                      | 6.               | $9\frac{1}{2}$ of that of 6 s. per Bushel.     |
| 3 : 6.                      | 64 ::                      | 6.               | $9\frac{1}{2}$ of that of 7 s. per Bushel.     |
| 3 : 6.                      | 64 ::                      | 18.              | $27\frac{1}{2}$ of that of 7 s. 6 d. per Bush. |

Sum (or Proof) 64 being the whole Mixture.

The Proportions by the second Way of placing the Differences are thus:

|        |       |     |                                            |
|--------|-------|-----|--------------------------------------------|
| s.     | d.    | d.  |                                            |
| 3 : 6. | 64 :: | 6.  | $9\frac{1}{2}$ Bushels of 5 s. per Bushel. |
| 3 : 6. | 64 :: | 12. | $18\frac{1}{2}$ Bushels of 6 s.            |
| 3 : 6. | 64 :: | 18. | $27\frac{1}{2}$ Bushels of 7 s.            |
| 3 : 6. | 64 :: | 6.  | $9\frac{1}{2}$ Bushels of 7 s. 6 d.        |

64 Sum for Proof.

And

And the Proportions by the third Way of placing the Differences are still various. Thus,

| s.      | d.  | d.        |                                               |
|---------|-----|-----------|-----------------------------------------------|
| As 7 or | 84. | 64 :: 18. | $13\frac{32}{41}$ Bushels at 5 s. per Bushel. |
|         | 84. | 64 :: 18. | $13\frac{32}{41}$ Bushels at 6 s.             |
|         | 84. | 64 :: 24. | $18\frac{34}{41}$ Bushels at 7 s.             |
|         | 84. | 64 :: 24. | $18\frac{34}{41}$ Bushels at 7 s. 6 d.        |

64 Sum as before.

In the 2d Way you see that the Difference between the mean Price and 5 s. is placed against the Price 7 s. and 6 s. against 7 s. 6 d. 7 s. against 5 s. and between the 6 s. 6 d. and 7 s. 6 d. against 6 s.

And in the 3d Variety,

|                | s. | d. |     | s. | d. |
|----------------|----|----|-----|----|----|
| The Difference | s. | d. |     | s. | d. |
| between —      | 6  | 6  | and | 7  | 6  |
|                | 5  | 6  |     | 7  | 6  |
|                | 7  | 6  |     | 5  | 6  |
|                | 7  | 6  |     | 5  | 6  |

is placed against the Prices —

So that the Difference between the mean Price and those less than it are placed against all those greater than it; and the Difference between the mean Price and those greater are placed against all those that are less than it. Then the Sum of the Differences in each Line are added together in the Column next the right hand; as 6 d. and 1 s. in the middle Column is 1 s. 6 d. in the third Column; and 1 s. 6 d. and 6 d. is 2 s. So the Sum of that third Column is 7 s. and thus ariseth the Numbers in porportion, in the third Way or Variety.

Hence you see, that there are three different Answers to one Question, and yet they are all true, as fully giving what is required in the Demand; whence (as well as from placing the Differences in each) it may justly be called *Alternate Alligation*. And for a thorow and intire Proof of this, I shall shew that 64 Bushels at the mean Price 6 s. 6 d. per Bushel, is (in the last or most abstruse, tho' best Way of the three Varities) the same Amount as each of the Quantities exhibited for Answers, being cast up at the Prices given, and added together, amount to. Thus,

64 Bushels

64 Bushels  $\left\{ \begin{array}{l} \text{the whole to be mix'd at the mean} \\ \text{Price 6 s. 6 d. amount to } \text{---} \end{array} \right\} l. 20 : 16 : 0$

So also  $13\frac{2}{8}$  Bush. of 5 s. per Bush. comes to  $l. 3 : 8\frac{2}{8}$   
 $13\frac{2}{8}$  — of 6 s. — — — — —  $l. 4 : 2\frac{2}{8}$   
*The Clear*  $18\frac{2}{8}$  — of 7 s. — — — — —  $l. 6 : 8$   
*Proof of the*  $18\frac{2}{8}$  — of 7 s. 6 d. — — — — —  $l. 6 : 17\frac{2}{8}$   
*Operation.*

64 Sum of the Sorts.

Sum =  $l. 20 : 16$  = Total Value *ut supra*.

I shall give one other Example, where there is only one Price less than the mean Price; and this I shall do according to the Method of the third Variety foregoing, that being the best, as making the equallest Mixture: where it is required to know much of 2, 5, 9, and 17 must be taken to make 100 of, or worth 4. See the Operation in the Margin.

|            |               |                                               |                             |
|------------|---------------|-----------------------------------------------|-----------------------------|
|            | Prices given. | Differences of the Prices and the mean Price. | Sum of each Line of Differ. |
|            | 2 s. —        | 1, 5, 13, =                                   | 19                          |
| 4 —        |               |                                               |                             |
| = the mean | 5 — 2         | =                                             | 2                           |
| Price.     | 9 — 2         | =                                             | 2                           |
|            | 17 — 2        | =                                             | 2                           |

And this is proved as before, thus:

76 at 2 = 152  
 8 at 5 = 40  
 8 at 9 = 72  
 8 at 17 = 136

Sum  $\frac{1}{4}$  400 = 100 at 4.

Sum of all the Differences } 25

Answer.

Now 25. 100 :: 19. 76 at 2 s.  
 25. 100 :: 2. 8 at 5  
 25. 100 :: 2. 8 at 9  
 25. 100 :: 2. 8 at 17

Note, That if the Prices given were ever so many, the Method is the same.



## SECT. XI. The Rule of False.

**T**HIS is so called not because it requires any more Conjunction, or is more difficult to comprehend or perform than some of the Rules preceeding; but it is properly called *The Rule of False Supposition*, because by supposing Numbers to be the Answers to Questions which are not really so, but are feigned or supposed at pleasure, we do by such false or fictitious ones discover the true Numbers required.

An Example or two will illustrate this, and I shall not insist farther, because the Solution of an easy simple Equation in Algebra answers not only any Question in this Rule, but also gives at the same time a Canon whereby any Question of the like kind is much more easily and speedily resolved.

*Quest. 1.]* Three Merchants built a Ship, which cost *l.* 1600. *A.* paid a Sum not known, *B.* paid double to *A.* within *l.* 50, *C.* paid as much as *A.* and *B.* wanting *l.* 100: What did each pay of the Cost?

|                                                                        | Operat. Colum. | Suppo- sitions. | Errors. |
|------------------------------------------------------------------------|----------------|-----------------|---------|
| 1st, Suppose <i>A.</i> paid _____                                      | 200            | 200             |         |
| Then, according to the Question, <i>B.</i> paid _____                  | 350            |                 |         |
| And <i>C.</i> paid _____                                               | 450            |                 |         |
| Sum                                                                    | 1000           |                 |         |
| But it should have been <i>l.</i> 1600, therefore the } Error is _____ |                |                 | 600     |
| 2d, Suppose <i>A.</i> paid _____                                       | 250            | 250             |         |
| Then it follows that <i>B.</i> paid _____                              | 450            |                 |         |
| And <i>C.</i> paid, (according to the Question) _____                  | 600            |                 |         |
| Sum                                                                    | 1300           |                 |         |
| Which is therefore still too little by _____                           |                |                 | 300     |
| Then multiplying the 1st Position by } the 2d Error gives _____        | 60000          |                 |         |
| And the 2d suppose in the 1st Error, gives _____                       | 150000         |                 |         |
| The Difference of the Product is = _____                               | 90000          |                 |         |
| Which divided by the Differ. of the Errors (300) _____                 |                |                 |         |
| The Answer is, that <i>A.</i> put in _____                             | 300            |                 |         |
| Then <i>B.</i> put in _____                                            | 550            |                 |         |
| And <i>C.</i> _____                                                    | 750            |                 |         |
| Sum for Proof                                                          | 1600           |                 |         |

Here

Here you see, that the Suppositions and Errors being multiplied, you divide the Difference of the Products by the Difference of the Errors, which you must always do when the Errors are both Surpluses, or both Deficiencies of the supposed Numbers.

But if the one Error is caused by supposing too much, the other by supposing too little; then you must divide the Sum of the said Products by the Sum of the Errors, and the Quotient is the Answer, as in the next Example appears: which Rule you may retain in mind by this Distich;

*When Errors are not both of the same kind,  
To add the Products, as the Errors, mind:  
But if they're both too small, or both too much,  
Subtraction must be us'd in Cases such.*

*Quest. 2.]* Admit a Church hath a Choir (or Chancel) 40 foot long; and that the Ground taken up by the Belfrey is  $\frac{1}{4}$  of the Chancel, and  $\frac{1}{5}$  of the Nave, or Body; the Nave (or Body of the Church) is 3 times the Length of the Belfrey, and  $\frac{1}{4}$  the Chancel: how long is the whole Church within the Walls, and every Part of it?

First find the Length of the Nave or main Body of the Church, for then the rest are discovered, thus:

|                                                                     | <i>The Operation.</i> | <i>Suppositions.</i> | <i>Errors.</i> |
|---------------------------------------------------------------------|-----------------------|----------------------|----------------|
| 1. Suppose the Nave be — — — — —                                    |                       | 150                  |                |
| $\frac{1}{5}$ of that is — — — — —                                  |                       | 25                   |                |
| $\frac{1}{4}$ of the Chancel — — — — —                              |                       | 10                   |                |
| Sum, is the Belfrey — — — — —                                       |                       | 35                   |                |
| Then the Nave (according to the Question) is 3 times that — — — — — | } =                   | 105                  |                |
| And $\frac{1}{4}$ of the Chancel, viz. — — — — —                    |                       | 30                   |                |
| Sum is but — — — — —                                                |                       | 135                  |                |
| Which being less than 150 supposed, the Error is —                  |                       |                      | 15             |
| 2. Suppose the Nave be — — — — —                                    |                       | 102                  |                |
| $\frac{1}{5}$ of that is — — — — —                                  |                       | 17                   |                |
| $\frac{1}{4}$ of the Chancel is — — — — —                           |                       | 10                   |                |
| The Sum or Belfrey is — — — — —                                     |                       | 27                   |                |
| 3 times that Belfrey is — — — — —                                   |                       | 81                   |                |
| $\frac{1}{4}$ of the Chancel is — — — — —                           |                       | 30                   |                |
| Sum is — — — — —                                                    |                       | 111                  |                |
| Which being more than the 102 supposed, the Error is —              |                       |                      | 9              |

S

Now

Now the Product of 150 by 9 is — — — 1350  
 And of the 102 by the Error 15, gives 1530

Which 2 Products I am to add according to  
 the Rule, because the 1st Error was the effect of  
 supposing too much, the 2d of supposing too  
 little; and the Sum is — — — — — } 2880

That divided by the Sum of the Errors = 24 } = 120  
 gives the Length of the Nave of the Church

And the Chancel being = 40

And the Belfrey  $\frac{1}{2}$  of 120 and  $\frac{1}{4}$  of 40 is = 30

The Length of the whole Church is = 190

And you'll find these 3 Dimensions (120, 40, and 30) to answer in all respects what is proposed in the Question, which is a Proof of the Work.

And thus you have a clear and perspicuous Method of solving Questions in this Rule of False Position; which may suffice till we come to Algebra, where the Reader will find a great Variety of Problems answered the best and shortest way, consistent with demonstrative Plainness.

*The Canon* for answering Questions of like nature with the 1st, is this: Divide the whole Value of the Ship, more twice what *B.* abateth of paying double to *A.* more what *C.* abateth of paying as much as *A.* and *B.* by 6, and the Quotient is what *A.* paid. And this Canon holds, vary the Numbers as you please, keeping to the Words.

*The Canon* answering Questions of like nature with the 2d, is, Whatsoever Length the Chancel of the Church is, if that be multiplied by 3, it gives the length of the Body of the Church proportioned as in the Question, as 3 times 40 is 120. But if you suppose the Chancel 60, then is the Nave 180, and the Belfrey 45, in all 285. For Proof  $\frac{1}{2}$  of the Nave, and  $\frac{1}{4}$  of the Chancel 60, is 45 = the Belfrey; and 3 times that (or 135) added to  $\frac{1}{4}$  of the Chancel, is 180.

#### Single Position.

But there are some Questions answered by one single Supposition, with the help of the Rule of Proportion. As for

*Example.]*



*Example.]* Three Men build a House, which cost £ 300. *A.* paid a Sum unknown, *B.* paid twice as much, and *C.* paid 3 times so much; what did each pay of the £ 300?

I suppose *A.* £ 40, then *B.* must pay 80, and *C.* = 120; the Sum of which is but £ 240, instead of £ 300. Then I say,

If 240 doth arise from supposing 40,

What Number will 300 be the Result of?

The Answer is 50; for

$$240. 40 :: 300. 50$$

Now that 50 is the Sum that *A.* paid, may be proved thus:

$$\begin{array}{rcl} A. & = & 50 \\ B. \text{ paid} & = & 100 \\ C. \text{ paid} & = & 150 \end{array} \left. \vphantom{\begin{array}{rcl} A. & = & 50 \\ B. \text{ paid} & = & 100 \\ C. \text{ paid} & = & 150 \end{array}} \right\} \text{Sum} = 300 \text{ for Proof.}$$

*Here ends VULGAR ARITHMETIC.*

## CHAP. III.

### DECIMAL ARITHMETIC.

#### SECT. I. *Notation and Numeration.*

**T**HIS kind of Arithmetic takes its Name from the Nature of the Denominator, which is always 10, or some Power of 10; in which only it differs from a Vulgar Fraction: for as that has any promiscuous Number for its Denominator, so a Decimal Fraction hath always 10, 100, 1000, 10000, &c. for its Denominator.

2. Hence 'tis easy (the Denominators of Decimals being so few in comparison of Vulgar, and so certain) to express a Decimal Fraction without its Denominator, by separating so many places of the Numerator, as the Denominator hath of Cyphers, by a Point: Thus  $\frac{5}{10}$  is wrote .5;  $\frac{25}{100}$  is .25;  $\frac{175}{1000}$  is .075, &c. And if (as in the last) there be not so many places in the Numerator as there are Cyphers in its Denominator, you must make up that Number by placing a Cypher or Cyphers towards the left hand of the Numerator.







shortness be wrote thus: The Decimal of  $\frac{1}{3}$  which is .33333333, &c. may be .3 $r$  that is 3 repeated infinitely; or if a Digit repeat after others are in the Quotient, they will stand thus:

.83  $r$  1 which shews that 1 (or the 3) is only repeated.

.285714285714, &c. hath 6 repeating, wrote thus .285714  $r$  6

And shews that 6 of the Digits next the left hand would be repeated by carrying on the Division *ad infinitum*. And these by some are called circulating Numbers, all which kinds of Decimals are produced by Division thus:

| Divi-<br>fors. | Digits<br>divided. | Decimals<br>arising. | Divi-<br>fors. | Digits<br>divided. | Decimals<br>arising. |
|----------------|--------------------|----------------------|----------------|--------------------|----------------------|
| 9              | 8.0                | .8 $r$               | 7              | 6.000000           | .857142 $r$ 6        |
|                | 7.0                | .7 $r$               |                | 5.000000           | .714285 $r$ 6        |
|                | 6.0                | .6 $r$               |                | 4.000000           | .571428 $r$ 6        |
|                | 5.0                | .5 $r$               |                | 1.000000           | .142857 $r$ 6        |
|                | 1.0                | .1 $r$               | 6              | 5.00               | .83 $r$ 1            |
| 8              | 7.000              | .875.                |                | 4.                 | .6 $r$               |
|                | 5.000              | .625.                |                | 1.00               | .16 $r$ 1            |
|                | 3.000              | .375.                | 5              | 4.0                | .8.                  |
|                | 1.000              | .125.                |                | 3.0                | .6.                  |

A mixt Number (or one composed of a whole Number and Decimal) is thus written 365.125.; 1728.34617, and 252.6 $r$ , &c.

And this shall suffice to shew the Nature of Decimals, and how to read and write any. I shall next pass on to Reduction, and the rest of the Rules, inserting only such Examples as are likely to accrue in Practice; and avoiding all those that are impertinent, and tend to perplex the Reader, and make this Part of Arithmetic abstruse and tedious, which in reality is most easy and obvious.

#### SECT. II. Reduction of Decimals.

**I**T is almost as improper to treat of Addition and the other Rules before this of Reduction in Fractions, as to teach Reduction before the Parts of single Arithmetic in Integers: for what can be more irregular, than to shew how to add, subtract, &c. those Numbers, which you neither know the Production of, nor how to discover

discover what Parts of Coin, Weight, &c. they represent or include? Therefore

I shall, as is usual, as well as justly and methodically done, in the next place shew, how to reduce Vulgar Fractions of inferior Denominations to Decimals of superior, considered as Integers of those Fractions; and, by way of Proof thereof, give Rules and Examples to find the Values of those Decimals or any other.

I. To reduce Vulgar Fractions into Decimals.

A Decimal of 4 places, nay generally of 3, is sufficient, when that Decimal is not to be multiplied by any Number: Therefore the Denominator of a Decimal of 4 places being (as shewed before) 10000, the Proportion will be for any Fraction in general, to find the Numerator of such a Decimal;

As the Denominator of the Fraction given

Is to the Numerator of that Vulgar Fraction ::

So is the Denominator of the Decimal (admit 10000)

To its Numerator or the Decimal required.

*Example.]* What is the Decimal of  $\frac{39}{100}$ ?

Here it may be ob-

served, *Rule* = As 39. 27 :: 10000. .6923 = the Answer.

1st, That a Cypher next the right hand of my Decimal may be omitted, because the same place in the Denominator is a Cypher also: Thus  $\frac{1120}{10000}$  is  $\frac{112}{1000}$ , and  $\frac{11200}{10000}$  is  $\frac{112}{1000}$ : that is,  $\frac{112}{1000}$  is equal to  $\frac{11200}{10000}$ , and  $\frac{112}{1000}$  to  $\frac{112000}{100000}$ , in their lowest Terms.

39) 2700000 (.6923

.....

360

90

120

30

2dly, *Note for a General Rule*, That you must always set off with a Point so many places of the Quotient for your Decimal, as the Number of Cyphers in the proposed Denominator (according to the beginning of the last Sect). And if there be not so many Figures in the Quotient, as there are Cyphers in the Denominator given, then that Number must be made up by putting Cyphers towards the left hand next the Point in the Quotient or Decimal.

3. Hence it follows, That to reduce Coin, Weight, Measure, &c. to the Decimals of an higher Denomination, may easily be done, by first

first representing them in Vulgar Fractions, and then reducing those Vulgar to Decimal, as above.

So to reduce  $3\frac{1}{4}d.$  to the Decimal of a Pound, I consider that this is 13 Farthings, and that a Farthing is  $\frac{1}{960}$  of a Pound: therefore 13 *grs.* is  $\frac{13}{960}$  of a Pound. So that

$$960. 13 :: 10000. .0135 = \text{the Decimal sought.}$$

Hereupon multiplying and dividing, I find the Quotient 135; but because there are 4 Cyphers in the Denominator (or 10000) given, therefore I must have 4 places towards the right hand of the Point for the Decimal, which 4th place I make up with a Cypher, as you see; and the Converse of this Proportion reduceth any Decimal to a Vulgar Fraction; or one Vulgar Fraction may be reduced to any other, whose Numerator or Denominator is given. For

$$135, \text{ £c. } 10000 :: 13. 960 \text{ or } 10000. 135, \text{ £c. } :: 960. 13$$

Or if you would reduce the given Vulgar Fraction to another whose Denominator is 12372, the Proportion holds thus:

$$960. 13 :: 12372. 167\frac{11}{12}$$

So that  $1\frac{11}{12}$  is near-equal to  $\frac{13}{960}$  or .0135

Which may suffice to shew how to reduce any one Fraction to another and consequently a Vulgar to another whose Denominator is 10, 100, &c. i. e. any Vulgar to a Decimal.

And this puts me in mind of a new kind of Decimal that might be contrived, which would not only admit of being expressed in one Line as the common Decimal; but whereas that Decimal saves much of Division (which is its Excellency) so this new one would save much trouble of Multiplication: and that is, by reducing all Vulgar Fractions to one whose Numerator is 10, 100, 1000, &c. And the Numerator might be shewn, by pointing over the Figure in the Denominator, where the 1 or place next the left hand of the Numerator would fall: thus  $14\frac{1}{2}$  is wrote  $14\dot{7}2$ :  $14\frac{10}{100}$  is  $14\dot{7}2$ ; and  $14\frac{100}{1000}$   $14\dot{7}2$ . But I shall leave this Notion to be improved by the Ingenious, if thought worth their while; and beg the Reader's pardon for this *small* Digression, which there is not room in this Book to enlarge on.



II. To reduce Weight, &c. to Decimals, as per Rules Parag. 2, 3, &c. above.

|                                       |                                       |           |                |  |  | Decimals<br>required. |
|---------------------------------------|---------------------------------------|-----------|----------------|--|--|-----------------------|
| 1. In Averdupois                      | 3 : 20, is $\frac{1}{20}$ C.          | And 112.  | 104 :: 10000.  |  |  | .9285                 |
|                                       | 18, is $\frac{1}{18}$ C.              | And 112.  | 18 :: 10000.   |  |  | .1607                 |
| 2. In Troy Weight                     | 11 : 12 : 10, is $\frac{1}{12}$ C.    | And 5760. | 5578 :: 10000. |  |  | .9684                 |
| 3. In Liquid Measure                  | 1 : 3 : is $\frac{1}{3}$ C.           | And 288.  | 11 :: 10000.   |  |  | .0382                 |
| this Vulgar Fraction of a Beer-Barrel |                                       |           |                |  |  |                       |
| 4. In Dry Measure                     | 1 Pint is $\frac{1}{16}$ of a Qt.     | And 512.  | 1 :: 10000.    |  |  | .0019                 |
| 5. In Long Meas.                      | 1 Yard is $\frac{1}{1760}$ of a Mile. | And 1760. | 1 :: 100000.   |  |  | .00056                |

And thus I have fully shewed the Fundamentals of making Decimals. I shall proceed in the third place,

III. To find the Values of Decimals.

Note, That .9 of any lowest Denomination, or .999 of an highest, may be taken for a Unit, as in the last of the 3 first Examples following, &c.

To find the Values (as in these Examples) you must multiply as in the 2d, 4th, 6th, and 8th Columns.

| Value.         | Decimals<br>of a L.<br>Sterl. as | Value.  | Decimals<br>of 1 C.<br>wt. as | Value.           | Decimals<br>of 1 lb<br>Troy, as | Value.   | Decimals of a<br>Barrel of<br>Beer, as |
|----------------|----------------------------------|---------|-------------------------------|------------------|---------------------------------|----------|----------------------------------------|
|                | .0135<br>by 20 s.                |         | .9285<br>by 4 qrs.            |                  | .9684<br>by 12                  |          | .0382<br>by 4                          |
|                | .2700<br>by 12 d.                | Qrs. 3. | .7140<br>by 7 l.              | 3 11             | .6208<br>by 20                  |          | .1528 Firk.<br>by 9 Gal.               |
| Pen. 3.        | .2400<br>by 4 qrs                | 4.      | .9980<br>by 4 l.              | dw. 12.          | .4160<br>by 6                   | Gal. 1.  | .3752<br>8 Pints.                      |
| or 1<br>Farth. | .9600<br>or 20<br>Pound.         | lb 19.  | .9920                         | 2.               | .4960<br>by 4                   | Pints 3. | .0016                                  |
|                |                                  |         |                               | grs. 9.<br>or 10 | .9840                           |          |                                        |
| 1              | 2                                | 3       | 4                             | 5                | 6                               | 7        | 8                                      |

And cut off as many Places from the right hand of each Product, as there are in the Decimal given. These prove the Examples above of the same Denominations.

*Note,* That in the 2d Example I multiply .7140 by 7, and that Product by 4, instead of multiplying .7140 by 28, the Pounds in a Quarter of an Hundred. And in the 3d Example, because I cannot multiply .4160 by 24, as by 1 Figure, therefore I first multiply it by 6, and that Product by 4; which Methods produce the same thing as multiplying .7140 by 28, and .4160 by 24, only there is Addition saved in each Example.

IV. *A second Way to find the Decimal of any thing, deduced from the four last Examples.*

From a due Consideration of the last Examples, you may, by a direct contrary way of Operation, find the Decimals of any Denomination, here dividing, whereas there you multiplied. For instance,

| <i>Divide by</i> | <i>To reduce to a Decimal of a Pound.</i> | <i>Quotients or Answers.</i> | <i>Divide by</i> | <i>To reduce to a Decimal of 1 C.</i> | <i>Quotients or Answers.</i> | <i>Divide by</i> | <i>To reduce to a Decimal of 1 Troy.</i> | <i>Quotients or Answers.</i> |
|------------------|-------------------------------------------|------------------------------|------------------|---------------------------------------|------------------------------|------------------|------------------------------------------|------------------------------|
|                  | l. 0                                      | .0135416 r 1 = 3d qu.        |                  | C. 0                                  | .92857 = 3d qu.              |                  | l.                                       | .9684 = 4 quo.               |
| 20               | s. 0                                      | .27083 r 1 = 2d quo.         | 4                | grs 3                                 | .71428 = 2d qu.              | 12               | oz. 11                                   | .62083 r 1 = 3q.             |
| 12               | Pence 3                                   | .25 = 1st quote.             | 7                | 5                                     | = 1st quote.                 | 20               | dw. 12                                   | .416 r 1 = 2d q.             |
| 4                | Farth. 1                                  |                              | 4                | lb. 20                                |                              | 6                | 2                                        | .5 = 1st quote.              |
|                  |                                           |                              |                  |                                       |                              | 4                | gr. 10                                   |                              |

1. Having put down your Denominations that you would reduce into the Decimal of an higher, as you see in these Examples, begin at the lowest to divide by the proper Divisor for reducing into the next superior Denomination, putting, or imagining, Cyphers to stand in the Decimal places toward the right hand. But,

2. You will observe from these Examples, that the mixt Numbers, as 3.25 Pence, 3.7428 Quarters of a C. are the Dividends, and that the Decimals are the Quotients: Thus I first divide 1 Farthing or 1.00 by 4, and put the Quote to the right hand of the 3 Pence for the Decimal of a Penny; then I divide 3.25 by 12, &c. But,

3. In the second Example, instead of dividing  $\text{lb } 20$  by 28, I divide it at twice by 4, and that Quote by 7; and also in the third Example, instead of dividing by 24 Grains, I (in this Case putting down only the Quotient, and doing the rest mentally) divide by the Digits 4 and 6, which amounts to the same thing.

V. *Some*

## V. Some brief Ways of Reduction of Decimals.

There are in many Instances much shorter Ways both of making Decimals and of finding their Value.

1. As to the first, any even Number of Shillings under 20 are reduced to the Decimal of a Pound, by taking half thereof; and thus you see in the Margin.

The Decimal of 6 s. is .3. l.

12 — .6.

14 — .7.

18 — .9.

2. Any odd Number of Shillings under 20 are made Decimals of a Pound by putting a Cypher towards the right, and then taking the half, as in the Margin.

s. l.

3.0 = .15

9.0 = .45

13.0 = .65

17.0 = .85

3. From hence it is plain, that any Number of Shillings may have their Decimals put down as soon as named, by taking half the even Number; and for the 1 s. (in an odd Number) you see that it is always 5 in the 2d place from the Point.

4. To put any Pence and Farthings down in the Decimal of a Pound, consider what Farthings they make, and put them down in the 2d and 3d places from the Point: But after the Farthings are 24 or more, add 1 to the third place in the Decimal. And thus 19 s. 7 d.  $\frac{1}{4}$  is put down in a Decimal immediately .982; for the 19 s. is .95, and the  $7\frac{1}{4}$  is 31 Farthings, more 1 because 1 must be added at every 24, is 32; which added to the 2d and 3d places from the Point in .95, the Sum is .982. And,

5. If you desire to be so exact to have a 4th place from the Point, you may make the Figure in the 4th place 1 more than a 3d of what Farthings are in the Farthings given, when they do not exceed 23. As in the 3d Example of the 4 in the Margin, for 16 s. I put 8; for 9 d.  $\frac{1}{2}$  or 38 Farthings I put 39 in the 2d and 3d places, because 38 is 13 above 25. Then for the 3d Place I take  $\frac{1}{3}$  of 13, which is 4; to which I add 1, and the Sum is 5 for the 4th place from the Point.

May be put

s. d. in Decimals.

8 :  $4\frac{1}{2}$  — .4187

13 :  $2\frac{1}{4}$  — .6594

16 :  $9\frac{1}{2}$  — .8395

19 :  $11\frac{1}{4}$  — .9989

So also in the 4th Example for the 19 s. I put .95, for the 6 d. of the  $11\frac{1}{4}$ , I put 25, (that is, 1 added at 6 d.) lastly, for the 5 d.  $\frac{1}{4}$  (the rest of the  $11\frac{1}{4}$ , or 23 qrs.) I put  $\frac{1}{3}$  of 24, more 1.

T 2

For



For note, That when the Farthings given are under, or so much as they are more than 24, you must take a third part of that Sum they are next to, which can be divided without a Remainder; as in the last, I take  $\frac{1}{3}$  of 24, because 23 is nearer 24 than it is to 21.

And thus the Decimal of a Pound may be wrote to 4 places at once, or mentally computed, in a fifth part of the time it can be found by the nearer of the Ways above, and that near enough the Truth, unless it be required to be multiplied, as is before mentioned, and as in the Table at the beginning of Division.

|                                                                                                                                                                                                                                                                                                                                                                                                                                   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                |                |    |    |        |      |     |                |        |     |                |  |        |      |   |  |         |      |                |  |         |      |   |  |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------|----------------|----|----|--------|------|-----|----------------|--------|-----|----------------|--|--------|------|---|--|---------|------|----------------|--|---------|------|---|--|
| 6. The Value also of any Decimal of a Pound may be found by Inspection: For if the place next the Point be doubled, it gives the Shillings, to which add 1 s. so often as the 2d place is 5 or more, for the reason under the 2d Rule above; then call what is under 5 (or more than 5) in the 2d place so many Tens of Farthings, and the Digit in the 3d place so many Units; and as often as they are 25, make them less by 1. | <table border="0"> <tr> <td>l.</td> <td>s.</td> <td>d.</td> <td>q.</td> </tr> <tr> <td>.912 =</td> <td>18 :</td> <td>2 :</td> <td><math>3\frac{1}{2}</math></td> </tr> <tr> <td>.123 =</td> <td>2 :</td> <td><math>5\frac{1}{2}</math></td> <td></td> </tr> <tr> <td>.754 =</td> <td>15 :</td> <td>1</td> <td></td> </tr> <tr> <td>.8766 =</td> <td>17 :</td> <td><math>6\frac{1}{2}</math></td> <td></td> </tr> <tr> <td>.9877 =</td> <td>19 :</td> <td>9</td> <td></td> </tr> </table> | l.             | s.             | d. | q. | .912 = | 18 : | 2 : | $3\frac{1}{2}$ | .123 = | 2 : | $5\frac{1}{2}$ |  | .754 = | 15 : | 1 |  | .8766 = | 17 : | $6\frac{1}{2}$ |  | .9877 = | 19 : | 9 |  |
| l.                                                                                                                                                                                                                                                                                                                                                                                                                                | s.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | d.             | q.             |    |    |        |      |     |                |        |     |                |  |        |      |   |  |         |      |                |  |         |      |   |  |
| .912 =                                                                                                                                                                                                                                                                                                                                                                                                                            | 18 :                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | 2 :            | $3\frac{1}{2}$ |    |    |        |      |     |                |        |     |                |  |        |      |   |  |         |      |                |  |         |      |   |  |
| .123 =                                                                                                                                                                                                                                                                                                                                                                                                                            | 2 :                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      | $5\frac{1}{2}$ |                |    |    |        |      |     |                |        |     |                |  |        |      |   |  |         |      |                |  |         |      |   |  |
| .754 =                                                                                                                                                                                                                                                                                                                                                                                                                            | 15 :                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | 1              |                |    |    |        |      |     |                |        |     |                |  |        |      |   |  |         |      |                |  |         |      |   |  |
| .8766 =                                                                                                                                                                                                                                                                                                                                                                                                                           | 17 :                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | $6\frac{1}{2}$ |                |    |    |        |      |     |                |        |     |                |  |        |      |   |  |         |      |                |  |         |      |   |  |
| .9877 =                                                                                                                                                                                                                                                                                                                                                                                                                           | 19 :                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | 9              |                |    |    |        |      |     |                |        |     |                |  |        |      |   |  |         |      |                |  |         |      |   |  |

Or more accurately thus; tho' the Rule above may be thought near enough, as not erring a Farthing:

If the Farthings under or above 5 in the 2d place from the Point, and those in the 3d place be — — — — —

|                                                   |
|---------------------------------------------------|
| 6, and under 19, deduct $\frac{1}{2}$ a Farthing. |
| 19, ————— 33, ————— 1                             |
| 33, ————— 45, ————— $1\frac{1}{2}$                |
| 45 and upward ————— 2                             |

And so often as the 4th place is 6 or more, you may add half a Farthing.

And after this manner you may read Decimals in Shillings, Pence, &c. as fast as if they were wrote so; as I have examined many a thousand, having the Species read to me looking on the Decimals, which if continued to ever so many places, they alter not the Rules above.

Note, That these Allowances or Deductions are made because the Rule supposeth 1000 Farthings in a Pound Sterling, whereas there are but 960; therefore if 1000 Farthings is 40 more than the Truth, 500 is 20 more, 250 is 10, or 25 is 1 too much. And it is sufficient if done to a Farthing, which Decimals may be valued without regarding the 4th place,

7. To find the Value of the Decimal of a Foot in Length by Inspection.  
Rule. For every 10th of a Foot reckon as many Inches and as many Quarters, under .5, (which is known 6 Inches.) 2dly, For every 2 in the 2ds place add 1 quarter of an Inch. And this will not err a quarter

quarter of an Inch in the generality of Decimals, if in any.

Here in the 2d Example

|                                | In. qr. | Feet. Inch. qr. | } Examples. |
|--------------------------------|---------|-----------------|-------------|
| .2 according to the Rule is    | 2 : 2   | .37 = 4 : 2     |             |
| and .08 is half so many qrs. = | 1 : —   | .28 = 3 : 2     |             |
|                                | —       | .73 = 8 : 3     |             |
| Sum =                          | 3 : 2   | .92 = 11 : 1    |             |

And in the 4th Example,

|                        |   |        |
|------------------------|---|--------|
| .9 ; .5 I say is       | — | 6 : —  |
| .4 is 4 inch. 4 qr. or | — | 5 : —  |
| .02 is                 | — | : 1    |
| Sum =                  | — | 11 : 1 |

*Note,* That in adding half, in the 2ds place, if odd, you may omit the odd, and take half the next lesser even Number, as in .07 and .03 I take only 3 qrs. and 1.

The immediate writing down any Inches and Quarters in the Decimal of a Foot is done so easily by the Rule above inverted, that there is no occasion for Examples.

8. To find the Value of the Decimal of a Gallon by Inspection.

Rule.] Multiply the Digit in the Primes place of the Decimal by 8, and the Digit next the left hand of the 2 Digits in the Product is so many Pints; then add the Digit next the right hand of the Product to that in the 2ds place of the Decimal given, and multiply that Sum by 4, and the place next the left hand of the Product are Quarters of Pints. Which is done with about a third of the trouble in the common Way; and gives the Answer accurately enough.

| Gall. Pints. qrs. | Gall. Pints. qrs. | Gall. Pints. qrs. |
|-------------------|-------------------|-------------------|
| .19 = 1 : 2       | .64 = 5 : —       | .34 = 2 : 3       |
| .28 = 2 : 1       | .73 = 5 : 3       | .71 = 5 : 2       |
| .37 = 3 : —       | .82 = 6 : 2       | .66 = 5 : 1       |
| .46 = 3 : 3       | .91 = 7 : 1       | .88 = 7 : —       |
| .55 = 4 : 2       | .11 = — : 3       | .99 = 8 : —       |

Thus in .66, 8 times 6 is 48, the 4 are Pints, and the 8 of the Product added to 6 in the 2ds place is 14; 4 times 14 is 56; the 5 are Quarters of Pints or  $1\frac{1}{4}$  Pints, which with the 4 is  $5\frac{1}{4}$  Pints, &c.

9. To

9. To find the Value of the Decimal of a Barrel of Beer very briefly, and accurately.

Rule.] Multiply the given Decimal by 4, cut off from the Product 1 less than the Decimal places given; then subtract every Digit in the Product from its right hand Digit, and the Remainder is the Value sought in Gallons and Parts. Thus

in the 3d Example, the Product of .9876 by 4 is 39.504 (cutting but 3 off, according to the Rule): Then I begin with 4, and

take that from (0) which I suppose next it towards the right hand, that is, from 10, rests 6; 1 borrow'd and 0 (in Tens place) is 1 from 4 (in Units place) there rests 3; 5 from 10 rests 5; 1 borrow'd and 9 is 10 from 15 rests 5; 1 borrow'd and 3 (next the left hand, Column 2.) is 4, from 9 rests 5, and 0 from the said 3 rests 3; which said Rests are the Answers in Column 3, viz. 35.5536.

10. The 2 short Rules above being observed, the Decimal of a Barrel is found in Gallons, Pints, and Quarters of a Pint, very concisely thus:

Example 1.  
Barrels Beer .19765

7.9060  
Rests Gallons 7.11540

Pints 0 : 3<sup>6</sup> { Quarters of a  
Pint and Parts.

Example 2.  
Barrels Beer .8764

35.056  
Rests Gallons 31.5504

Pints = 4 : 2 quarters Pints.

I have contrived these 2 last for the Use of his Majesty's Officers of Excise, where they may almost by Inspection see the Value of the Decimal of a Barrel, to the 10th part of a Pint.

11. A most concise and easy Way to find the Value of the Decimal of a Pound Averdupois Weight exactly.

Rule 1.] This may be done by this Rule: Multiply the Decimal given by 3, and cut off the Product 2 places fewer than are in the Decimal given; from that Product subtract a 7th of itself; then take a 16 of the Remainder, (which you may do, only putting down



down the Quotient, which are Ounces; and the Remainder (less 1) are Drams, *per* this Example.

2. But the Rule to be insisted upon as more brief and accurate, is this: Multiply the given Number by 8, and double the Product of every 2 Digits as you go on, and the Product is Ounces and Parts. Then multiply in like manner the Parts of an Ounce, and the Product is Drams and Parts.

.924

277.2

39.6

237.6

3 14:12 3.

Example 1.

Given = lb .924

Answer { Ounces = 14.784  
Drams = 12.544

Example 2.

lb .567

Oun. = 9.072

Dr. = 1.152

Example 3.

lb .234

Oun. = 3.744

Dr. = 11.904

Or if it be thought burthensome to the Memory to multiply by 8, &c. as *per* the Rule above, it will be more easily done thus:

Rule 3.] Multiply the Decimal given by 6, and after you have added the Tens carried (as usual) add thereto the Digits next towards the right hand of that which you last multiplied, and put down what is above 10 (as commonly): But when you have multiplied the Digit next the left hand, add the Tens carried to that Digit. For instance, in the first Example above say 6 times 4 is 24, put down 4, and carry 2; 6 times 2 (the next Digit) is 12, and 2 carry'd is 14, and the 4 in Units place is 18, put down the 8, and carry 1; 6 times 9 is 54, and 1 carry'd is 55, and 2 (the middle Digit) is 57; put down 7, and add the 5 carry'd to the 9 last multiplied, and that makes 14: which I take to be the most concise and easy Method that this thing is capable of, unless as by the following

Rule 4.] Take the Primes place for so many Ounces, Halfs and Tenths of an Ounce; Seconds place for so many 2.5 Drams; and Thirds place for so many Quarters of a Dram.

Example. In .924 lb .9 is 9 3, 9 Halfs, and .9 = 3 14: 6 3. 2 in Seconds place is 2 times 2.5 or 5 3; and 4 in Thirds place is 1 3 = 14 3 12 3.

12. The most easy and short Way of finding the Value of a Decimal of 1 lb Troy Weight in 3, dw. and grs.

1st, Multiply the Decimal given by 2, and add what is carry'd, and the Digit next the right hand (as in the third Rule) gives the Ounces.

2dly, Twice Primes place in the next Decimal of Ounces is so many Penny-Weight, and every 5 in the Seconds place is 1 more.

3dly,

3dly, What the Seconds place is under or above 5, and the Thirds place; take half thereof for Grains. But if they are 25 or upward, deduct 1; if 38 or upward, deduct 2. And if the Digit in Thirds place be an odd one, you may deduct only the odd one at 25, and 1 and the odd one at 38, &c.

Examples.  $\text{℥}$ . dw. gr.  
 $\text{℔}$  .5642 or 6 : 15 : 10  
 $\text{℥}$  6.7704 do this by Inspection.

$\text{℔}$  .9782 or 11 : 14 : 18 | dw. gr.  
 $\text{℥}$  11.7384 by Inspection is 14 : 18

### SECT. III. Addition of Decimals.

**T**HIS Rule has no more difficulty than that of one Denomination in whole Numbers: observing,

1. To place the Decimal Places next the Point, or the several Places next the left hand, and the Points themselves, &c. one under another, as in the Examples.

2. If the Decimals to be added exceed not 5 in number, it is sufficient if they consist of 5 places where the Sum is not to be multiplied by any thing; but where it is to be multiplied, you must observe the Rules given in the beginning of Division, *Seet. 6.* of this Chapter. And if Decimals of a Pound Troy are to be added, the Decimals to be added ought to consist of at least 6 places: But if the Decimals be compleat, it is no matter how few the places be.

#### Example 1.

.925.  
 .0775.  
 .5.  
 .25.  
 .75.  
 .05.  
 .125.

Sum 2.6775.

#### Example 2.

| Decimals of a<br>Pound Sterl. | Value in<br>Specie. |
|-------------------------------|---------------------|
| .9. ———                       | 18 s. — d.          |
| .05. ———                      | 1 : —               |
| .125. ———                     | 2 : 6               |
| .75. ———                      | 15 : —              |
| .775. ———                     | 15 : 6              |
| .65. ———                      | 13 : —              |
| .825. ———                     | 16 : 6              |
| .15. ———                      | 3 : —               |

Sum 4.225. = l. 4 : 4 : 6 Sum Proof.

3. But

3. But because the Decimals that occur in Practice are generally, if not always promiscuous, that is, neither all compleat Decimals, nor infinite, (certain or uncertain) therefore Examples of such mixt will be the most useful.

| <i>Example 3.</i><br><i>Of a mixt Number.</i> | <i>Example 4.</i><br><i>Promiscuous Decimals.</i> | <i>Example 5.</i><br><i>Promiscuous Decimals,</i><br><i>&amp;c.</i> |
|-----------------------------------------------|---------------------------------------------------|---------------------------------------------------------------------|
| 198765.                                       | .17565                                            | .9666666                                                            |
| 19876.5.                                      | .6 r                                              | .12345                                                              |
| 1987.65.                                      | .72939                                            | 7.6777777                                                           |
| 198.765.                                      | .5.                                               | 4.00276                                                             |
| 19.8765.                                      | .17234                                            | .9879879                                                            |
| 1.98765.                                      | .97548                                            | .12729                                                              |
| <hr/>                                         | .9 r                                              | <hr/>                                                               |
| Sum 220849.77915.                             | .87654                                            | .9.                                                                 |
|                                               | .25.                                              |                                                                     |
|                                               | .09842                                            | 14.78593 Sum.                                                       |
|                                               |                                                   | or 14.7859324 r 3                                                   |
|                                               | { Sum=5.44447<br>or = 5.444486 r 1                |                                                                     |

By the 3 first Examples you see how easy the Addition of Decimals is, when placed as they ought to be, and so many are cut off for Decimals in the Sum, as are the greatest Number of places of those given; which is sufficient for the understanding of this Rule.

But because some have made a difficulty where there was none, by talking much of repeating Numbers; I have given the two last Examples, to shew that the common Method is accurate enough, without taking Notice of repeating Digits as differing from others.

In Example 4. there being 2 Decimals consisting of repeating Digits, I put down only those next the Point, and sum up them, 1st saying 9 and 6 is 15, and 1 carry'd (supposing the 9 and 6 which repeat were placed 2 places more to the right hand than 5 places, the greatest Number given) is 6: which I put down as you see in the lowest Line, and say 1 and 2 is 3, and 4 is 7, and 8 is 15, and 4 is 19, and 9 is 28, and 5 is 33, and (coming downward again at every Series you add upward, with the repeating Digits) 6 is 39, and 9 is 48; put down 8 and carry 4 to the next Series, which makes it 29, and the 6 and 9 repeating is 44; put down 4, and carry 4 to the third Series, which makes it 39, and the 6 and 9 Repeaters is 54; put down 4, and so proceed in like manner to the Digits next the Points, where the 6 and 9 are added in Course, as being there placed; so the Sum

U

is



is 5.444486 & 1. And if you had filled only 5 places each with the 2 Digits that repeat, and had added them as the 3 former Examples, the Sum would have been the same, wanting only .000016, which supposing even the Decimal of a Pound Troy, is less than a 10th of a Grain. And in the 5th Example the Difference between the Sums having regard to the repeated Digits, and the Sum the common way is so inconsiderable, as appears by the 2 Sums; not worth notice, if the 2d Rule of this Section be observed.

I have demonstrated in the 2d Example, by the Addition of the Specie answering the Decimals, that the Rules for adding Decimals are right: I shall here shew that the Rule for adding these is agreeable to that for Vulgar Fractions. For example, in adding  $\frac{1365}{10000}$  to  $\frac{9600}{10000}$ , the common Denominator is 10000, the Numerators 36500 and 96000, the Sum of which is 132500; so the Answer, cutting off the Cyphers as useless, is  $\frac{1325}{1000}$ : and dividing the Numerator by the Denominator, that is, cutting off 3 Figures, the Answer is 1.325: But the superfluous trouble of putting down the Cyphers being omitted, the Work of Addition of Decimals is as above, and as per Margin.

.365

.96

1.325

## SECT. IV. Subtraction of Decimals.

THERE is no difference between the Method of this and Intire Numbers, observing to place the Point of the Subtrahend exactly under that of the greater Decimal. Two or three Examples will suffice to shew it.

Example 1.

From .9876  
Take .87654321

Remains = .11105679

Example 2.

From 98.76  
Take .08765

Remains = 98.67235

Example 3.

From 9876.  
Take .1234

Rem. = 9875.8766

Example 4.

From 123.45  
Take 97.

Rests = 26.45

Example 5.

From l. 1.0976 — or from 1 : 1 : 8.9856  
Take .987 — or take 0 : 15 : 12.672

Difference .1106 — — or = 0 : 1 : 12.3136

Averdupois Weight.

lb 3 3

The

The Reason of the Work of Subtraction of Decimals is the same as is said of Addition, only subtracting instead of adding the Numerators; agreeing exactly with the Deduction of Vulgar Fractions, in which Method if the Cyphers be neglected in the Results, the Method falls just into that of Subtraction of Decimals, where the cutting off from the Sums, Remainders, Products, &c. is the same as dividing by a Unit with Cyphers (which is the Nature of the Denominators of Decimals) as is shewed in Division of Intire Numbers.

SECT. V. *Multiplication of Decimals.*

**T**HERE is no Difference between the Operations here and by Integers, but observe this Rule:

After the Work is over, you must set off so many Figures towards the right hand of the Product, as you have Decimals in both the Factors.

But if so many places are not in the Product (as it will happen when you multiply Decimals of small Value) then you must make up that Number by placing Cyphers towards the left hand of the Product next the Point.

*Example 1.*  
*A Decimal by an Integer.*

$$\begin{array}{r} .012345. \\ 932 \\ \hline 24690 \\ 37035 \\ 111105 \\ \hline 11.505540 \end{array}$$

*Example 2.*  
*A mixt by a mixt Number.*

$$\begin{array}{r} 9.37241. \\ 25.324. \\ \hline 3748964 \\ 1874482 \\ 2811723 \\ 4686205 \\ 1874482 \\ \hline 237.34691084 \end{array}$$

*Example 3.*  
*A mixt Number by a Decimal.*

$$\begin{array}{r} 36.252. \\ .00032. \\ \hline 72504 \\ 108756 \\ \hline .01160064 \end{array}$$

*Example 4.*  
*A Decimal by a Decimal.*

$$\begin{array}{r} .12564 r 1 \\ .00009 \\ \hline .0000113079 r 1 \end{array}$$

*Note,* That in the fourth Example, because the 4 is repeated *ad infinitum*, therefore I say, 9 times 4 is 36, and 3 (which would be carried if you actually put down another 4) is 39; put down 9 and carry 3.

U 2

Now

Now if you had put down and multiplied 100 Fours of those repeated, so many Nines would also be repeated in the Product; but for brevity sake I only put down one of each with an r.

*To multiply mixt Numbers, Decimals, &c. and to have only so many Decimals in the Product as you assign: And how to avoid all unnecessary Figures in such Operations.*

**Case 1.** A mixt Number by a mixt, as 1.234504 by 9.2123, and to have only 4 Decimals in the Product.

1.234504  
3212.9. = the Multiplier inverted.

IIIIIO5  
2469  
123  
25  
4

11.3726 = the Product.

**Case 2.** A Decimal by a whole Number, as .1234504 by 92123, and to have 3 places in the Product Decimals.

.1234504  
32129 = the multiplier inverted.

IIIIIO536  
246901  
12345  
2469  
370

11372.621 = the Product.

**Case 3.** A Decimal by a Decimal, and to have only 4 Decimals places in the Product; as .12345 by .92123

.12345  
.32129. = the Multiplier inverted.

IIII  
25  
1

.1137 = Product require d.

2. To



1. To perform these Operations, it is plain that the Multiplier is inverted, and Units place put next the left hand, &c. when you place it down to multiply.

2. The due placing of it is the next thing, which is to put Units place of the intire part under that place in the Decimals of the Multiplicand, which answers that place next the right hand of the Decimals you would have in the Product: As in the first Example, because I would have 4 places Decimal in the Product, therefore I put 9 (the Units given) under the 4th place from the Point in the Multiplicand. And in the 2d Example, I put 3 (the Units place of the Multiplier given) under the 3d Decimal of the Multiplicand; and in the 3d Example, because I would have 4 Decimals in the Product, I put the Digit next the Point in the Multiplier under the 3d place of the Multiplicand, because there is no intire Number to put under the 4th place; and place all the other Figures of the Multipliers in a reverse Order, as *per* the Example.

3. Begin to multiply the Figure next the right hand of the Multiplier, when placed as *per* Rules above, into the Figure standing over it, &c. as in the common Way; observing at the same time what Tens would be carry'd if you multiplied the next Figure or two toward the right hand, and adding such Tens and half Tens or upward as Ten, and so proceed with each Figure in the Multiplier that hath another standing over it, (omitting the rest) and place the 1st Product of each Figure, *viz.* what the same is above ten or tens Units, under the Units of the first Line, &c. and not as in common Multiplication; which Lines add up, and the Aggregate is the Product required. The 3 Examples above make it plain, and that much Trouble and many superfluous Figures are prevented.

Case 4. *But it may sometimes happen*, That, as in the following Example, so many places are not in the Sum as you proposed for Decimals: in which case add 1 or more Cyphers.

Thus to multiply .12345 by .234, see the Work in the Margin, to have 4 Decimal places in the Product; where the making of 13 Digits are saved.

Case 5. *Also it may happen*, That there are not so many Decimal places in the Multiplicand as you propose shall be in the Multiplier; so that you cannot proceed as *per* Rule 2. above: in this Case you may sup-

$$\begin{array}{r}
 .12345 \\
 432. \\
 \hline
 247 \\
 37 \\
 5 \\
 \hline
 .0289 = \text{Product.}
 \end{array}$$

ply the place by Points : thus, to multiply 32.34. by 7.93245 see the Work in the Margin, to bring out 4 places Decimal.

These are all the Variety I could think of as to this matter, which are mostly not extant before that I know of: And the following Compendiums are intirely my own Contrivance; and if any of them are extant, I can assure the Reader I borrow'd nothing therefrom.

32.34. .

54239.7

22638. .

29106. .

9702 .

647

129

16

256.5354

### New COMPENDIUMS in Multiplication.

1. To multiply by any Factor under 20 at once.

*Example* 3.4567  
by .19.

*The common Way* 3.4567  
.19.

.656773 Product.

311103

34567

The Probation .656773

2. To multiply by any Factor between 100 and 110 (exclusive) at once.

*Example* 34567  
.108

*The common Way* 34567  
.108

3733.236 Product.

276536

34567

Prob. = 3733.236

3. To multiply by any Factor between 111 and 119 (inclusive) at once.

*Example* .34567  
11.7.

*The common Way* .34567  
11.7.

4.044339 Product.

241969

34567

34567

Prob. = 4.044339

4. To

# SECT. V. *Multiplication of Decimals.*

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4. To multiply by any Number between 1111 and 1119 (inclusive) at once.

*Example* 345 67  
11.14.

*The common Way* = 345 67  
11.14.

385076.38 = Product

138268  
34567.  
34567  
34567

Prob. = 385076.38

5. To multiply by a Factor between 11111 and 11119 (inclusive) putting only the Product.

*Example* 345 67  
111.16.

*The common Way* = 345 67  
111.16.

3842467.72 = Product.

207402  
34567  
34567  
34567  
34567

Prob. = 3842467.72

6. To multiply by a Factor between 20 and 100 exclusive.

*Example* 3.4567.  
65

*The common Way* = 3.4567  
by .. 65

224.6855 = Product.

172835  
207402

Prob. = 224.6855.

7. To multiply by a Factor consisting of a Cypher between two Digits.

*Example* .34567  
605

*The common Way* = .34567  
605

209.13035 = Product

172835  
207402

Prob. = 209.13035

8. To



8. To multiply by a Factor consisting of 2, &c. Cyphers between two Digits.

*Example* 
$$\begin{array}{r} 34567 \\ 6005 \\ \hline \end{array}$$

Product = 207574835

*The common Way* 
$$\begin{array}{r} 34567 \\ 6005 \\ \hline \end{array}$$

$$\begin{array}{r} 172835 \\ 207402 \\ \hline \end{array}$$

Prob. = 207574835

9. To multiply by a Factor consisting of so many Cyphers between two Digits, as there are Places in the Multiplicand.

$$\begin{array}{r} 34567 \\ 6000005 \\ \hline \end{array}$$

Product = 207402172835

10. To multiply by any Factor between 100 and 200; only putting down the Product.

*Example* 
$$\begin{array}{r} 23456 \\ 154 \\ \hline \end{array}$$

Product = 3612224

*The common Way* 
$$\begin{array}{r} 23456 \\ 154 \\ \hline \end{array}$$

$$\begin{array}{r} 93824 \\ 117280 \\ 23456 \\ \hline \end{array}$$

Prob. = 3612224

11. To multiply by a Factor consisting of a Unit between any 2 Digits.

*Example* 
$$\begin{array}{r} 23456 \\ 416 \\ \hline \end{array}$$

Product = 9757696

*The common Way* = 
$$\begin{array}{r} 23456 \\ 416 \\ \hline \end{array}$$

$$\begin{array}{r} 140736 \\ 23456 \\ 93824 \\ \hline \end{array}$$

Prob. = 9757696

Thus far by only putting down the Figures of the Product.

12. From

12. From what is above, it is plain that any Number may be multiplied by a Factor, being the Square of any in the Propositions above, or the Rectangle of any two of them, by only making 1 Line besides the Product. For knowing that 1235 is the Product of 65 by 19, multiply for example 34567 by 19 (as *per* Case 1.) produceth 656773; and that by 65 (as *per* Case 6.) produceth 42690245, the same as 34567. by 1235.

|                                                                                                                |                                                                                                                        |
|----------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------|
| <i>Example 1.</i>                                                                                              | <i>Example 2.</i>                                                                                                      |
| $  \begin{array}{r}  34567 \\  \times 19 \\  \hline  656773 \\  \times 65 \\  \hline  42690245  \end{array}  $ | $  \begin{array}{r}  34567 \\  \times 11116 \\  \hline  384246772 \\  \times 6005 \\  \hline  42690245  \end{array}  $ |
| $42690245 \mid \text{Prod.} = 2307401865860$                                                                   |                                                                                                                        |

$$\begin{array}{r}
 \text{or } 34567 \\
 \text{by } 66751580 \\
 \hline
 276536 \\
 172835 \\
 34567 \\
 172835 \\
 \times 241969 \\
 \hline
 207402 \\
 207402 \\
 \hline
 \text{Proof} = 2307401865860
 \end{array}$$

The Factor 66751580 being the Rectangle of the 2 towards the left hand; therefore 34567 is multiplied by 66751580, by only making 9 Figures besides the Product; which is 32 fewer than must be put down and added in the common way.

13. Or if a Factor or Multiplier consist of the Figures in any two of the former Digits, &c. put down in course; the same may be performed as follows:

|                                                                                                                                                                                                                                                  |                                                                                                                                                                                                                                                                     |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $  \begin{array}{r}  12.3456732. \\  \times 111166005 \\  \hline  741357675660 = \text{the multipl. by } 6005. \\  1372345032912 = \text{the multiplied by } 11116. \mid \\  \hline  = 1372419168.6795660 = \text{Product.} \mid  \end{array}  $ | $  \begin{array}{r}  \text{The common Way } 12.3456732. \\  \times 111166005 \\  \hline  617283660 \\  740740392 \\  740740392 \\  \times 123456732 \\  \hline  123456732 \\  123456732 \\  123456732 \\  \hline  \text{Proof} = 1372419168.6795660  \end{array}  $ |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

By this Method there are put down and added 38 Figures fewer than in the common way.

See also the Proof of the last Example in the Book save one, where 9 Places are multiplied by, and only 5 Lines made.

14. To multiply by a whole Number consisting of any Digit repeated any number of times, as suppose by 7 repeated 8 times.

|                                                                                                 |                                                                                                                                                                                                        |
|-------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $\begin{array}{r} 12345678932 \\ 77777777 \\ \hline 86419752524 \\ \dots\dots\dots \end{array}$ | <i>Common Way</i> $1234.5678932.$<br>$77777777$<br><hr/> $86419752524$<br>$86419752524$<br>$86419752524$<br>$86419752524$<br>$86419752524$<br>$86419752524$<br>$86419752524$<br>$86419752524$<br><hr/> |
|-------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

960219462886694164 Product.

*Note,* This Invention is so much shorter than the common Way, that in this Example there are 77 Figures fewer.

Proof = 960219462886694164

15. To multiply by a Decimal consisting of any Digit repeated *ad infinitum*, very easily, briefly, and accurately.

*Note,* This is done with 84 Figures fewer than the common Way, and is about 100 nearer the Truth.

In this Case cut one place more from the Product than the Decimal Places in the Multiplicand, for Decimals of the Product; or so many more than one as there are Cyphers before the repeated Digit of the Multiplier.

$$\begin{array}{r} 12345678932 \\ .77777777, \&c. \\ \hline 86419752524 \end{array}$$

Product 9602194724.8 r 1

16. To multiply by a mixt Number, consisting of any Number of Digits repeated.

*Example.*

$$\begin{array}{l} \text{as } 12345678932 \\ \text{by } = 7777.7777, \&c. \end{array}$$

$$\text{by } 7 = 86419752524$$

$$\text{Prod.} = 96021947248888.8 r 1$$

*For Proof of this.*

$$\begin{array}{r} 12345678932 \\ \text{by } 7777. \end{array}$$

$$= 96012345054164$$

$$\text{and by } .7, \&c. = 9602194724.8$$

$$\text{Sum} = 96021947248888.8 r 1$$

*Directions*



*Directions concerning the 16 Compendiums above.*

As to *Compend. 1.]* It is performed as in common Multiplication by 9, only adding the Digit toward the right hand of every one that you multiply by the 9; and when you have multiplied the last in order, add what you carry to that Digit next the left hand, and that gives the 2 last in the Product.

*Compend. 2.]* Work as in the last, only with this difference, that you must add the 7 in Units place to the Product of 8 by 5, and the 6 to the Product of 8 and 4, &c. Lastly, add what you carry to the 4, is 7, and then put down the 3 next the left hand.

*Compend 3.]* When there is 1 in Units and 1 in Tens place, besides a place Decimal, or 114, 116, 117, &c. you must add to each Product of every Digit multiplied by 7, the 2 next Digits standing towards the right hand, when there are so many. As to the Product of 6 by 7, I add the 7 next the 6; and then say, 7 times 5 is 35, and 5 carried is 40, and 6 is 46, and 7 is 53; put down 3, and carry 5, &c. Lastly, when you have multiplied the 3 by 7, and added the 4 and 5, add the 3 carried to the 3 and 4 is 10, and 1 carried and 3 is 4.

*Compend 4.]* This is performed as the last, only adding 3 Figures (when you have so many) standing toward the right hand of every Digit which you multiply by the 4. And at last the 3 carried is added to 3, 4 and 5 is 15, and the 1 carried (the 5 being put in the Product) added to the 3 and 4 make 8, which put down, and the 3, as you see in the Example.

*Compend. 5.]* Whereas here you have 4 Units towards the left hand, you must add 4 Figures (when you have so many) standing towards the right hand of every Digit you multiply by 6. And what is carried must be added to the 4 places next the left hand of the Multiplicand at last, and then 3 and then 2, and then put the 3 in the Multiplicand down.

*Compend 6.]* Say 5 times 7 is 35, 5 and carry 3; 5 times 6 is 30, and 3 is 33, and 6 times 7 makes 75; put down 5, and carry 7, and say 5 times 5 is 25, and 7 is 32, and 6 times 6 makes 68; put down 8, and carry 6, and say 5 times 4 is 20, and 6 is 26, and 6 times 5 makes 56; put down the 6 and carry 5, and say 5 times 3 is 15, and 5 carried is 20, and 6 times 4 makes 44; put down 4, and say 6 times 3 is 18, and 4 is 22.

*Compend. 7.]* In this Example, because there is a Cypher between, you must multiply the two first as usual, and to the Product of the 5 in the 3d add what is carried, and the Product of the 6 in the

first or Units place of the Multiplicand. Then to the Product of 5 in the 4th add what is carried, and the Product of 6 in the 2d or 6, &c. and lastly multiply 4, then 3 by the 6, and put the Product down.

*Compend 8.* Multiply as usual to multiply the 4, then say 5 times 4 is 20, and 2 is 22, and 6 times the 7 is 64, 4 and carry 6; 5 times 3 is 15, and 6 is 21, and 6 times the 6 (or 36) makes 57; put down the 7, and multiply the 5, 4 and 3 by the 6, adding the 5 carried.

*Compend. 9.]* In this and the like Cases there is no more difficulty than in multiplying by a single Digit; for the Number of Cyphers being equal to the places in the Multiplicand, the Product by the 6 falls just to the left hand of the Product by the 5. But if the Product of the Digit next the left hand by that in Units place, and what is carried be less than 10, then a Cypher must be put down between the two Products.

*Compend. 10.]* This is performed by considering the Nature of the Operations of the first and 6th Compendiums. For 4 times 6 is 24, 4 and carry 2; 4 times 5 is 20, and 2 carried and 5 times 6 makes 52, 2 and carry 5; 4 times 4 is 16, and 5 is 21, and 5 times 5 is 46, and 6 in Units place makes 52, 2 and carry 5; 4 times 3 is 12, and 5 is 17, and 5 times 4 makes 37, and 5 (in Tens place of the Multiplicand) is 42, 2 and carry 4; 4 times 2 is 8, and 4 is 12, and 5 times 3 makes 27, and 4 (in Hundreds place) is 31; put down 1 and carry 3. Then (having multiplied all by the first Figure) I say 5 times 2 is 10, and 3 carried is 13, and 3 in Thousands place is 16; 1 time 2 is 2, and 1 carried is 3.

*Compend. 11.]* This is performed as the last, only instead of adding the Digit next the right hand, &c. to the Product of the second Figure in the Multiplier, you must here add it to the Product of the first Figure, because the 1 stands next that Digit; and then you keep to the 1st and 6th Compendiums, as in the last.

*Compend 12.]* By this Compendium is shewn how to multiply by 66751580, or other large Number produced by the Multiplication of 2 or Square of one known Number, by only making 1 Line besides the Product. The Example is that of the 5th and 8th.

*Compend. 13.]* This is likewise an Example shewing how the Work of Multiplication by large Numbers may be shorten'd, by dividing such large Number into 2, 3, &c. such Numbers as fall within some of the abovesaid Rules: as 111166005 I divide into two Multipliers, 11116 and 6005, &c. done at twice.

*Compend.*

*Compend. 14.]* This Compendium is performed, first, by multiplying the given Number by one of the repeated Digits, as in the Example above the Product is 86419752524, from which the general Product is found thus; Add that Product from the right hand toward the left, and say 4 and 2 is 6, and 5 is 11, (put down the 4, 6 and 1, and for every 10 you arrive at, put 1 to the Sum more than what is wrote down) and 1 is 12, and 2 is 14, and 5 is 19, and 7 is 26, put down the 4, 9 and 6; then adding 1 more for the 2d 10, I say, and 1 is 27, and 9 is 36, and 1 (for that 3d 10) is 37, and 1 next the 9 is 38; so I put down the 6 and 8.

Then because I have added as many places as are in the Multiplier, I leave out Units place (4) and add from 2 in Tens place to 4, the 3d Figure from the left hand inclusive, which with the 3 carried (part of the said 38) makes another 38; put down that 8, and carry 3, so adding that to 5, &c. to 6, the 2d Figure from the left hand (leaving out the 2 first Figures next the right hand) that Sum is 42; I put down the 2, and carry 4, adding that to 2, (the 4th from the right hand, leaving out the 524) and so forward to 8, next the left hand inclusive, the Sum is 46: I put down the 6, and add the 4 to the Figures between 5 and 8 inclusive, (leaving out the 2524) and the Sum is 44; I put down 4, and add 4 to the Figures between 7 and 8 inclusive, (leaving out the 52524, and so every time I leave out 1 more) as 4 and 7 is 11, and 9 is 20, &c. to 8 makes 39; put down the 9, and add 3 to 86419, as they stand from the right to left, as before, and the Sum is 31: put down 1, and add 3 to 8641, as taught before, and the Sum is 22; put down the 2, and add 2 to the 864 as aforesaid, and the Sum is 20; 0, and carry 2 and 6 is 8, and 8 is 16; put down 6, and say 1 and 8 is 9: so have you the Answer only by easy Additions.

*Compend. 15.]* If the repeated Figure be a Decimal for the Multiplier, I multiply as before to gain the 86419752524; whence the Product is found, first by adding that whole Line from the right to the left hand, which makes 53: I add the 5 to the 3 is 8, which I put down for the repeating Digit of the Product. Then add the 5 carried to 2 (in Tens place) and that to the rest of the Line towards the left hand makes 54; put down the 4, and carry 5 to the 5, (in Hundreds place) &c. adding all toward the left hand as before, leaving out 1 toward the right for every Digit put in the Sum: which is a most easy and brief Way of answering all Questions of that kind.



*Compend. 16.]* If the Multiplier be a repeated Digit and a mixt Number, and the Multiplicand an intire Number, you have nothing to do more than in the last Compendium: only to put so many of the repeated Digits toward the right hand of the integral part of the Product, as there are places in the integral part of the Multiplier, as in the Example 16. above.

A second Example of the 16th Compendium, to multiply 1234 by 333.3333333, &c. or 333.3 *r*. See the Operation.

$$\begin{array}{r} \text{Multiply } 1234 \\ \text{by } \text{---} 333.3333333, \text{ \&c.} \\ \hline 3702 \end{array}$$

$$411333.3 \text{ } r = \text{Product.}$$

$$\begin{array}{l} \text{Proved thus --- } 1234 \\ \text{as per 15th by } .3 \quad .3 \text{ } r \\ \hline \text{is } = 411.3 \text{ } r \text{ } 1 \\ \text{and by } 333 = 410922 \end{array}$$

$$\text{Sum} = \text{Proof} = 411333.3 \text{ } r$$

In this (as the last) having multiplied the 1234 by 3, it produces 3702, which I add from the right hand towards the left, and the Sum is 12; so 2 and the 1 is 3 = the repeating Digit, and 1 carried added to 0, 7 and 3 is 11: I put 1 down, and carry 1 and 7 is 8, and 3 is 11; put 1 down, and carry 1 to 3 makes 411.3 *r*; and because there are 3 places Integers of the Multiplier, I therefore put 3 places of the repeating Figure towards the right hand of the 411, makes 411333.3 *r* the true Answer, as by the first Example and Rules for working the same under the last mentioned Compendium.

Or if there be Decimal places in the Multiplicand: you may omit so many of the repeating Integers in the Product: as in the above Supposition 1.234, then the Product will be 411.3 *r* as per the Rule under the 15th Compendium.

17. Or if it should be supposed (tho' such a thing does not often happen) that the Decimal Digits repeat in both the Factors: then work as under the 15th Compendium, and to that Result add the Product by the next, &c. Digits, putting down what would be carried if the Decimal Digit were really repeated.

$$\begin{array}{r} \text{Example} \quad 3.6 \text{ } r \\ \quad \quad 4.3 \text{ } r \\ \hline \text{Product} = 110 \end{array}$$

$$\begin{array}{l} \text{Sum} = 1.22 \text{ } r \\ \text{Product} = 14.6 \text{ } r \end{array} \}$$

$$\text{Sum} = \text{Answer} = 15.8 \text{ } r \text{ } 1$$

$$\text{Proof } 3\frac{2}{3} = 1\frac{1}{3}$$

$$4\frac{1}{3} = 1\frac{1}{3}$$

$$\text{and } \frac{1}{3} \text{ by } \frac{1}{3} = \frac{1}{9} = 15.8 \text{ } r \text{ } 1$$

18. But

18. But if the repeating Digits in both Factors begin after the Primes place, (altho' the Case may rarely happen) you may proceed as in the following Example.

To multiply 27.16 *r* 1 by 5.23 *r* 1.

27.1666666, &c.

5.23333333, &c.

.8150 = Product by .03 in 27.16 *r* 1

.905 *r* 1 = Sum of the last Line, as *per* 15th, &c. Compend.

5.433 *r* 1 = 27.16 *r* 1 in .2

135.833 *r* 1 = 27.16 *r* 1 in 5

142.172 *r* 1 = the true Product, or Sum of the three last Lines.

But if the Product of 3 by 6 and what is carried had not amounted to even Tens (as here 20) then the Digit in Units place of the 1st Product will repeat, and must be so considered in adding them thus

27.17 *r*  
by 5.23 *r*

.8153 *r* 1

.90592 *r* 3

5.435 *r* 1

135.8 *r* 1

142.23037 *r* 3

In this Case I suppose 3 repeated so often in .8153 (as .81533) till by adding, it brings out the .90592, and shews the repeating of the 592.

I need say no more to illustrate the last Example; and having now finished what I thought might be of real use in Operations of Multiplication, whether by Decimals or Integers; I shall not trouble my Reader with useless as well as troublesome and tedious Speculations. But for the Solution of all Questions in Practice, as of Concrete Numbers, I refer to the third Rule in the next Sect. (6.) for making your Decimal longer or shorter, as occasion requires, according to the Greatness of the Number by which such Decimal is to be multiplied: for which purpose a Table is there inserted, to direct on almost all Occasions.

These

These 18 Compendiums, as well as the 3 or 4 Cases preceeding them, are new to me; tho' I am since my writing them informed, that one or two of the first and most easy of these Compendiums are in Mr *Leyburn's* Arithmetic; however that be, I have no reason to omit any, as being my own Contrivance.

These Operations, altho' at first they may seem difficult, and to burthen the Memory; yet I can assure the Reader, that by Use they will become as easy as common Multiplication, (as I know by Experience) and that they very much abbreviate the Operations, cannot be deny'd: nay, many of these new Compendiums are not only shorter, but easier and more accurate than the common Way. And I doubt not but these, and those many in Reduction of Decimals, being new and great Improvements, will be accordingly received by the Ingenious.

*That Multiplication of Decimals is done by the same Rule as Vulgar Fractions.*

To instance in multiplying .126 by .74: According to the Rule of Vulgar Fractions, the Product of the Numerators is 9324, of the Denominators 100000, the new Fraction  $\frac{9324}{100000}$  as a Vulgar; or as a Decimal, by the Rule in Notation, .09324, that is, 9324 being divided by 100000, quotes .09324.

Hence appears wherein the Preference of the Decimal Fraction consists: As,

1. In the easy Multiplication of the Denominators, which is only putting the Cyphers of the one toward the right hand of the other.

2. In that the Decimal can be expressed without its Denominator. And,

3. In Case of finding the Value of the Decimal Fraction, the dividing by the Denominator is done by Inspection, which in Vulgar Fractions often requires a tedious Operation, &c.

$$\begin{array}{r} .126 \\ .74 \end{array} \} \text{Numerators.}$$

9324 Product.

$$\text{Denominators } \left\{ \begin{array}{l} 1000 \\ 100 \end{array} \right.$$

$$\text{Product} = 100000$$

that is  $\frac{9324}{100000}$  as Vulgar.  
or .09324 as Decimal.



## SECT. VI. Division of Decimals.

**T**HIS Rule is the same, as to the Operation, with Division of Intire Numbers; but it must be observed,

1. To know how many Cyphers to add to the Dividend, that so the Quotient may have a competent Number of Decimal Places, take this

Rule. Consider how many Decimals will suffice to be in your Quotient, (upon the Foundation of the Rule of Decimals, as per the little Table following the next Rule) and add that Number to the Decimal places in the Divisor, and then make the Decimal places in the Dividend equal to that Sum, by putting Cyphers toward the right hand, if need be.

2. To know (when the Work of Division is ended) how many Places of the Quotient are Decimal.

Rule. The difference between the Decimal places in the Dividend and Divisor, are the Decimal places toward the right hand of the Quotient.

But if so many Places be not in the Quote as the said difference is, you must make up that Number by prefixing Cyphers towards the left hand, as per the Rule in Reduction.

I could here give another Rule for this purpose, but it not being so intelligible and useful in all Cases, the inserting of it may only serve to hinder the Learner in his Progress: I therefore omit it.

A few Examples will enable the Reader to apply the Rules above, and fully to understand Division of Decimals; as in the 9 Cases following. But first observe,

3. The Number of Places in a Decimal sufficient, when to be multiplied as followeth.

| In the Decimal of<br>l. s. Sterling. |                                                   | In the Decimal<br>of 1 Shilling. |                                                   | In the Decimal<br>of 1 lb. Troy. |                                                   | In the Decimal<br>of 1 lb. Averd. |                                                   | In the Decimal of<br>1 Barrel Beer. |                                                   |                                                                                                    |
|--------------------------------------|---------------------------------------------------|----------------------------------|---------------------------------------------------|----------------------------------|---------------------------------------------------|-----------------------------------|---------------------------------------------------|-------------------------------------|---------------------------------------------------|----------------------------------------------------------------------------------------------------|
| Pla-<br>ces<br>suffi-<br>cient.      | When to be<br>multiplied<br>by these or<br>under. | Pla-<br>ces<br>suffi-<br>cient.  | When to be<br>multiplied<br>by these or<br>under. | Pla-<br>ces<br>suffi-<br>cient.  | When to be<br>multiplied<br>by these or<br>under. | Pla-<br>ces<br>suffi-<br>cient.   | When to be<br>multiplied<br>by these or<br>under. | Pla-<br>ces<br>suffi-<br>cient.     | When to be<br>multiplied<br>by these or<br>under. |                                                                                                    |
| 9                                    | 1000000                                           | 9                                | 20000000                                          | 9                                | 190000                                            | 9                                 | 4000000                                           | 9                                   | 3800000                                           | Without lo-<br>sing a Unit<br>of the least<br>Denomina-<br>tion.<br>Vid. also<br>Case 5.<br>above. |
| 8                                    | 100000                                            | 8                                | 2000000                                           | 8                                | 19000                                             | 8                                 | 400000                                            | 8                                   | 380000                                            |                                                                                                    |
| 7                                    | 10000                                             | 7                                | 200000                                            | 7                                | 1900                                              | 7                                 | 40000                                             | 7                                   | 38000                                             |                                                                                                    |
| 6                                    | 1000                                              | 6                                | 20000                                             | 6                                | 190                                               | 6                                 | 4000                                              | 6                                   | 3800                                              |                                                                                                    |
| 5                                    | 100                                               | 5                                | 2000                                              | 5                                | 19                                                | 5                                 | 400                                               | 5                                   | 380                                               |                                                                                                    |
| 4                                    | 10                                                | 4                                | 200                                               | 4                                | 1.9                                               | 4                                 | 40                                                | 4                                   | 38                                                |                                                                                                    |
| 3                                    | 1.0                                               | 3                                | 20                                                |                                  |                                                   | 3                                 | 4.0                                               | 3                                   | 3.8                                               |                                                                                                    |
|                                      |                                                   | 2                                | 2.0                                               |                                  |                                                   |                                   |                                                   |                                     |                                                   |                                                                                                    |

I have inserted this Table, that you may not run your Decimal to more places than are absolutely necessary.

To apply the first and second Rules above.

*Example 1.* To divide a mixt Number by a mixt, as  $237.34691084$  by  $25.324$ .

*Example 2.* To divide  $11.50554$  by  $932$ , so that six places Decimal may be in the Product.

$932 \overline{) 11.505540} \text{ (.012345 = Quotient as per the two Rules above, adding one Cypher to the Dividend.)}$

$$\begin{array}{r} 2185 \\ \underline{3215} \\ 4194 \\ \underline{4660} \\ 0 \end{array}$$

$$25.324 \overline{) 237.34691084} (9.37241.$$

$$\begin{array}{r} 94309 \\ \underline{183371} \\ 61030 \\ \underline{103828} \\ 25324 \\ \underline{\phantom{0}} \\ 0 \end{array}$$

*Example 3.* To divide  $.1234567$  by  $.321$ , a Decimal by a Decimal, and to have seven Decimal Places in the Quote.

Here according to the first Rule above, to have 7 places in the Quotient, I must have 10 in the Dividend, which I make the other places up by adding 3 Cyphers.

2. According to the 2d Rule, I point off 7 Decimal places in the Quote, because that is the difference between those places in the Dividend and Divisor.

*Example 4.* To divide a Decimal by a whole Number, as  $.463$  by  $3214$ , and to have 8 Decimal places in the Quotient.

$$.321 \overline{) .1234567000} (.3846003.$$

$$\begin{array}{r} 2715 \\ \underline{1476} \\ 1927 \\ \underline{1000} \\ 37 \end{array}$$

*Example 5.* To divide a whole by a mixt Number, so as to have 9 places in the Decimal Part of the Quote; as 9 by  $3.214$ .

$$3.214 \overline{) 9.000000000000} (2.800248911$$

$$\begin{array}{r} 25720 \\ \underline{8000} \\ 15720 \\ \underline{28640} \\ 29280 \\ \underline{3550} \\ 3260 \end{array}$$

46 Rests.

$$3214 \overline{) .46300000} (.00014405.$$

$$\begin{array}{r} 14160 \\ \underline{13040} \\ 18400 \\ \underline{2130} \end{array}$$

*Example*

# SECT. VI. Division of Decimals.

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*Example 6.* To divide a whole Number by a whole, and to have 10 Decimal Places in the Quotient; as 5 by 365.

$$365) 5.0000000000 (.0136986301$$

.....

*Example 7.* To divide a mixt Number by a Decimal; as 1.25999 by .002: and to have 4 Decimal Places in the Quotient.

$$.002) 1.2599900 (629.9950$$

Half the Dividend gives the Figures of the Quote.

*Example 8.* To divide a whole Number by a Decimal, as 999 by .00013; and to have only 3 Decimal places in the Quote.

$$.00013) 999.0000000 (7684615.384$$

.....

*Example 9.* To divide a Decimal by a mixt Number, as .5 by 7.5; and to have 2 places Decimal in the Quotient: which are sufficient in case of Decimals of a Gallon, and the Quotient not required to be multiplied by any thing; as follows.

$$7.5).500 (.06 r$$

$$\begin{array}{r} \text{---} \\ 50 \end{array}$$

$$\begin{array}{r} \text{---} \\ 89 \\ \text{---} \\ 110 \\ \text{---} \\ 60 \\ \text{---} \\ 80 \\ \text{---} \\ 20 \\ \text{---} \\ 70 \\ \text{---} \\ 50 \\ \text{---} \\ 110 \\ \text{---} \\ 60 \\ \text{---} \\ 8 \text{ Rests} \end{array}$$

These are all the Varieties that can happen in Division of Decimals: And the Exemplification of the two Rules first above, appear at one View thus.



| The 9 Cases. | Divisors. | Dividends given. | Cyphers added as per Rule 1. above. | Quotients; having Decimals, as per Rule 2. |
|--------------|-----------|------------------|-------------------------------------|--------------------------------------------|
| 1            | 25.324    | 237.34691084     |                                     | 9.37241.                                   |
| 2            | 932.      | 11.50554         | 0                                   | .012345.                                   |
| 3            | .321      | .1234567         | 000                                 | .3846003                                   |
| 4            | 3214.     | .463             | 00000                               | .00014405                                  |
| 5            | 3.214     | 9.               | 000000000000                        | 2.800248911                                |
| 6            | 365.      | 5.               | 0000000000                          | .0136986301                                |
| 7            | .002      | 1.25999          | 00                                  | 629.9950                                   |
| 8            | .00013    | 999.             | 00000000                            | 7684615.384                                |
| 9            | 7.5       | .5               | 00                                  | .0611                                      |

*Of Decimals by way of Vulgar Fractions.*

To work any Question so as to give the Answer exquisitely (without omitting any thing) by Decimals as well as by Vulgar Fractions. [Note, This is my own Thought, as the rest of what is new in Decimals is.]

It has been one great Objection against Decimals, that where the Decimals in Quotients happen not to be compleat, but infinite, the Answer cannot be exhibited so accurately, as to be truly said to be the whole Truth: But this is easily solved, by considering the Nature of the Remainders in the dividing and making a Fraction of that and the Divisor.

So that the Proof of this depends both upon Vulgar and Decimal Fractions worked promiscuously, or of Decimal worked by way of Vulgar Fractions.

To instance in Example 3. foregoing, the true Quote (as in the Margin below is proved) is =  $.3846003 \frac{0000000000}{321}$ : For the 37 is what remained in dividing the last part of the Dividend; and besides the 2 places next the right hand thereof, there being 8 more Decimal places towards the left hand, therefore so many Cyphers are in the Numerator of that part expressed by way of Vulgar Fractions, which must always be observed.

The Truth of this appears by reducing this mixt Number to one Fraction: for the Product of  $.3846003$  multiplied by the Denominator  $.321$ , produceth  $.1234566963$ ; to which adding the Numerator, the Sum is  $.1234567$ : which placed over the Denominator



If 10 divide 98765.4 the Quote is 9876.54

100 divide it, — — — — 987.654

1000 — — — — 98.7654

10000 — — — — 9.87654

100000 — — — — .987654

1000000 — — — — .0987654

4. Or if 60 divide 98765.4 the Quote is 1646.09

600 — — — — 164.609

6000 — — — — 16.4609

60000 — — — — 1.64609

600000 — — — — .164609

dividing only by the 6, and cutting of so many from the Quote (besides the Decimal Places in the Dividend) as there are Cyphers in the Divisor.

5. That Division of Decimals is performed as that of Vulgar Fractions.

As if  $\frac{18888888}{18888888}$  be divided

by  $\frac{18888}{18888}$ , the Quote will be

$\frac{18888}{18888} \frac{18888888}{18888888} \frac{18888888}{18888888}$

$\frac{18888888888}{18888888888}$  equal to the De-

321) 1234567000 (.3846003

cimal Quote .3846003, by di-

viding the Numerator by 321, and cutting as many from the Quotient as there are Cyphers to the right of 321, as *per* Margin.

#### SECT. VII. The Rule of Proportion by Decimals.

THIS being the same as in whole Numbers, observing Multiplication and Division of Decimals for the proper Rules of pointing out the Products and Quotients; I need only give one Example.

What is the Interest of  $l. 325 : 15 : 0$  for a Year at  $4l. 10s. per Cent. per Annum$ ?

The Answer is  $l. 14.65775$ , as *per* 100. 4.5 :: 325.75

Compend. 6, &c. or  $l. 14 : 13 : 1\frac{1}{2}$ , by the 6th Rule of the Valuation of Decimals.

4.5

l. 14.65775 Answer.

And this puts me in mind of

SECT.



SECT. VIII. *The Use of Decimals in the calculating Interest Simple and Compound.*

**I.** SIMPLE Interest is the Interest of the Principal only, for the Time proposed.

**II.** The true Discount of Money is in proportion to its Principal: As 5 is to 105 (at 5 per Cent.) or as 6 to 106 at 6 per Cent. &c. So that the Discount of any Sum for any time is less than the Interest for that time, and consequently the Discount of any Sum at any rate is found for a Year by this Proportion:

As 100 and the Rate: is to the Rate: so any Sum to the Discount for the same time.

**III.** Compound Interest is the Interest of the Principal and of the Interest put together, (the later not being paid when due, but becomes Principal.)

1. As to Simple Interest, there are Laws in most Countries to ascertain it, that no one shall take more, under the penalty of forfeiting the Principal, &c. as now ours in 5 per Cent. for what is lent for the future.

*To find the Simple Interest of any Sum for a Year.*

(1.) What is the Interest of £. 364 : 17 : 5½ at 5 per Cent ?

As 100. 5 :: 364 : 17 : 5½

Answ. £. 18 : 4 : 10½

Multiply the whole by 5,  
and divide by 100 continually,  
as you see here.

£. 18.24 : 7 : 2½.

20

5. 4.87

12

d. 10.46

4

qr. 1.85

(2.) But if the Shillings, &c. are easily by Inspection reducible to a Compleat Decimal, the best way is to do such Questions decimally.

*Example.*

*Example.* What is the Interest of  $l. 364 : 18 : 0$  for a Year, at 5 per Cent?

See the Operation in the Margin,  
the Answer being  $l. 18 : 4 : 10\frac{1}{4}$ .

$$\begin{array}{r} 100. 5 :: 364.9 \\ \hline 5 \end{array}$$

18.245 Answer.

(3.) If the Interest of any Sum be required for any Number of Days (or part of a Year) 'tis best done by Decimals.

*Example.* Suppose the Interest of the Principal in the last Case is required only for 145 Days:

$$\text{1st say, } 100. 5 :: 364.9$$

the Interest = 18.245 for 1 Year.

Days.  $l.$  Days.

$$\text{2dly, } 365. 18.245 :: 145$$

145

$l.$

$$365)2645.525(7.248\frac{22}{100} = \text{Answer.}$$

905

1752

2925

5

2. And consequently the Interest of the said  $l. 364 : 17 : 11\frac{1}{4}$  for 314 Days is found, reducing the odd Money to Decimals, as per the second Rule in Reduction, which is  $l. .871875$  : Therefore

$$\begin{array}{r} 100. 5 :: 364.871875. 18.2435937 = \text{Interest for a Year.} \\ \text{Days. } l. \quad \text{Days. } l. \end{array}$$

$$365. 18.2435937 :: 314. 15.694 = \text{the Answer.}$$

3. By the same Rule with the two last, the Interest of  $l. 1$  for one Day at any rate is found, and they are Multipliers for finding the Simple Interest of any Number of Pounds for any Days:

$$\left\{ \begin{array}{l} \text{For } 100. 5 :: 1. .05 \\ \text{And as } 365. .05 :: 1. .000136, \text{ \&c.} \end{array} \right.$$

Thus

|                                                   |                                                       |
|---------------------------------------------------|-------------------------------------------------------|
| Thus 1 l. for 1 Day at 4 per Cent. = .00010958904 | } Factors for<br>finding the<br>Interest for<br>Days. |
| 5 per Cent. = .0001369863                         |                                                       |
| 6 per Cent. = .00016438356                        |                                                       |
| 7 per Cent. = .00019178082                        |                                                       |
| 8 per Cent. = .00021917808                        |                                                       |

These Numbers may be thus used: Suppose I would know the Interest of *l.* 400 for 297 Days at 5 per Cent. I multiply .0001369863 by 297, and the Product is the Interest of *l.* 1 for 297 Days; which Product multiplied by *l.* 400, gives the Interest of *l.* 400 for 297 Days = the Answer, *l.* 16 : 5 : 5 $\frac{1}{2}$ .

4. If you divide 100 by the Rate of Interest, it gives the Years Purchase in Fee of Lands Value in many places. And consequently if 100 be divided by the Years Purchase that the generality of Lands are valued at, the Quote gives the proportionable Rate of Interest. Which is a Rule that many go by, to know whether Interest is too high or low, by observing how Lands sell: It happens right with us at present, for 5 per Cent. agrees with 20 Years Purchase of Land.

5. It is easy to find the Simple Interest of any Sum for any Number of Years, as suppose *l.* 100 : 15, for 7 Years at 5 per Cent. say

$$100. 5 :: 100.75. \quad 5.0375 = \text{Interest for 1 Year.}$$

7 mult.

$$35.2625 = \text{Answer, or } l. 35 : 5 : 3$$

6. And consequently for the Interest of a Sum under *l.* 1, as 15 s.

$$100. 5 :: 75. .0375 \text{ for 1 Year; or } .2625 l. \text{ for 7 Years.}$$

7. Finding the Interest of *l.* 732 for 5 Years to be *l.* 183, what is that per Cent. per Ann.? I first divide *l.* 183 by 5 Years, and the Quote is *l.* 36.6. Then say, As 732. 36.6. :: 100. 5 per Cent. Answer.

Here followeth Tables of Interest, Simple, Compound, and Discount, with their Calculation and Use.

Z

A TABLE



# A TABLE of SIMPLE INTEREST at 4 per Cent.

| Principal. | 1 Day.      |    | 2 Days. |    | 3 Days. |    | 4 Days. |    | 5 Days. |    | 6 Days. |    | 7 Days. |    | 8 Days. |    |
|------------|-------------|----|---------|----|---------|----|---------|----|---------|----|---------|----|---------|----|---------|----|
|            | s.          | d. | s.      | d. | s.      | d. | s.      | d. | s.      | d. | s.      | d. | s.      | d. | s.      | d. |
| 1          | 0.000109589 |    |         |    |         |    |         |    |         |    |         |    |         |    |         |    |
| 2          |             |    |         |    |         |    |         |    |         |    |         |    |         |    |         |    |
| 3          |             |    |         |    |         |    |         |    |         |    |         |    |         |    |         |    |
| 4          |             |    |         |    |         |    |         |    |         |    |         |    |         |    |         |    |
| 5          |             |    |         |    |         |    |         |    |         |    |         |    |         |    |         |    |
| 6          |             |    |         |    |         |    |         |    |         |    |         |    |         |    |         |    |
| 7          |             |    |         |    |         |    |         |    |         |    |         |    |         |    |         |    |
| 8          |             |    |         |    |         |    |         |    |         |    |         |    |         |    |         |    |
| 9          |             |    |         |    |         |    |         |    |         |    |         |    |         |    |         |    |
| 10         |             |    |         |    |         |    |         |    |         |    |         |    |         |    |         |    |
| 20         |             |    |         |    |         |    |         |    |         |    |         |    |         |    |         |    |
| 30         |             |    |         |    |         |    |         |    |         |    |         |    |         |    |         |    |
| 40         |             |    |         |    |         |    |         |    |         |    |         |    |         |    |         |    |
| 50         |             |    |         |    |         |    |         |    |         |    |         |    |         |    |         |    |
| 60         |             |    |         |    |         |    |         |    |         |    |         |    |         |    |         |    |
| 70         |             |    |         |    |         |    |         |    |         |    |         |    |         |    |         |    |
| 80         |             |    |         |    |         |    |         |    |         |    |         |    |         |    |         |    |
| 90         |             |    |         |    |         |    |         |    |         |    |         |    |         |    |         |    |
| 100        |             |    |         |    |         |    |         |    |         |    |         |    |         |    |         |    |
| 200        |             |    |         |    |         |    |         |    |         |    |         |    |         |    |         |    |
| 300        |             |    |         |    |         |    |         |    |         |    |         |    |         |    |         |    |
| 400        |             |    |         |    |         |    |         |    |         |    |         |    |         |    |         |    |
| 500        |             |    |         |    |         |    |         |    |         |    |         |    |         |    |         |    |
| 600        |             |    |         |    |         |    |         |    |         |    |         |    |         |    |         |    |
| 700        |             |    |         |    |         |    |         |    |         |    |         |    |         |    |         |    |
| 800        |             |    |         |    |         |    |         |    |         |    |         |    |         |    |         |    |
| 900        |             |    |         |    |         |    |         |    |         |    |         |    |         |    |         |    |
| 1000       |             |    |         |    |         |    |         |    |         |    |         |    |         |    |         |    |
| ditto      | 1.109589    |    | 2.19178 |    | 3.28767 |    | 4.38356 |    | 5.47945 |    | 6.57534 |    | 7.67123 |    | 8.76712 |    |

## SIMPLE INTEREST at 4 per Cent.

| Principal.<br>l. | 9 Days. |    |    | 10 Days. |    |    | 20 Days. |         |    | 30 Days. |    |          | 40 Days. |    |    | 50 Days. |          |    | 60 Days. |    |          |    |    |    |          |    |   |   |          |
|------------------|---------|----|----|----------|----|----|----------|---------|----|----------|----|----------|----------|----|----|----------|----------|----|----------|----|----------|----|----|----|----------|----|---|---|----------|
|                  | s.      | d. | q. | l.       | s. | d. | q.       | l.      | s. | d.       | q. | l.       | s.       | d. | q. | l.       | s.       | d. | q.       | l. | s.       | d. | q. |    |          |    |   |   |          |
| 1                |         |    |    | 1        |    |    |          | 1       |    |          |    | 2        |          |    |    | 1        | 0        |    | 1        | 1  |          |    | 1  | 2  |          |    |   |   |          |
| 2                |         |    |    | 2        |    |    |          | 2       |    |          |    | 1        | 0        |    |    | 2        | 0        |    | 2        | 2  |          |    | 3  | 1  |          |    |   |   |          |
| 3                |         |    |    | 3        |    |    |          | 3       |    |          |    | 1        | 2        |    |    | 3        | 1        |    | 4        | 0  |          |    | 4  | 3  |          |    |   |   |          |
| 4                |         |    |    | 1        | 0  |    |          | 1       | 0  |          |    | 2        | 1        |    |    | 3        | 1        |    | 4        | 1  |          |    | 6  | 1  |          |    |   |   |          |
| 5                |         |    |    | 1        | 1  |    |          | 1       | 1  |          |    | 2        | 3        |    |    | 4        | 0        |    | 5        | 1  |          |    | 7  | 3  |          |    |   |   |          |
| 6                |         |    |    | 1        | 2  |    |          | 1       | 2  |          |    | 3        | 1        |    |    | 4        | 3        |    | 6        | 1  |          |    | 9  | 2  |          |    |   |   |          |
| 7                |         |    |    | 1        | 3  |    |          | 1       | 3  |          |    | 3        | 3        |    |    | 5        | 2        |    | 7        | 1  |          |    | 11 | 1  |          |    |   |   |          |
| 8                |         |    |    | 2        | 0  |    |          | 2       | 1  |          |    | 4        | 1        |    |    | 6        | 1        |    | 8        | 2  |          |    | 1  | 00 | 2        |    |   |   |          |
| 9                |         |    |    | 2        | 1  |    |          | 2       | 2  |          |    | 4        | 3        |    |    | 7        | 0        |    | 9        | 2  |          |    | 1  | 2  | 1        |    |   |   |          |
| 10               |         |    |    | 2        | 2  |    |          | 2       | 3  |          |    | 5        | 1        |    |    | 7        | 3        |    | 10       | 2  |          |    | 1  | 3  | 3        |    |   |   |          |
| 20               |         |    |    | 4        | 3  |    |          | 5       | 1  |          |    | 10       | 2        |    |    | 1        | 3        | 3  | 1        | 9  | 0        |    | 2  | 2  | 1        |    |   |   |          |
| 30               |         |    |    | 7        | 0  |    |          | 7       | 3  |          |    | 1        | 3        | 3  |    | 2        | 7        | 2  | 3        | 3  | 2        |    | 3  | 11 | 2        |    |   |   |          |
| 40               |         |    |    | 9        | 2  |    |          | 10      | 2  |          |    | 1        | 9        | 0  |    | 3        | 7        | 2  | 4        | 4  | 3        |    | 5  | 3  | 1        |    |   |   |          |
| 50               |         |    |    | 1        | 00 | 0  |          | 1       | 10 | 0        |    | 2        | 2        | 1  |    | 3        | 3        | 2  | 4        | 4  | 3        |    | 5  | 5  | 3        |    |   |   |          |
| 60               |         |    |    | 1        | 2  | 1  |          | 1       | 3  | 3        |    | 2        | 7        | 2  |    | 3        | 11       | 2  | 5        | 3  | 1        |    | 6  | 7  | 0        |    |   |   |          |
| 70               |         |    |    | 1        | 4  | 2  |          | 1       | 6  | 2        |    | 3        | 00       | 3  |    | 4        | 7        | 1  | 6        | 1  | 2        |    | 7  | 8  | 0        |    |   |   |          |
| 80               |         |    |    | 1        | 7  | 0  |          | 1       | 9  | 0        |    | 3        | 6        | 0  |    | 5        | 3        | 1  | 7        | 00 | 0        |    | 8  | 9  | 1        |    |   |   |          |
| 90               |         |    |    | 1        | 9  | 1  |          | 1       | 11 | 3        |    | 3        | 11       | 2  |    | 5        | 11       | 0  | 7        | 10 | 3        |    | 9  | 10 | 2        |    |   |   |          |
| 100              |         |    |    | 1        | 11 | 3  |          | 2       | 2  | 1        |    | 4        | 4        | 3  |    | 6        | 7        | 0  | 8        | 9  | 1        |    | 10 | 11 | 2        |    |   |   |          |
| 200              |         |    |    | 3        | 11 | 2  |          | 4       | 4  | 3        |    | 8        | 9        | 1  |    | 13       | 1        | 3  | 17       | 6  | 2        | 1  | 1  | 11 | 0        |    |   |   |          |
| 300              |         |    |    | 5        | 11 | 0  |          | 6       | 7  | 0        |    | 13       | 1        | 3  |    | 19       | 8        | 3  | 1        | 6  | 3        | 3  | 1  | 12 | 10       | 2  |   |   |          |
| 400              |         |    |    | 7        | 10 | 3  |          | 8       | 9  | 1        |    | 17       | 6        | 2  |    | 1        | 6        | 3  | 3        | 1  | 15       | 00 | 3  | 2  | 3        | 10 | 0 |   |          |
| 500              |         |    |    | 9        | 10 | 2  |          | 10      | 11 | 2        |    | 1        | 11       | 0  |    | 1        | 12       | 10 | 2        | 2  | 3        | 10 | 0  | 2  | 14       | 9  | 2 |   |          |
| 600              |         |    |    | 11       | 10 | 0  |          | 13      | 1  | 3        |    | 6        | 3        | 3  |    | 1        | 19       | 5  | 2        | 2  | 12       | 7  | 1  | 3  | 5        | 9  | 0 |   |          |
| 700              |         |    |    | 13       | 9  | 2  |          | 15      | 4  | 0        |    | 10       | 8        | 1  |    | 2        | 6        | 00 | 1        | 3  | 1        | 4  | 2  | 3  | 16       | 8  | 2 |   |          |
| 800              |         |    |    | 15       | 9  | 2  |          | 17      | 6  | 2        |    | 15       | 00       | 3  |    | 2        | 12       | 7  | 1        | 3  | 10       | 1  | 2  | 4  | 7        | 8  | 0 |   |          |
| 900              |         |    |    | 17       | 9  | 0  |          | 19      | 8  | 3        |    | 19       | 5        | 2  |    | 2        | 19       | 2  | 0        | 3  | 18       | 11 | 0  | 4  | 18       | 7  | 2 |   |          |
| 1000             |         |    |    | 19       | 8  | 3  |          | 1       | 11 | 0        |    | 2        | 3        | 10 | 0  |          | 3        | 5  | 9        | 0  | 4        | 7  | 8  | 0  | 5        | 9  | 7 | 0 |          |
| ditto            |         |    |    | 1.986301 |    |    |          | 1.09589 |    |          |    | 2.191781 |          |    |    |          | 3.287671 |    |          |    | 4.383562 |    |    |    | 5.479452 |    |   |   | 6.575342 |

## SIMPLE INTEREST at 4 per Cent.

| Principal. | 70 Days. |      |      |    | 80 Days. |     |    |       | 90 Days. |    |        |    | 100 Days. |        |     |    | 110 Days. |     |    |    | 120 Days. |    |    |    |    |   |
|------------|----------|------|------|----|----------|-----|----|-------|----------|----|--------|----|-----------|--------|-----|----|-----------|-----|----|----|-----------|----|----|----|----|---|
|            | l.       | s.   | d.   | q. | l.       | s.  | d. | q.    | l.       | s. | d.     | q. | l.        | s.     | d.  | q. | l.        | s.  | d. | q. | l.        | s. | d. | q. |    |   |
| 1          |          |      | 1    | 3  |          |     | 2  | 0     |          |    | 2      | 1  |           |        | 2   | 2  |           |     | 3  | 0  |           |    | 3  | 1  |    |   |
| 2          |          |      | 3    | 3  |          |     | 4  | 1     |          |    | 4      | 3  |           |        | 5   | 1  |           |     | 5  | 3  |           |    | 6  | 1  |    |   |
| 3          |          |      | 5    | 2  |          |     | 6  | 1     |          |    | 7      | 0  |           |        | 7   | 3  |           |     | 8  | 3  |           |    | 9  | 3  |    |   |
| 4          |          |      | 7    | 1  |          |     | 8  | 2     |          |    | 9      | 2  |           |        | 10  | 2  |           |     | 11 | 3  |           | 1  | 00 | 2  |    |   |
| 5          |          |      | 9    | 1  |          |     | 10 | 2     |          | 1  | 00     | 0  |           | 1      | 1   | 1  |           | 1   | 2  | 2  |           | 1  | 3  | 3  |    |   |
| 6          |          |      | 11   | 0  |          |     | 1  | 00    | 2        |    | 1      | 2  | 1         |        | 1   | 3  | 3         |     | 1  | 5  | 2         |    | 1  | 7  | 0  |   |
| 7          |          | 1    | 00   | 3  |          |     | 1  | 2     | 3        |    | 1      | 4  | 3         |        | 1   | 6  | 2         |     | 1  | 8  | 1         |    | 1  | 10 | 0  |   |
| 8          |          | 1    | 2    | 3  |          |     | 1  | 5     | 0        |    | 1      | 7  | 0         |        | 1   | 9  | 0         |     | 1  | 11 | 1         |    | 2  | 1  | 1  |   |
| 9          |          | 1    | 4    | 3  |          |     | 1  | 7     | 0        |    | 1      | 9  | 1         |        | 1   | 11 | 3         |     | 2  | 2  | 0         |    | 2  | 4  | 2  |   |
| 10         |          | 1    | 6    | 2  |          |     | 1  | 9     | 0        |    | 1      | 11 | 3         |        | 2   | 00 | 0         |     | 2  | 5  | 0         |    | 2  | 7  | 2  |   |
| 20         |          | 3    | 00   | 3  |          |     | 3  | 6     | 0        |    | 3      | 11 | 2         |        | 4   | 4  | 3         |     | 4  | 10 | 0         |    | 5  | 3  | 1  |   |
| 30         |          | 4    | 7    | 1  |          |     | 5  | 3     | 1        |    | 5      | 11 | 0         |        | 6   | 7  | 0         |     | 7  | 2  | 3         |    | 7  | 10 | 3  |   |
| 40         |          | 6    | 1    | 2  |          |     | 7  | 00    | 0        |    | 7      | 11 | 0         |        | 8   | 9  | 1         |     | 9  | 7  | 3         |    | 10 | 6  | 1  |   |
| 50         |          | 7    | 8    | 0  |          |     | 8  | 9     | 1        |    | 9      | 10 | 2         |        | 10  | 11 | 2         |     | 12 | 00 | 2         |    | 13 | 1  | 3  |   |
| 60         |          | 9    | 2    | 2  |          |     | 10 | 6     | 1        |    | 11     | 10 | 0         |        | 13  | 1  | 3         |     | 14 | 5  | 2         |    | 15 | 9  | 2  |   |
| 70         |          | 10   | 8    | 3  |          |     | 12 | 3     | 1        |    | 13     | 9  | 3         |        | 15  | 4  | 1         |     | 16 | 10 | 2         |    | 18 | 4  | 3  |   |
| 80         |          | 12   | 3    | 1  |          |     | 14 | 00    | 1        |    | 15     | 9  | 1         |        | 17  | 6  | 1         |     | 19 | 3  | 2         |    | 1  | 1  | 00 | 2 |
| 90         |          | 13   | 9    | 3  |          |     | 15 | 9     | 2        |    | 17     | 9  | 0         |        | 19  | 8  | 3         |     | 1  | 1  | 8         | 1  | 1  | 3  | 8  | 0 |
| 100        |          | 15   | 4    | 1  |          |     | 17 | 6     | 2        |    | 19     | 8  | 3         |        | 1   | 1  | 11        | 0   | 1  | 4  | 1         | 1  | 1  | 6  | 3  | 3 |
| 200        | 1        | 10   | 8    | 1  |          |     | 15 | 00    | 3        |    | 19     | 5  | 1         |        | 2   | 3  | 10        | 0   | 2  | 8  | 2         | 2  | 2  | 12 | 7  | 1 |
| 300        | 2        | 6    | 00   | 12 |          |     | 12 | 7     | 12       |    | 19     | 2  | 1         |        | 3   | 5  | 9         | 0   | 3  | 12 | 4         | 0  | 3  | 18 | 11 | 0 |
| 400        | 3        | 1    | 4    | 23 |          |     | 10 | 1     | 23       |    | 18     | 11 | 0         |        | 4   | 7  | 8         | 0   | 4  | 16 | 5         | 1  | 5  | 5  | 2  | 2 |
| 500        | 3        | 16   | 8    | 24 |          |     | 7  | 8     | 04       |    | 18     | 7  | 2         |        | 5   | 9  | 7         | 0   | 6  | 00 | 6         | 2  | 6  | 11 | 6  | 0 |
| 600        | 4        | 12   | 00   | 35 |          |     | 5  | 2     | 25       |    | 18     | 4  | 1         |        | 6   | 11 | 6         | 0   | 7  | 4  | 7         | 3  | 7  | 17 | 9  | 3 |
| 700        | 5        | 7    | 4    | 36 |          |     | 2  | 8     | 36       |    | 18     | 1  | 0         |        | 7   | 13 | 5         | 0   | 8  | 8  | 9         | 1  | 9  | 4  | 1  | 1 |
| 800        | 6        | 2    | 8    | 37 |          |     | 00 | 3     | 17       |    | 17     | 9  | 3         |        | 8   | 15 | 4         | 0   | 9  | 12 | 10        | 2  | 10 | 10 | 5  | 0 |
| 900        | 6        | 18   | 1    | 07 |          |     | 17 | 9     | 38       |    | 17     | 6  | 2         |        | 9   | 17 | 3         | 0   | 10 | 16 | 11        | 3  | 11 | 16 | 8  | 2 |
| 1000       | 7        | 13   | 5    | 08 |          |     | 15 | 4     | 19       |    | 17     | 3  | 1         |        | 10  | 19 | 2         | 1   | 12 | 1  | 1         | 0  | 13 | 3  | 00 | 0 |
| ditto      | l.       | 7.67 | 1233 |    | 8.767    | 123 |    | 9.863 | 014      |    | 10.958 | 9  |           | 12.054 | 794 |    | 13.150    | 684 |    |    |           |    |    |    |    |   |





## SIMPLE INTEREST at 4 per Cent.

|            | 190 Days. |    |    |    | 200 Days. |    |    |    | 210 Days. |    |    |    | 220 Days. |    |    |    | 230 Days. |    |    |    | 240 Days. |    |    |    |   |
|------------|-----------|----|----|----|-----------|----|----|----|-----------|----|----|----|-----------|----|----|----|-----------|----|----|----|-----------|----|----|----|---|
| Principal. | l.        | s. | d. | q. | l.        | s. | d. | q. | l.        | s. | d. | q. | l.        | s. | d. | q. | l.        | s. | d. | q. | l.        | s. | d. | q. |   |
| 1          |           |    | 5  | 0  |           |    | 5  | 1  |           |    | 5  | 2  |           |    | 5  | 3  |           |    | 6  | 0  |           |    | 6  | 1  |   |
| 2          |           |    | 10 | 0  |           |    | 10 | 2  |           |    | 11 | 0  |           |    | 11 | 3  |           |    | 1  | 0  | 0         |    | 1  | 0  | 2 |
| 3          |           | 1  | 3  | 0  |           | 1  | 3  | 3  |           | 1  | 4  | 3  |           | 1  | 5  | 2  |           | 1  | 6  | 0  |           | 1  | 7  | 0  |   |
| 4          |           | 1  | 8  | 0  |           | 1  | 9  | 0  |           | 1  | 10 | 0  |           | 1  | 11 | 1  |           | 2  | 0  | 0  |           | 2  | 1  | 1  |   |
| 5          |           | 2  | 1  | 0  |           | 2  | 2  | 1  |           | 2  | 3  | 3  |           | 2  | 5  | 0  |           | 2  | 6  | 1  |           | 2  | 7  | 2  |   |
| 6          |           | 2  | 6  | 0  |           | 2  | 7  | 2  |           | 2  | 9  | 1  |           | 2  | 10 | 3  |           | 3  | 0  | 1  |           | 3  | 1  | 3  |   |
| 7          |           | 2  | 11 | 0  |           | 3  | 0  | 3  |           | 3  | 2  | 3  |           | 3  | 4  | 2  |           | 3  | 6  | 1  |           | 3  | 8  | 1  |   |
| 8          |           | 3  | 4  | 0  |           | 3  | 6  | 0  |           | 3  | 8  | 1  |           | 3  | 10 | 1  |           | 4  | 0  | 1  |           | 4  | 2  | 2  |   |
| 9          |           | 3  | 9  | 0  |           | 3  | 11 | 2  |           | 4  | 1  | 3  |           | 4  | 4  | 0  |           | 4  | 6  | 2  |           | 4  | 8  | 3  |   |
| 10         |           | 4  | 2  | 0  |           | 4  | 4  | 3  |           | 4  | 7  | 1  |           | 4  | 10 | 0  |           | 5  | 0  | 2  |           | 5  | 3  | 1  |   |
| 20         |           | 8  | 4  | 0  |           | 8  | 9  | 1  |           | 9  | 2  | 2  |           | 9  | 7  | 3  |           | 10 | 1  | 0  |           | 10 | 6  | 1  |   |
| 30         |           | 12 | 6  | 0  |           | 13 | 1  | 3  |           | 13 | 9  | 2  |           | 14 | 5  | 3  |           | 15 | 1  | 2  |           | 15 | 9  | 1  |   |
| 40         |           | 16 | 7  | 3  |           | 17 | 6  | 2  |           | 18 | 5  | 0  |           | 19 | 3  | 2  |           | 1  | 0  | 2  | 0         | 1  | 1  | 0  | 2 |
| 50         | 1         | 0  | 10 | 0  | 1         | 1  | 11 | 0  | 1         | 3  | 0  | 0  | 1         | 4  | 1  | 1  | 1         | 5  | 2  | 2  | 1         | 6  | 3  | 3  |   |
| 60         | 1         | 5  | 0  | 0  | 1         | 6  | 3  | 3  | 1         | 7  | 7  | 1  | 1         | 8  | 11 | 1  | 1         | 10 | 3  | 0  | 1         | 11 | 6  | 3  |   |
| 70         | 1         | 9  | 1  | 3  | 1         | 10 | 8  | 1  | 1         | 12 | 2  | 2  | 1         | 13 | 9  | 0  | 1         | 15 | 3  | 2  | 1         | 16 | 10 | 0  |   |
| 80         | 1         | 13 | 4  | 3  | 1         | 15 | 0  | 3  | 1         | 16 | 10 | 0  | 1         | 18 | 7  | 0  | 2         | 0  | 4  | 0  | 2         | 2  | 1  | 0  |   |
| 90         | 1         | 17 | 5  | 3  | 1         | 19 | 5  | 2  | 2         | 1  | 5  | 0  | 2         | 3  | 4  | 3  | 2         | 5  | 4  | 2  | 2         | 7  | 4  | 1  |   |
| 100        | 2         | 1  | 7  | 3  | 2         | 3  | 10 | 0  | 2         | 6  | 0  | 1  | 2         | 8  | 2  | 2  | 2         | 10 | 5  | 0  | 2         | 12 | 7  | 1  |   |
| 200        | 4         | 3  | 3  | 2  | 4         | 7  | 8  | 0  | 4         | 12 | 0  | 2  | 4         | 16 | 5  | 1  | 5         | 0  | 10 | 0  | 5         | 5  | 2  | 2  |   |
| 300        | 6         | 4  | 11 | 1  | 6         | 11 | 6  | 0  | 6         | 18 | 1  | 0  | 7         | 4  | 7  | 3  | 7         | 11 | 2  | 3  | 7         | 17 | 9  | 3  |   |
| 400        | 8         | 6  | 6  | 3  | 8         | 15 | 4  | 0  | 9         | 4  | 1  | 1  | 9         | 12 | 10 | 2  | 10        | 1  | 7  | 3  | 10        | 10 | 5  | 0  |   |
| 500        | 10        | 8  | 2  | 2  | 10        | 19 | 2  | 0  | 11        | 10 | 1  | 2  | 12        | 1  | 1  | 0  | 12        | 12 | 0  | 3  | 13        | 3  | 0  | 0  |   |
| 600        | 12        | 9  | 10 | 2  | 13        | 3  | 0  | 0  | 13        | 16 | 2  | 0  | 14        | 9  | 3  | 3  | 15        | 2  | 5  | 2  | 15        | 15 | 7  | 1  |   |
| 700        | 14        | 11 | 6  | 0  | 15        | 6  | 10 | 1  | 16        | 2  | 2  | 1  | 16        | 17 | 6  | 2  | 17        | 12 | 10 | 2  | 18        | 8  | 2  | 2  |   |
| 800        | 16        | 13 | 1  | 3  | 17        | 10 | 8  | 1  | 18        | 8  | 2  | 2  | 19        | 5  | 9  | 0  | 20        | 3  | 3  | 2  | 21        | 0  | 10 | 0  |   |
| 900        | 18        | 14 | 9  | 2  | 19        | 14 | 6  | 1  | 20        | 14 | 3  | 0  | 21        | 13 | 11 | 3  | 22        | 13 | 8  | 1  | 23        | 13 | 5  | 0  |   |
| 1000       | 20        | 16 | 5  | 1  | 21        | 18 | 4  | 1  | 23        | 0  | 3  | 1  | 24        | 2  | 2  | 1  | 25        | 4  | 1  | 1  | 26        | 6  | 0  | 1  |   |
| ditto      | 20        | 82 | 19 | 14 | 21        | 91 | 78 | 95 | 23        | 01 | 36 | 95 | 24        | 10 | 95 | 85 | 25        | 20 | 54 | 76 | 26        | 30 | 13 | 66 |   |

## SIMPLE INTEREST at 4 per Cent.

|            | 250 Days.   |    |    |    | 260 Days. |    |    |    | 270 Days. |    |    |    | 280 Days. |    |    |    | 290 Days. |    |    |    | 300 Days. |    |    |    |
|------------|-------------|----|----|----|-----------|----|----|----|-----------|----|----|----|-----------|----|----|----|-----------|----|----|----|-----------|----|----|----|
| Principal. | l.          | s. | d. | q. | l.        | s. | d. | q. | l.        | s. | d. | q. | l.        | s. | d. | q. | l.        | s. | d. | q. | l.        | s. | d. | q. |
| 1          |             |    | 6  | 2  |           |    | 6  | 3  |           |    | 7  | 0  |           |    | 7  | 1  |           |    | 7  | 2  |           |    | 7  | 3  |
| 2          |             | 1  | 1  | 0  |           | 1  | 1  | 2  |           | 1  | 2  | 1  |           | 1  | 2  | 3  |           | 1  | 3  | 1  |           | 1  | 3  | 3  |
| 3          |             | 1  | 7  | 3  |           | 1  | 8  | 2  |           | 1  | 9  | 1  |           | 1  | 10 | 0  |           | 1  | 11 | 0  |           | 1  | 11 | 3  |
| 4          |             | 2  | 2  | 1  |           | 2  | 3  | 1  |           | 2  | 4  | 2  |           | 2  | 5  | 2  |           | 2  | 6  | 2  |           | 2  | 7  | 2  |
| 5          |             | 2  | 8  | 3  |           | 2  | 10 | 1  |           | 2  | 11 | 2  |           | 3  | 0  | 3  |           | 3  | 2  | 0  |           | 3  | 3  | 2  |
| 6          |             | 3  | 3  | 2  |           | 3  | 5  | 0  |           | 3  | 6  | 2  |           | 3  | 8  | 1  |           | 3  | 9  | 3  |           | 3  | 11 | 2  |
| 7          |             | 3  | 10 | 0  |           | 4  | 0  | 0  |           | 4  | 1  | 3  |           | 4  | 3  | 2  |           | 4  | 5  | 1  |           | 4  | 7  | 1  |
| 8          |             | 4  | 4  | 2  |           | 4  | 6  | 3  |           | 4  | 8  | 3  |           | 4  | 11 | 0  |           | 5  | 1  | 0  |           | 5  | 3  | 1  |
| 9          |             | 4  | 11 | 1  |           | 5  | 1  | 2  |           | 5  | 4  | 0  |           | 5  | 6  | 1  |           | 5  | 8  | 3  |           | 5  | 11 | 0  |
| 10         |             | 5  | 3  | 1  |           | 5  | 8  | 1  |           | 5  | 11 | 0  |           | 6  | 1  | 2  |           | 6  | 4  | 1  |           | 6  | 7  | 0  |
| 20         |             | 10 | 11 | 2  |           | 11 | 4  | 3  |           | 11 | 10 | 0  |           | 12 | 3  | 1  |           | 12 | 8  | 2  |           | 13 | 1  | 3  |
| 30         |             | 16 | 5  | 1  |           | 17 | 1  | 0  |           | 17 | 9  | 0  |           | 18 | 5  | 0  |           | 19 | 0  | 3  |           | 19 | 8  | 3  |
| 40         | 1           | 1  | 11 | 0  | 1         | 2  | 9  | 2  | 1         | 3  | 8  | 0  | 1         | 4  | 6  | 2  | 1         | 5  | 5  | 0  | 1         | 6  | 3  | 3  |
| 50         | 1           | 7  | 4  | 3  | 1         | 8  | 6  | 0  | 1         | 9  | 7  | 0  | 1         | 10 | 8  | 1  | 1         | 11 | 9  | 2  | 1         | 12 | 10 | 2  |
| 60         | 1           | 12 | 10 | 3  | 1         | 14 | 2  | 1  | 1         | 15 | 6  | 0  | 1         | 16 | 10 | 0  | 1         | 18 | 1  | 2  | 1         | 19 | 5  | 2  |
| 70         | 1           | 18 | 4  | 1  | 1         | 19 | 10 | 3  | 2         | 1  | 5  | 0  | 2         | 2  | 11 | 2  | 2         | 4  | 6  | 0  | 2         | 6  | 0  | 1  |
| 80         | 2           | 3  | 10 | 0  | 2         | 5  | 7  | 0  | 2         | 7  | 4  | 0  | 2         | 9  | 1  | 1  | 2         | 10 | 10 | 1  | 2         | 12 | 7  | 1  |
| 90         | 2           | 9  | 3  | 3  | 2         | 11 | 3  | 2  | 2         | 13 | 3  | 1  | 2         | 15 | 2  | 3  | 2         | 17 | 2  | 2  | 2         | 19 | 2  | 0  |
| 100        | 2           | 14 | 9  | 2  | 2         | 17 | 0  | 0  | 2         | 19 | 2  | 1  | 3         | 1  | 4  | 2  | 3         | 3  | 6  | 3  | 3         | 5  | 9  | 0  |
| 200        | 5           | 9  | 7  | 0  | 5         | 13 | 11 | 3  | 5         | 18 | 4  | 1  | 6         | 2  | 8  | 3  | 6         | 7  | 1  | 2  | 6         | 11 | 6  | 0  |
| 300        | 8           | 4  | 4  | 2  | 8         | 10 | 11 | 2  | 8         | 17 | 6  | 2  | 9         | 4  | 1  | 1  | 9         | 10 | 8  | 1  | 9         | 17 | 3  | 1  |
| 400        | 10          | 19 | 2  | 0  | 11        | 7  | 11 | 2  | 11        | 16 | 8  | 2  | 12        | 5  | 5  | 3  | 12        | 14 | 3  | 0  | 13        | 3  | 0  | 0  |
| 500        | 13          | 13 | 11 | 3  | 14        | 4  | 11 | 1  | 14        | 15 | 10 | 3  | 15        | 6  | 10 | 1  | 15        | 17 | 9  | 3  | 16        | 8  | 9  | 1  |
| 600        | 16          | 8  | 9  | 1  | 17        | 1  | 11 | 0  | 17        | 15 | 0  | 3  | 18        | 8  | 2  | 2  | 19        | 1  | 4  | 2  | 19        | 14 | 6  | 1  |
| 700        | 19          | 3  | 6  | 3  | 19        | 18 | 11 | 0  | 20        | 14 | 3  | 0  | 21        | 9  | 7  | 0  | 22        | 4  | 11 | 1  | 23        | 0  | 3  | 1  |
| 800        | 21          | 18 | 4  | 1  | 22        | 15 | 10 | 3  | 23        | 13 | 5  | 0  | 24        | 10 | 11 | 2  | 25        | 8  | 6  | 0  | 26        | 6  | 0  | 1  |
| 900        | 24          | 13 | 1  | 3  | 25        | 12 | 10 | 2  | 26        | 12 | 7  | 1  | 27        | 12 | 4  | 0  | 28        | 12 | 0  | 3  | 29        | 11 | 9  | 2  |
| 1000       | 27          | 7  | 11 | 2  | 28        | 9  | 10 | 1  | 29        | 11 | 9  | 2  | 30        | 13 | 8  | 1  | 31        | 15 | 7  | 1  | 32        | 17 | 6  | 2  |
| ditto      | l.27.397256 |    |    |    | 28.49315  |    |    |    | 29.58904  |    |    |    | 30.68493  |    |    |    | 31.78082  |    |    |    | 32.87671  |    |    |    |



## SIMPLE INTEREST *at 4 per Cent.*

| Principal.<br>l. | 310 Days. |    |    |    | 320 Days. |    |    |    | 330 Days. |    |    |    | 340 Days. |    |    |    | 350 Days. |     |    |    | 360 Days. |    |    |    | 365 Days,<br>or 1 Year. |    |    |    |
|------------------|-----------|----|----|----|-----------|----|----|----|-----------|----|----|----|-----------|----|----|----|-----------|-----|----|----|-----------|----|----|----|-------------------------|----|----|----|
|                  | l.        | s. | d. | q. | l.        | s. | d. | q. | l.        | s. | d. | q. | l.        | s. | d. | q. | l.        | s.  | d. | q. | l.        | s. | d. | q. | l.                      | s. | d. | q. |
| 1                |           |    | 8  | 1  |           |    | 8  | 2  |           |    | 8  | 3  |           |    | 9  | 0  |           |     | 9  | 1  |           |    | 9  | 2  |                         |    | 9  | 3  |
| 2                |           | 1  | 4  | 1  |           | 1  | 5  | 0  |           | 1  | 5  | 2  |           | 1  | 6  | 0  |           | 1   | 6  | 2  |           | 1  | 7  | 0  |                         | 1  | 7  | 1  |
| 3                |           | 2  | 0  | 2  |           | 2  | 1  | 1  |           | 2  | 2  | 0  |           | 2  | 2  | 3  |           | 2   | 3  | 3  |           | 2  | 4  | 2  |                         | 2  | 5  | 0  |
| 4                |           | 2  | 8  | 2  |           | 2  | 9  | 3  |           | 2  | 10 | 3  |           | 2  | 11 | 3  |           | 3   | 0  | 3  |           | 3  | 1  | 3  |                         | 3  | 2  | 2  |
| 5                |           | 3  | 4  | 3  |           | 3  | 6  | 0  |           | 3  | 7  | 1  |           | 3  | 8  | 3  |           | 3   | 10 | 0  |           | 3  | 11 | 2  |                         | 4  | 0  | 0  |
| 6                |           | 4  | 0  | 3  |           | 4  | 2  | 2  |           | 4  | 4  | 0  |           | 4  | 5  | 3  |           | 4   | 7  | 1  |           | 4  | 8  | 3  |                         | 4  | 9  | 3  |
| 7                |           | 4  | 9  | 0  |           | 4  | 11 | 0  |           | 5  | 0  | 3  |           | 5  | 2  | 2  |           | 5   | 4  | 2  |           | 5  | 6  | 1  |                         | 5  | 7  | 1  |
| 8                |           | 5  | 5  | 1  |           | 5  | 7  | 1  |           | 5  | 9  | 2  |           | 5  | 11 | 2  |           | 6   | 1  | 2  |           | 6  | 3  | 3  |                         | 6  | 5  | 0  |
| 9                |           | 5  | 1  | 1  |           | 6  | 3  | 3  |           | 6  | 6  | 0  |           | 6  | 8  | 2  |           | 6   | 11 | 0  |           | 7  | 1  | 1  |                         | 7  | 2  | 2  |
| 10               |           | 6  | 9  | 2  |           | 7  | 0  | 0  |           | 7  | 2  | 3  |           | 7  | 5  | 2  |           | 7   | 8  | 0  |           | 7  | 10 | 3  |                         | 8  |    |    |
| 20               |           | 13 | 7  | 0  |           | 14 | 0  | 1  |           | 14 | 5  | 2  |           | 14 | 11 | 0  |           | 15  | 4  | 0  |           | 15 | 9  | 2  |                         | 16 |    |    |
| 30               |           | 1  | 0  | 4  | 3         |    | 1  | 1  | 0         | 2  |    | 1  | 1         | 8  | 1  |    | 1         | 2   | 4  | 1  |           | 1  | 3  | 0  | 0                       |    | 1  | 4  |
| 40               |           | 1  | 7  | 2  | 0         |    | 1  | 8  | 0         | 3  |    | 1  | 8         | 11 | 1  |    | 1         | 5   | 9  | 3  |           | 1  | 10 | 8  | 1                       |    | 1  | 11 |
| 50               |           | 1  | 18 | 11 | 3         |    | 1  | 15 | 0         | 3  |    | 1  | 16        | 2  | 0  |    | 1         | 17  | 3  | 1  |           | 1  | 18 | 4  | 1                       |    | 1  | 19 |
| 60               |           | 2  | 6  | 9  | 1         |    | 2  | 2  | 0         | 1  | 0  |    | 2         | 3  | 4  | 3  |           | 2   | 4  | 8  | 2         |    | 2  | 6  | 0                       | 1  |    | 2  |
| 70               |           | 2  | 7  | 6  | 3         |    | 2  | 9  | 1         | 1  |    | 2  | 10        | 7  | 2  |    | 2         | 12  | 2  | 0  |           | 2  | 13 | 8  | 1                       |    | 2  |    |
| 80               |           | 2  | 14 | 4  | 1         |    | 2  | 16 | 1         | 1  |    | 2  | 17        | 10 | 2  |    | 2         | 19  | 7  | 1  |           | 3  | 1  | 4  | 2                       |    | 3  |    |
| 90               |           | 3  | 1  | 1  | 3         |    | 3  | 3  | 1         | 2  |    | 3  | 5         | 1  | 1  |    | 3         | 7   | 0  | 3  |           | 3  | 9  | 0  | 2                       |    | 3  |    |
| 100              |           | 3  | 7  | 11 | 2         |    | 3  | 10 | 1         | 3  |    | 3  | 12        | 4  | 0  |    | 3         | 14  | 6  | 1  |           | 3  | 16 | 8  | 2                       |    | 3  |    |
| 200              |           | 6  | 15 | 10 | 3         |    | 7  | 0  | 3         | 1  |    | 7  | 4         | 7  | 3  |    | 7         | 9   | 0  | 2  |           | 7  | 13 | 5  | 0                       |    | 7  |    |
| 300              |           | 10 | 3  | 10 | 0         |    | 10 | 10 | 5         | 0  |    | 10 | 16        | 11 | 3  |    | 11        | 3   | 6  | 3  |           | 11 | 10 | 1  | 2                       |    | 11 |    |
| 400              |           | 13 | 11 | 9  | 2         |    | 14 | 0  | 6         | 2  |    | 14 | 9         | 4  | 3  |    | 14        | 18  | 1  | 0  |           | 15 | 6  | 10 | 1                       |    | 15 |    |
| 500              |           | 16 | 19 | 8  | 3         |    | 17 | 10 | 8         | 1  |    | 18 | 1         | 7  | 3  |    | 18        | 12  | 7  | 1  |           | 19 | 3  | 6  | 2                       |    | 19 |    |
| 600              |           | 20 | 7  | 8  | 0         |    | 21 | 0  | 10        | 0  |    | 21 | 13        | 11 | 3  |    | 22        | 7   | 1  | 2  |           | 23 | 0  | 3  | 1                       |    | 23 |    |
| 700              |           | 23 | 15 | 7  | 1         |    | 24 | 10 | 11        | 2  |    | 25 | 6         | 3  | 3  |    | 26        | 1   | 7  | 3  |           | 26 | 17 | 0  | 0                       |    | 27 |    |
| 800              |           | 27 | 3  | 6  | 3         |    | 28 | 1  | 11        | 1  |    | 28 | 18        | 7  | 2  |    | 29        | 16  | 2  | 0  |           | 30 | 13 | 8  | 1                       |    | 31 |    |
| 900              |           | 30 | 11 | 6  | 0         |    | 31 | 11 | 2         | 3  |    | 32 | 10        | 11 | 2  |    | 33        | 10  | 8  | 1  |           | 34 | 10 | 5  | 0                       |    | 35 |    |
| 1000             |           | 33 | 19 | 5  | 2         |    | 35 | 1  | 4         | 2  |    | 36 | 3         | 3  | 2  |    | 37        | 5   | 2  | 2  |           | 38 | 7  | 1  | 2                       |    | 39 |    |
| ditto            |           | l. | 33 | 97 | 26        |    | 35 | 06 | 84        | 9  |    | 36 | 16        | 43 | 8  |    | 7         | 260 | 27 |    |           | 38 | 35 | 61 | 6                       |    | 39 | 45 |
|                  |           |    |    |    |           |    |    |    |           |    |    |    |           |    |    |    |           |     |    |    |           |    |    |    |                         |    |    |    |



## SIMPLE INTEREST at 5 per Cent.

|            | 9 Days. |    |    |    | 10 Days. |    |    |    | 20 Days. |    |    |    | 30 Days. |    |    |    | 40 Days. |    |    |    | 50 Days. |    |    |    | 60 Days. |    |    |    |     |       |    |    |
|------------|---------|----|----|----|----------|----|----|----|----------|----|----|----|----------|----|----|----|----------|----|----|----|----------|----|----|----|----------|----|----|----|-----|-------|----|----|
| Principal. | l.      | s. | d. | q. | l.       | s. | d. | q. | l.       | s. | d. | q. | l.       | s. | d. | q. | l.       | s. | d. | q. | l.       | s. | d. | q. | l.       | s. | d. | q. |     |       |    |    |
| 1          |         |    |    | 1  |          |    |    | 1  |          |    |    | 2  |          |    |    | 1  | 0        |    |    | 1  | 1        |    |    | 1  | 2        |    |    | 2  | 0   |       |    |    |
| 2          |         |    |    | 2  |          |    |    | 2  |          |    |    | 1  | 1        |    |    | 2  | 0        |    |    | 2  | 2        |    |    | 3  | 1        |    |    | 4  | 0   |       |    |    |
| 3          |         |    |    | 3  |          |    |    | 1  | 0        |    |    | 2  | 0        |    |    | 3  | 0        |    |    | 4  | 0        |    |    | 5  | 0        |    |    | 5  | 3   |       |    |    |
| 4          |         |    |    | 1  | 1        |    |    | 1  | 1        |    |    | 2  | 2        |    |    | 4  | 0        |    |    | 5  | 1        |    |    | 6  | 2        |    |    | 7  | 3   |       |    |    |
| 5          |         |    |    | 1  | 2        |    |    | 1  | 2        |    |    | 3  | 1        |    |    | 5  | 0        |    |    | 6  | 2        |    |    | 8  | 1        |    |    | 9  | 3   |       |    |    |
| 6          |         |    |    | 1  | 3        |    |    | 2  | 0        |    |    | 4  | 0        |    |    | 5  | 3        |    |    | 7  | 3        |    |    | 10 | 0        |    |    | 11 | 3   |       |    |    |
| 7          |         |    |    | 2  | 0        |    |    | 2  | 1        |    |    | 4  | 3        |    |    | 6  | 3        |    |    | 9  | 1        |    |    | 11 | 2        |    |    | 1  | 13  |       |    |    |
| 8          |         |    |    | 2  | 1        |    |    | 2  | 2        |    |    | 5  | 1        |    |    | 7  | 3        |    |    | 10 | 2        |    |    | 1  | 10       |    |    | 1  | 33  |       |    |    |
| 9          |         |    |    | 2  | 3        |    |    | 3  | 0        |    |    | 5  | 3        |    |    | 8  | 3        |    |    | 11 | 3        |    |    | 1  | 23       |    |    | 1  | 53  |       |    |    |
| 10         |         |    |    | 3  | 0        |    |    | 3  | 1        |    |    | 6  | 2        |    |    | 10 | 0        |    |    | 1  | 10       |    |    | 1  | 42       |    |    | 1  | 73  |       |    |    |
| 20         |         |    |    | 5  | 3        |    |    | 6  | 2        |    |    | 1  | 10       |    |    | 1  | 7        | 3  |    | 2  | 21       |    |    | 2  | 83       |    |    | 3  | 32  |       |    |    |
| 30         |         |    |    | 8  | 3        |    |    | 10 | 0        |    |    | 1  | 7        | 3  |    | 2  | 5        | 2  |    | 3  | 32       |    |    | 4  | 11       |    |    | 4  | 111 |       |    |    |
| 40         |         |    |    | 11 | 3        |    |    | 1  | 10       |    |    | 2  | 21       |    |    | 3  | 3        | 2  |    | 4  | 43       |    |    | 5  | 52       |    |    | 6  | 63  |       |    |    |
| 50         |         |    |    | 1  | 2        | 3  |    | 1  | 4        | 2  |    | 2  | 8        | 3  |    | 4  | 1        | 1  |    | 5  | 53       |    |    | 6  | 100      |    |    | 8  | 22  |       |    |    |
| 60         |         |    |    | 1  | 5        | 2  |    | 1  | 7        | 3  |    | 3  | 3        | 2  |    | 4  | 11       | 1  |    | 6  | 63       |    |    | 8  | 22       |    |    | 9  | 101 |       |    |    |
| 70         |         |    |    | 1  | 8        | 3  |    | 1  | 11       | 0  |    | 3  | 10       | 0  |    | 5  | 9        | 0  |    | 7  | 80       |    |    | 9  | 70       |    |    | 11 | 60  |       |    |    |
| 80         |         |    |    | 1  | 11       | 2  |    | 2  | 2        | 1  |    | 4  | 4        | 3  |    | 6  | 6        | 3  |    | 8  | 91       |    |    | 10 | 112      |    |    | 13 | 13  |       |    |    |
| 90         |         |    |    | 2  | 2        | 2  |    | 2  | 5        | 2  |    | 4  | 11       | 1  |    | 7  | 4        | 3  |    | 9  | 102      |    |    | 12 | 40       |    |    | 14 | 92  |       |    |    |
| 100        |         |    |    | 2  | 5        | 2  |    | 2  | 8        | 3  |    | 5  | 5        | 3  |    | 8  | 2        | 2  |    | 10 | 112      |    |    | 13 | 80       |    |    | 16 | 51  |       |    |    |
| 200        |         |    |    | 4  | 11       | 0  |    | 5  | 5        | 2  |    | 10 | 11       | 2  |    | 16 | 5        | 11 |    | 1  | 111      | 0  |    | 1  | 7        | 43 |    |    | 1   | 12102 |    |    |
| 300        |         |    |    | 7  | 4        | 2  |    | 8  | 2        | 2  |    | 16 | 5        | 11 |    | 1  | 4        | 7  | 3  | 1  | 12       | 10 | 2  |    | 1        | 10 |    |    | 2   | 9     | 33 |    |
| 400        |         |    |    | 9  | 10       | 1  |    | 10 | 11       | 2  |    | 1  | 11       | 0  |    | 1  | 12       | 10 | 2  | 2  | 3        | 10 | 0  |    | 2        | 14 |    |    | 3   | 5     | 90 |    |
| 500        |         |    |    | 12 | 4        | 0  |    | 13 | 8        | 11 |    | 7  | 4        | 3  | 2  | 1  | 1        | 10 | 2  | 2  | 14       | 9  | 2  |    | 8        | 5  | 3  |    | 4   | 2     | 21 |    |
| 600        |         |    |    | 14 | 9        | 2  |    | 16 | 5        | 0  |    | 12 | 10       | 2  | 2  | 9  | 3        | 3  | 3  | 5  | 9        | 0  |    | 4  | 2        | 21 |    |    | 4   | 18    | 72 |    |
| 700        |         |    |    | 17 | 3        | 0  |    | 19 | 2        | 0  |    | 18 | 4        | 1  | 2  | 17 | 6        | 1  | 3  | 16 | 8        | 2  |    | 15 | 10       | 3  |    |    | 5   | 15    | 03 |    |
| 800        |         |    |    | 19 | 8        | 3  |    | 1  | 11       | 10 | 2  | 3  | 10       | 0  | 3  | 5  | 9        | 0  | 4  | 7  | 8        | 0  |    | 5  | 9        | 70 |    |    | 6   | 11    | 60 |    |
| 900        |         |    |    | 1  | 2        | 2  | 1  | 1  | 4        | 7  | 3  | 2  | 9        | 3  | 3  | 3  | 13       | 11 | 3  | 4  | 18       | 7  | 2  |    | 5        | 3  | 3  |    | 7   | 7     | 11 |    |
| 1000       |         |    |    | 1  | 4        | 7  | 3  | 1  | 7        | 4  | 3  | 2  | 14       | 9  | 2  | 4  | 2        | 2  | 1  | 5  | 9        | 7  | 0  |    | 6        | 16 | 11 |    |     | 8     | 4  | 42 |
| ditto      | l.      | 1. | 23 | 28 | l.       | 1. | 36 | 98 | l.       | 2. | 73 | 96 | l.       | 4. | 10 | 94 | l.       | 5. | 47 | 92 | l.       | 6. | 84 | 90 | l.       | 8. | 21 | 88 |     |       |    |    |



## SIMPLE INTEREST at 5 per Cent.

|            | 70 Days. |        |    |    | 80 Days. |         |    |    | 90 Days. |    |         |    | 100 Days. |    |         |    | 110 Days. |    |         |    | 120 Days. |    |         |    |   |
|------------|----------|--------|----|----|----------|---------|----|----|----------|----|---------|----|-----------|----|---------|----|-----------|----|---------|----|-----------|----|---------|----|---|
| Principal. | l.       | s.     | d. | q. | l.       | s.      | d. | q. | l.       | s. | d.      | q. | l.        | s. | d.      | q. | l.        | s. | d.      | q. | l.        | s. | d.      | q. |   |
| 1          |          |        | 2  | 1  |          |         | 2  | 3  |          |    | 3       | 0  |           |    | 3       | 1  |           |    | 3       | 2  |           |    | 4       | 0  |   |
| 2          |          |        | 4  | 2  |          |         | 5  | 1  |          |    | 5       | 3  |           |    | 6       | 2  |           |    | 7       | 1  |           |    | 7       | 3  |   |
| 3          |          |        | 6  | 3  |          |         | 7  | 3  |          |    | 8       | 3  |           |    | 9       | 3  |           |    | 10      | 3  |           |    | 11      | 3  |   |
| 4          |          |        | 9  | 1  |          |         | 10 | 2  |          |    | 11      | 3  |           | 1  | 1       | 0  |           | 1  | 2       | 2  |           | 1  | 3       | 3  |   |
| 5          |          |        | 11 | 2  |          | 1       | 1  | 0  |          | 1  | 2       | 3  |           | 1  | 4       | 2  |           | 1  | 6       | 0  |           | 1  | 7       | 3  |   |
| 6          |          | 1      | 1  | 3  |          | 1       | 3  | 3  |          | 1  | 5       | 3  |           | 1  | 7       | 3  |           | 1  | 9       | 2  |           | 1  | 11      | 2  |   |
| 7          |          | 1      | 4  | 0  |          | 1       | 6  | 1  |          | 1  | 8       | 3  |           | 1  | 11      | 0  |           | 2  | 1       | 1  |           | 2  | 3       | 2  |   |
| 8          |          | 1      | 6  | 1  |          | 1       | 9  | 0  |          | 1  | 11      | 2  |           | 2  | 2       | 1  |           | 2  | 5       | 0  |           | 2  | 7       | 2  |   |
| 9          |          | 1      | 8  | 3  |          | 1       | 11 | 2  |          | 2  | 2       | 2  |           | 2  | 5       | 2  |           | 2  | 8       | 2  |           | 2  | 11      | 2  |   |
| 10         |          | 1      | 11 | 0  |          | 2       | 2  | 1  |          | 2  | 5       | 2  |           | 2  | 8       | 3  |           | 3  | 0       | 0  |           | 3  | 3       | 2  |   |
| 20         |          | 3      | 10 | 0  |          | 4       | 4  | 2  |          | 4  | 11      | 0  |           | 5  | 5       | 3  |           | 6  | 0       | 1  |           | 6  | 6       | 3  |   |
| 30         |          | 5      | 9  | 0  |          | 6       | 6  | 3  |          | 7  | 4       | 3  |           | 8  | 2       | 2  |           | 3  | 0       | 2  |           | 9  | 10      | 1  |   |
| 40         |          | 7      | 8  | 0  |          | 8       | 9  | 1  |          | 9  | 10      | 1  |           | 10 | 11      | 2  |           | 12 | 0       | 2  |           | 13 | 1       | 3  |   |
| 50         |          | 9      | 7  | 0  |          | 10      | 11 | 2  |          | 12 | 4       | 0  |           | 13 | 8       | 1  |           | 15 | 0       | 3  |           | 16 | 5       | 1  |   |
| 60         |          | 11     | 6  | 0  |          | 13      | 1  | 3  |          | 14 | 9       | 2  |           | 16 | 5       | 1  |           | 18 | 1       | 0  |           | 19 | 8       | 2  |   |
| 70         |          | 13     | 5  | 0  |          | 15      | 4  | 0  |          | 17 | 3       | 1  |           | 19 | 2       | 0  |           | 1  | 1       | 1  | 0         | 1  | 3       | 0  | 0 |
| 80         |          | 15     | 4  | 0  |          | 17      | 6  | 1  |          | 19 | 8       | 2  |           | 1  | 1       | 11 | 0         | 1  | 4       | 1  | 1         | 1  | 6       | 3  | 2 |
| 90         |          | 17     | 3  | 0  |          | 19      | 8  | 3  |          | 1  | 2       | 2  | 1         | 1  | 4       | 7  | 3         | 1  | 7       | 1  | 2         | 1  | 9       | 7  | 0 |
| 100        |          | 19     | 2  | 0  |          | 1       | 1  | 11 | 0        | 1  | 4       | 7  | 3         | 1  | 7       | 4  | 3         | 1  | 10      | 1  | 2         | 1  | 12      | 10 | 2 |
| 200        | 1        | 18     | 4  | 1  |          | 2       | 3  | 10 | 0        | 2  | 9       | 3  | 3         | 2  | 14      | 9  | 2         | 3  | 0       | 3  | 1         | 3  | 5       | 9  | 0 |
| 300        | 2        | 17     | 6  | 1  |          | 3       | 5  | 9  | 0        | 3  | 13      | 11 | 2         | 4  | 2       | 2  | 1         | 4  | 10      | 5  | 0         | 4  | 18      | 7  | 2 |
| 400        | 3        | 16     | 8  | 2  |          | 4       | 7  | 8  | 0        | 4  | 18      | 7  | 2         | 5  | 9       | 7  | 0         | 6  | 0       | 6  | 2         | 6  | 11      | 6  | 0 |
| 500        | 4        | 15     | 10 | 3  |          | 5       | 9  | 7  | 0        | 6  | 3       | 3  | 2         | 6  | 16      | 11 | 3         | 7  | 10      | 8  | 1         | 8  | 4       | 4  | 2 |
| 600        | 5        | 15     | 0  | 3  |          | 6       | 11 | 6  | 0        | 7  | 7       | 11 | 1         | 8  | 4       | 4  | 2         | 9  | 0       | 9  | 3         | 9  | 17      | 3  | 0 |
| 700        | 6        | 14     | 3  | 0  |          | 7       | 13 | 5  | 0        | 8  | 12      | 7  | 1         | 9  | 11      | 9  | 1         | 10 | 10      | 11 | 2         | 11 | 10      | 1  | 2 |
| 800        | 7        | 13     | 5  | 0  |          | 8       | 15 | 4  | 0        | 9  | 17      | 3  | 0         | 10 | 19      | 2  | 0         | 12 | 1       | 1  | 0         | 13 | 3       | 0  | 0 |
| 900        | 8        | 12     | 7  | 1  |          | 9       | 17 | 3  | 0        | 11 | 1       | 11 | 0         | 12 | 6       | 6  | 3         | 13 | 11      | 2  | 3         | 14 | 15      | 10 | 2 |
| 1000       | 9        | 11     | 9  | 1  |          | 10      | 19 | 2  | 2        | 12 | 6       | 6  | 3         | 13 | 13      | 11 | 2         | 15 | 1       | 4  | 1         | 16 | 8       | 9  | 0 |
| ditto      | l.       | 9.5889 |    |    | l.       | 10.9587 |    |    |          | l. | 12.3286 |    |           | l. | 13.5984 |    |           | l. | 15.0682 |    |           | l. | 16.4379 |    |   |

## SIMPLE INTEREST at 5 per Cent.

|            | 130 Days.  |    |    |    | 140 Days.  |    |    |    | 150 Days.  |    |    |    | 160 Days.  |    |    |    | 170 Days.  |    |    |    | 180 Days.  |    |    |    |
|------------|------------|----|----|----|------------|----|----|----|------------|----|----|----|------------|----|----|----|------------|----|----|----|------------|----|----|----|
| Principal. | l.         | s. | d. | q. | l.         | s. | d. | q. | l.         | s. | d. | q. | l.         | s. | d. | q. | l.         | s. | d. | q. | l.         | s. | d. | q. |
| 1          |            |    | 4  | 1  |            |    | 4  | 2  |            |    | 4  | 3  |            |    | 5  | 1  |            |    | 5  | 2  |            |    | 5  | 3  |
| 2          |            |    | 8  | 2  |            |    | 9  | 0  |            |    | 9  | 3  |            |    | 10 | 2  |            |    | 11 | 1  |            |    | 11 | 3  |
| 3          |            | 1  | 0  | 3  |            | 1  | 1  | 3  |            | 1  | 2  | 3  |            | 1  | 3  | 3  |            | 1  | 4  | 3  |            | 1  | 5  | 3  |
| 4          |            | 1  | 5  | 0  |            | 1  | 6  | 1  |            | 1  | 7  | 3  |            | 1  | 9  | 0  |            | 1  | 10 | 1  |            | 1  | 11 | 2  |
| 5          |            | 1  | 9  | 1  |            | 1  | 11 | 0  |            | 2  | 0  | 2  |            | 2  | 2  | 1  |            | 2  | 4  | 0  |            | 2  | 5  | 2  |
| 6          |            | 2  | 1  | 2  |            | 2  | 3  | 2  |            | 2  | 5  | 2  |            | 2  | 7  | 2  |            | 2  | 9  | 2  |            | 2  | 11 | 2  |
| 7          |            | 2  | 5  | 3  |            | 2  | 8  | 1  |            | 2  | 10 | 2  |            | 3  | 0  | 3  |            | 3  | 3  | 1  |            | 3  | 5  | 2  |
| 8          |            | 2  | 10 | 0  |            | 3  | 0  | 3  |            | 3  | 3  | 2  |            | 3  | 6  | 0  |            | 3  | 8  | 3  |            | 3  | 11 | 1  |
| 9          |            | 3  | 2  | 2  |            | 3  | 5  | 1  |            | 3  | 8  | 1  |            | 3  | 11 | 1  |            | 4  | 2  | 1  |            | 4  | 5  | 1  |
| 10         |            | 3  | 6  | 3  |            | 3  | 10 | 0  |            | 4  | 1  | 1  |            | 4  | 4  | 2  |            | 4  | 7  | 3  |            | 4  | 11 | 1  |
| 20         |            | 7  | 1  | 2  |            | 7  | 8  | 0  |            | 8  | 2  | 2  |            | 8  | 9  | 0  |            | 9  | 3  | 3  |            | 9  | 10 | 1  |
| 30         |            | 10 | 8  | 1  |            | 11 | 6  | 0  |            | 12 | 3  | 3  |            | 13 | 1  | 3  |            | 13 | 11 | 3  |            | 14 | 9  | 2  |
| 40         |            | 14 | 3  | 0  |            | 15 | 4  | 0  |            | 16 | 5  | 1  |            | 17 | 6  | 1  |            | 18 | 7  | 2  |            | 19 | 8  | 3  |
| 50         |            | 17 | 9  | 2  |            | 19 | 2  | 0  | 1          | 0  | 6  | 2  | 1          | 1  | 11 | 0  | 1          | 3  | 3  | 2  | 1          | 4  | 7  | 3  |
| 60         | 1          | 1  | 4  | 1  | 1          | 3  | 0  | 0  | 1          | 4  | 7  | 3  | 1          | 6  | 3  | 2  | 1          | 7  | 11 | 1  | 1          | 9  | 7  | 0  |
| 70         | 1          | 4  | 11 | 0  | 1          | 6  | 10 | 1  | 1          | 8  | 9  | 0  | 1          | 10 | 8  | 1  | 1          | 12 | 7  | 1  | 1          | 14 | 6  | 1  |
| 80         | 1          | 8  | 5  | 3  | 1          | 10 | 8  | 1  | 1          | 12 | 10 | 2  | 1          | 15 | 0  | 3  | 1          | 17 | 3  | 0  | 1          | 19 | 5  | 1  |
| 90         | 1          | 12 | 0  | 2  | 1          | 14 | 6  | 1  | 1          | 16 | 11 | 3  | 1          | 19 | 5  | 2  | 2          | 1  | 11 | 0  | 2          | 4  | 4  | 2  |
| 100        | 1          | 15 | 7  | 1  | 1          | 18 | 4  | 1  | 2          | 1  | 1  | 0  | 2          | 3  | 10 | 0  | 2          | 6  | 6  | 3  | 2          | 9  | 3  | 3  |
| 200        | 3          | 11 | 2  | 3  | 3          | 16 | 8  | 2  | 4          | 2  | 2  | 1  | 4          | 7  | 8  | 0  | 4          | 13 | 1  | 3  | 4          | 18 | 7  | 2  |
| 300        | 5          | 6  | 10 | 0  | 5          | 15 | 0  | 3  | 6          | 3  | 3  | 2  | 6          | 11 | 6  | 0  | 6          | 19 | 8  | 2  | 7          | 7  | 11 | 1  |
| 400        | 7          | 2  | 5  | 2  | 7          | 13 | 5  | 0  | 8          | 4  | 4  | 2  | 8          | 15 | 4  | 0  | 9          | 6  | 3  | 2  | 9          | 17 | 3  | 0  |
| 500        | 8          | 18 | 0  | 3  | 9          | 11 | 9  | 1  | 10         | 5  | 5  | 3  | 10         | 19 | 2  | 0  | 11         | 12 | 10 | 2  | 12         | 6  | 6  | 3  |
| 600        | 10         | 13 | 8  | 1  | 11         | 10 | 1  | 2  | 12         | 6  | 6  | 3  | 13         | 3  | 0  | 0  | 13         | 19 | 5  | 2  | 14         | 15 | 10 | 2  |
| 700        | 12         | 9  | 3  | 3  | 13         | 8  | 5  | 3  | 14         | 7  | 8  | 0  | 15         | 6  | 10 | 1  | 16         | 6  | 0  | 1  | 17         | 5  | 2  | 2  |
| 800        | 14         | 4  | 11 | 0  | 15         | 6  | 10 | 0  | 16         | 8  | 9  | 0  | 17         | 10 | 8  | 1  | 18         | 12 | 7  | 1  | 19         | 14 | 6  | 0  |
| 900        | 16         | 0  | 6  | 2  | 17         | 5  | 2  | 1  | 18         | 9  | 10 | 1  | 19         | 14 | 6  | 1  | 20         | 19 | 2  | 0  | 22         | 3  | 10 | 0  |
| 1000       | 17         | 16 | 1  | 3  | 19         | 3  | 6  | 2  | 20         | 10 | 11 | 2  | 21         | 18 | 4  | 1  | 23         | 5  | 9  | 0  | 24         | 13 | 1  | 3  |
| ditto      | l. 17.8078 |    |    |    | l. 19.1776 |    |    |    | l. 20.5479 |    |    |    | l. 21.9177 |    |    |    | l. 23.2875 |    |    |    | l. 24.6573 |    |    |    |

## SIMPLE INTEREST at 5 per Cent.

|            | 190 Days. |    |      |      | 200 Days. |    |      |      | 210 Days. |    |      |      | 220 Days. |    |      |      | 230 Days. |    |      |      | 240 Days. |    |      |      |   |
|------------|-----------|----|------|------|-----------|----|------|------|-----------|----|------|------|-----------|----|------|------|-----------|----|------|------|-----------|----|------|------|---|
| Principal. | l.        | s. | d.   | q.   | l.        | s. | d.   | q.   | l.        | s. | d.   | q.   | l.        | s. | d.   | q.   | l.        | s. | d.   | q.   | l.        | s. | d.   | q.   |   |
| 1          |           |    | 6    | 1    |           |    | 6    | 2    |           |    | 7    | 0    |           |    | 7    | 1    |           |    | 7    | 2    |           |    | 7    | 3    |   |
| 2          |           | 1  | 0    | 2    |           | 1  | 1    | 0    |           | 1  | 1    | 3    |           | 1  | 2    | 2    |           | 1  | 3    | 0    |           | 1  | 3    | 3    |   |
| 3          |           | 1  | 6    | 3    |           | 1  | 7    | 3    |           | 1  | 8    | 3    |           | 1  | 9    | 2    |           | 1  | 10   | 3    |           | 1  | 11   | 3    |   |
| 4          |           | 2  | 1    | 0    |           | 2  | 2    | 1    |           | 2  | 3    | 2    |           | 2  | 5    | 0    |           | 2  | 6    | 1    |           | 2  | 7    | 2    |   |
| 5          |           | 2  | 7    | 1    |           | 2  | 8    | 3    |           | 2  | 10   | 2    |           | 3  | 0    | 1    |           | 3  | 1    | 3    |           | 3  | 3    | 2    |   |
| 6          |           | 3  | 1    | 2    |           | 3  | 3    | 2    |           | 3  | 5    | 2    |           | 3  | 7    | 2    |           | 3  | 9    | 1    |           | 3  | 11   | 1    |   |
| 7          |           | 3  | 7    | 3    |           | 3  | 10   | 0    |           | 4  | 0    | 1    |           | 4  | 2    | 2    |           | 4  | 5    | 0    |           | 4  | 7    | 1    |   |
| 8          |           | 4  | 2    | 0    |           | 4  | 4    | 2    |           | 4  | 7    | 1    |           | 4  | 9    | 3    |           | 5  | 0    | 2    |           | 5  | 3    | 0    |   |
| 9          |           | 4  | 8    | 1    |           | 4  | 11   | 1    |           | 5  | 2    | 1    |           | 5  | 5    | 0    |           | 5  | 8    | 0    |           | 5  | 11   | 0    |   |
| 10         |           | 5  | 2    | 2    |           | 5  | 5    | 3    |           | 5  | 9    | 0    |           | 6  | 0    | 1    |           | 6  | 3    | 2    |           | 6  | 7    | 0    |   |
| 20         |           | 10 | 5    | 0    |           | 10 | 11   | 2    |           | 11 | 6    | 0    |           | 12 | 0    | 2    |           | 12 | 7    | 1    |           | 13 | 1    | 3    |   |
| 30         |           | 15 | 7    | 1    |           | 16 | 5    | 1    |           | 17 | 3    | 0    |           | 18 | 1    | 0    |           | 18 | 10   | 3    |           | 19 | 8    | 2    |   |
| 40         |           | 1  | 0    | 9    | 3         |    | 1    | 11   | 0         |    | 1    | 3    | 0         | 0  |      | 1    | 4         | 1  | 1    |      | 1         | 5  | 2    | 2    |   |
| 50         |           | 1  | 6    | 0    | 1         |    | 1    | 7    | 4         | 3  |      | 1    | 8         | 9  | 0    |      | 1         | 10 | 1    | 3    |           | 1  | 11   | 6    | 0 |
| 60         |           | 1  | 11   | 2    | 3         |    | 1    | 12   | 10        | 2  |      | 1    | 14        | 6  | 1    |      | 1         | 16 | 2    | 0    |           | 1  | 17   | 9    | 2 |
| 70         |           | 1  | 16   | 5    | 1         |    | 1    | 18   | 4         | 1  |      | 2    | 0         | 3  | 1    |      | 2         | 2  | 2    | 1    |           | 2  | 4    | 1    | 1 |
| 80         |           | 2  | 1    | 7    | 3         |    | 2    | 3    | 10        | 0  |      | 2    | 6         | 0  | 1    |      | 2         | 8  | 2    | 2    |           | 2  | 10   | 5    | 0 |
| 90         |           | 2  | 6    | 10   | 1         |    | 2    | 9    | 3         | 3  |      | 2    | 11        | 9  | 1    |      | 2         | 14 | 3    | 0    |           | 2  | 16   | 8    | 2 |
| 100        |           | 2  | 12   | 0    | 2         |    | 2    | 14   | 9         | 2  |      | 2    | 17        | 6  | 2    |      | 3         | 0  | 3    | 1    |           | 3  | 3    | 0    | 0 |
| 200        |           | 5  | 4    | 1    | 1         |    | 5    | 9    | 7         | 0  |      | 5    | 15        | 0  | 3    |      | 6         | 0  | 6    | 2    |           | 6  | 6    | 0    | 1 |
| 300        |           | 7  | 16   | 2    | 0         |    | 8    | 4    | 4         | 3  |      | 8    | 12        | 7  | 1    |      | 9         | 0  | 9    | 3    |           | 9  | 2    | 0    | 2 |
| 400        |           | 10 | 8    | 2    | 2         |    | 10   | 19   | 2         | 1  |      | 11   | 10        | 1  | 1    |      | 12        | 1  | 1    | 1    |           | 12 | 12   | 0    | 2 |
| 500        |           | 13 | 0    | 3    | 1         |    | 13   | 13   | 11        | 3  |      | 14   | 7         | 8  | 0    |      | 15        | 1  | 4    | 2    |           | 15 | 15   | 0    | 3 |
| 600        |           | 15 | 12   | 4    | 0         |    | 16   | 8    | 9         | 1  |      | 17   | 5         | 2  | 2    |      | 18        | 1  | 7    | 3    |           | 18 | 18   | 1    | 0 |
| 700        |           | 18 | 4    | 4    | 2         |    | 19   | 3    | 6         | 3  |      | 20   | 2         | 8  | 3    |      | 21        | 1  | 1    | 0    |           | 22 | 1    | 1    | 0 |
| 800        |           | 20 | 16   | 5    | 1         |    | 21   | 18   | 4         | 1  |      | 23   | 0         | 3  | 1    |      | 24        | 2  | 2    | 1    |           | 25 | 4    | 1    | 1 |
| 900        |           | 23 | 8    | 5    | 3         |    | 24   | 13   | 1         | 3  |      | 25   | 17        | 9  | 2    |      | 27        | 2  | 5    | 2    |           | 28 | 7    | 1    | 2 |
| 1000       |           | 26 | 0    | 6    | 2         |    | 27   | 7    | 11        | 1  |      | 28   | 15        | 4  | 0    |      | 30        | 2  | 8    | 3    |           | 31 | 10   | 1    | 2 |
| ditto      |           | l. | 26.0 | 27 1 |           | l. | 27.3 | 96 9 |           | l. | 28.7 | 67 1 |           | l. | 30.1 | 36 8 |           | l. | 31.5 | 06 7 |           | l. | 32.8 | 76 5 |   |



## SIMPLE INTEREST at 5 per Cent.

|            | 250 Days. |    |         |    | 260 Days. |    |         |    | 270 Days. |    |         |    | 280 Days. |    |         |    | 290 Days. |    |         |    | 300 Days. |    |         |    |   |
|------------|-----------|----|---------|----|-----------|----|---------|----|-----------|----|---------|----|-----------|----|---------|----|-----------|----|---------|----|-----------|----|---------|----|---|
| Principal. | l.        | s. | d.      | q. | l.        | s. | d.      | q. | l.        | s. | d.      | q. | l.        | s. | d.      | q. | l.        | s. | d.      | q. | l.        | s. | d.      | q. |   |
| 1          |           |    | 8       | 1  |           |    | 8       | 2  |           |    | 8       | 3  |           |    | 9       | 1  |           |    | 9       | 2  |           |    | 9       | 3  |   |
| 2          |           | 1  | 4       | 2  |           | 1  | 5       | 0  |           | 1  | 5       | 3  |           | 1  | 6       | 2  |           | 1  | 7       | 0  |           | 1  | 7       | 3  |   |
| 3          |           | 2  | 0       | 3  |           | 2  | 1       | 3  |           | 2  | 2       | 2  |           | 2  | 3       | 2  |           | 2  | 4       | 2  |           | 2  | 5       | 2  |   |
| 4          |           | 2  | 8       | 3  |           | 2  | 10      | 1  |           | 2  | 11      | 2  |           | 3  | 0       | 3  |           | 3  | 2       | 1  |           | 3  | 3       | 2  |   |
| 5          |           | 3  | 5       | 1  |           | 3  | 6       | 3  |           | 3  | 8       | 1  |           | 3  | 10      | 0  |           | 3  | 11      | 3  |           | 4  | 1       | 1  |   |
| 6          |           | 4  | 1       | 1  |           | 4  | 3       | 1  |           | 4  | 5       | 1  |           | 4  | 7       | 1  |           | 4  | 9       | 0  |           | 4  | 11      | 1  |   |
| 7          |           | 4  | 9       | 2  |           | 4  | 11      | 3  |           | 5  | 2       | 1  |           | 5  | 4       | 2  |           | 5  | 6       | 3  |           | 5  | 9       | 0  |   |
| 8          |           | 5  | 5       | 3  |           | 5  | 8       | 2  |           | 5  | 11      | 0  |           | 6  | 1       | 2  |           | 6  | 4       | 1  |           | 6  | 7       | 0  |   |
| 9          |           | 6  | 2       | 0  |           | 6  | 5       | 0  |           | 6  | 7       | 3  |           | 6  | 11      | 0  |           | 7  | 1       | 3  |           | 7  | 4       | 3  |   |
| 10         |           | 6  | 10      | 0  |           | 7  | 1       | 2  |           | 7  | 4       | 3  |           | 7  | 8       | 0  |           | 7  | 11      | 1  |           | 8  | 2       | 2  |   |
| 20         |           | 13 | 8       | 2  |           | 14 | 3       | 0  |           | 14 | 9       | 2  |           | 15 | 4       | 0  |           | 15 | 10      | 3  |           | 16 | 5       | 1  |   |
| 30         |           | 1  | 0       | 6  | 2         |    | 1       | 1  | 4         | 2  |         | 1  | 2         | 2  | 1       |    | 1         | 3  | 0       | 0  |           | 1  | 4       | 8  | 0 |
| 40         |           | 1  | 7       | 4  | 3         |    | 1       | 8  | 5         | 3  |         | 1  | 9         | 7  | 0       |    | 1         | 10 | 8       | 1  |           | 1  | 12      | 10 | 2 |
| 50         |           | 1  | 14      | 3  | 0         |    | 1       | 15 | 7         | 1  |         | 1  | 16        | 11 | 3       |    | 1         | 18 | 4       | 1  |           | 1  | 19      | 8  | 3 |
| 60         |           | 2  | 1       | 1  | 0         |    | 2       | 2  | 8         | 3  |         | 2  | 4         | 4  | 2       |    | 2         | 6  | 0       | 1  |           | 2  | 7       | 8  | 0 |
| 70         |           | 2  | 7       | 11 | 1         |    | 2       | 9  | 10        | 1  |         | 2  | 11        | 9  | 1       |    | 2         | 13 | 8       | 2  |           | 2  | 15      | 7  | 1 |
| 80         |           | 2  | 14      | 9  | 2         |    | 2       | 16 | 11        | 3  |         | 2  | 19        | 2  | 0       |    | 3         | 1  | 4       | 1  |           | 3  | 3       | 6  | 3 |
| 90         |           | 3  | 1       | 7  | 3         |    | 3       | 4  | 1         | 1  |         | 3  | 6         | 7  | 0       |    | 3         | 9  | 0       | 2  |           | 3  | 11      | 6  | 0 |
| 100        |           | 3  | 8       | 5  | 3         |    | 3       | 11 | 2         | 3  |         | 3  | 13        | 11 | 3       |    | 3         | 16 | 8       | 2  |           | 3  | 19      | 5  | 2 |
| 200        |           | 6  | 16      | 11 | 3         |    | 7       | 2  | 5         | 2  |         | 7  | 7         | 11 | 1       |    | 7         | 13 | 5       | 0  |           | 7  | 18      | 10 | 3 |
| 300        |           | 10 | 5       | 5  | 3         |    | 10      | 13 | 8         | 2  |         | 11 | 1         | 11 | 0       |    | 11        | 10 | 1       | 2  |           | 11 | 18      | 4  | 1 |
| 400        |           | 13 | 13      | 11 | 2         |    | 14      | 4  | 11        | 1  |         | 14 | 15        | 10 | 2       |    | 15        | 6  | 10      | 1  |           | 15 | 17      | 9  | 2 |
| 500        |           | 17 | 2       | 5  | 2         |    | 17      | 16 | 2         | 0  |         | 18 | 9         | 10 | 1       |    | 19        | 3  | 6       | 3  |           | 16 | 17      | 3  | 0 |
| 600        |           | 20 | 10      | 11 | 2         |    | 21      | 7  | 4         | 3  |         | 22 | 3         | 10 | 0       |    | 23        | 0  | 3       | 1  |           | 23 | 16      | 8  | 2 |
| 700        |           | 23 | 19      | 5  | 1         |    | 24      | 18 | 7         | 2  |         | 25 | 17        | 9  | 2       |    | 26        | 16 | 11      | 3  |           | 27 | 16      | 2  | 0 |
| 800        |           | 27 | 7       | 11 | 1         |    | 28      | 9  | 10        | 1  |         | 29 | 11        | 9  | 1       |    | 30        | 13 | 8       | 1  |           | 31 | 15      | 7  | 1 |
| 900        |           | 30 | 16      | 5  | 1         |    | 32      | 1  | 1         | 0  |         | 33 | 5         | 9  | 0       |    | 34        | 10 | 5       | 0  |           | 35 | 15      | 0  | 3 |
| 1000       |           | 24 | 4       | 11 | 1         |    | 35      | 12 | 4         | 0  |         | 36 | 19        | 8  | 3       |    | 38        | 7  | 1       | 2  |           | 39 | 14      | 6  | 1 |
| ditto      |           | l. | 34.2466 |    |           | l. | 35.6164 |    |           | l. | 36.9862 |    |           | l. | 38.3561 |    |           | l. | 39.7259 |    |           | l. | 41.0957 |    |   |

## SIMPLE INTEREST at 5 per Cent.

| Principal. | 310 Days. |    |    |    | 320 Days. |    |    |    | 330 Days. |    |    |    | 340 Days. |    |    |    | 350 Days. |    |    |    | 360 Days. |    |    |    | 365 Days,<br>or 1 Year. |    |
|------------|-----------|----|----|----|-----------|----|----|----|-----------|----|----|----|-----------|----|----|----|-----------|----|----|----|-----------|----|----|----|-------------------------|----|
|            | l.        | s. | d. | q. | l.        | s. | d. | q. | l.        | s. | d. | q. | l.        | s. | d. | q. | l.        | s. | d. | q. | l.        | s. | d. | q. | l.                      | s. |
| 1          | 100       |    |    |    | 102       |    |    |    | 103       |    |    |    | 111       |    |    |    | 112       |    |    |    | 113       |    |    |    | 1                       |    |
| 2          | 181       |    |    |    | 190       |    |    |    | 193       |    |    |    | 1101      |    |    |    | 1110      |    |    |    | 1113      |    |    |    | 2                       |    |
| 3          | 262       |    |    |    | 272       |    |    |    | 282       |    |    |    | 292       |    |    |    | 2102      |    |    |    | 2112      |    |    |    | 3                       |    |
| 4          | 343       |    |    |    | 360       |    |    |    | 371       |    |    |    | 383       |    |    |    | 3100      |    |    |    | 3111      |    |    |    | 4                       |    |
| 5          | 430       |    |    |    | 442       |    |    |    | 461       |    |    |    | 473       |    |    |    | 492       |    |    |    | 4111      |    |    |    | 5                       |    |
| 6          | 510       |    |    |    | 530       |    |    |    | 550       |    |    |    | 570       |    |    |    | 590       |    |    |    | 5110      |    |    |    | 6                       |    |
| 7          | 5111      |    |    |    | 612       |    |    |    | 640       |    |    |    | 661       |    |    |    | 682       |    |    |    | 6103      |    |    |    | 7                       |    |
| 8          | 692       |    |    |    | 700       |    |    |    | 723       |    |    |    | 752       |    |    |    | 780       |    |    |    | 7103      |    |    |    | 8                       |    |
| 9          | 773       |    |    |    | 7103      |    |    |    | 812       |    |    |    | 842       |    |    |    | 872       |    |    |    | 8102      |    |    |    | 9                       |    |
| 10         | 860       |    |    |    | 890       |    |    |    | 902       |    |    |    | 933       |    |    |    | 970       |    |    |    | 9101      |    |    |    | 10                      |    |
| 20         | 16113     |    |    |    | 1761      |    |    |    | 1810      |    |    |    | 1872      |    |    |    | 1920      |    |    |    | 1983      |    |    |    | 10                      |    |
| 30         | 1553      |    |    |    | 1632      |    |    |    | 1712      |    |    |    | 17111     |    |    |    | 1820      |    |    |    | 1970      |    |    |    | 10                      |    |
| 40         | 113113    |    |    |    | 11503     |    |    |    | 11620     |    |    |    | 11730     |    |    |    | 11841     |    |    |    | 11952     |    |    |    | 20                      |    |
| 50         | 2252      |    |    |    | 23100     |    |    |    | 2522      |    |    |    | 2663      |    |    |    | 27111     |    |    |    | 2933      |    |    |    | 20                      |    |
| 60         | 210112    |    |    |    | 21271     |    |    |    | 21430     |    |    |    | 215102    |    |    |    | 21761     |    |    |    | 21921     |    |    |    | 30                      |    |
| 70         | 21952     |    |    |    | 3141      |    |    |    | 3332      |    |    |    | 3522      |    |    |    | 3712      |    |    |    | 3902      |    |    |    | 30                      |    |
| 80         | 37111     |    |    |    | 31012     |    |    |    | 31240     |    |    |    | 31461     |    |    |    | 31682     |    |    |    | 318103    |    |    |    | 40                      |    |
| 90         | 31651     |    |    |    | 318103    |    |    |    | 4142      |    |    |    | 43100     |    |    |    | 4632      |    |    |    | 4890      |    |    |    | 40                      |    |
| 100        | 44111     |    |    |    | 4780      |    |    |    | 41050     |    |    |    | 41313     |    |    |    | 415102    |    |    |    | 41872     |    |    |    | 50                      |    |
| 200        | 89101     |    |    |    | 81540     |    |    |    | 9093      |    |    |    | 9632      |    |    |    | 91191     |    |    |    | 91730     |    |    |    | 100                     |    |
| 300        | 121492    |    |    |    | 13300     |    |    |    | 131123    |    |    |    | 131951    |    |    |    | 14780     |    |    |    | 1415102   |    |    |    | 150                     |    |
| 400        | 161983    |    |    |    | 171081    |    |    |    | 18173     |    |    |    | 181270    |    |    |    | 19363     |    |    |    | 191461    |    |    |    | 200                     |    |
| 500        | 21473     |    |    |    | 211841    |    |    |    | 221202    |    |    |    | 23590     |    |    |    | 231951    |    |    |    | 241313    |    |    |    | 250                     |    |
| 600        | 25970     |    |    |    | 26601     |    |    |    | 27252     |    |    |    | 2718103   |    |    |    | 281540    |    |    |    | 29119130  |    |    |    | 300                     |    |
| 700        | 291461    |    |    |    | 301381    |    |    |    | 311210    |    |    |    | 321202    |    |    |    | 331123    |    |    |    | 341050    |    |    |    | 350                     |    |
| 800        | 331951    |    |    |    | 35141     |    |    |    | 36332     |    |    |    | 37522     |    |    |    | 38712     |    |    |    | 390240    |    |    |    | 400                     |    |
| 900        | 38442     |    |    |    | 39901     |    |    |    | 401381    |    |    |    | 411841    |    |    |    | 43300     |    |    |    | 44780     |    |    |    | 450                     |    |
| 1000       | 42933     |    |    |    | 431682    |    |    |    | 45411     |    |    |    | 461160    |    |    |    | 4718102   |    |    |    | 4963250   |    |    |    | 500                     |    |
| ditto      | 42.4655   |    |    |    | 43.8353   |    |    |    | 45.2051   |    |    |    | 46.5749   |    |    |    | 47.9447   |    |    |    | 49.3150   |    |    |    | 50                      |    |

A TABLE shewing the Number of Days between any Day of one Month, and the same Day of any other.

| Jan. 1, &c. | Febr.      | March.     | April.     | May.       | June.     |
|-------------|------------|------------|------------|------------|-----------|
| Febr. 31.   | Marc. 28.  | April 31.  | May 30.    | June 31.   | July 30.  |
| Marc. 59.   | April 59.  | May 61.    | June 61.   | July 61.   | Aug. 61.  |
| April 90.   | May 89.    | June 92.   | July 91.   | Aug. 92.   | Sept. 92. |
| May 120.    | June 120.  | July 122.  | Aug. 122.  | Sept. 123. | Oct. 122. |
| June 151.   | July 150.  | Aug. 153.  | Sept. 153. | Oct. 153.  | Nov. 153. |
| July 181.   | Aug. 181.  | Sept. 184. | Oct. 183.  | Nov. 184.  | Dec. 183. |
| Aug. 212.   | Sept. 212. | Oct. 214.  | Nov. 214.  | Dec. 214.  | Jan. 214. |
| Sept. 243.  | Oct. 242.  | Nov. 245.  | Dec. 244.  | Jan. 245.  | Feb. 245. |
| Oct. 273.   | Nov. 273.  | Dec. 275.  | Jan. 275.  | Feb. 276.  | Mar. 273. |
| Nov. 304.   | Dec. 303.  | Jan. 306.  | Feb. 306.  | Mar. 304.  | Apr. 304. |
| Dec. 334.   | Jan. 334.  | Feb. 337.  | Mar. 334.  | Apr. 335.  | May 334.  |
| Jan. 365.   | Feb. 365.  | Mar. 365.  | Apr. 365.  | May 365.   | June 365. |

| July.     | August.    | Septemb.   | October.    | Novemb.    | Decemb.    |
|-----------|------------|------------|-------------|------------|------------|
| Aug. 31.  | Sept. 31.  | Octob. 30. | Nov. 31.    | Dec. 30.   | Jan. 31.   |
| Sept. 62. | Octob. 61. | Nov. 61.   | Dec. 61.    | Jan. 61.   | Febr. 62.  |
| Oct. 92.  | Nov. 92.   | Dec. 91.   | Jan. 92.    | Febr. 92.  | Mar. 90.   |
| Nov. 123. | Dec. 122.  | Jan. 122.  | Feb. 123.   | Mar. 120.  | Apr. 121.  |
| Dec. 153. | Jan. 153.  | Feb. 153.  | Mar. 151.   | Apr. 151.  | May 151.   |
| Jan. 184. | Feb. 184.  | Mar. 181.  | Apr. 182.   | May 181.   | June 182.  |
| Feb. 215. | Mar. 212.  | Apr. 212.  | May 212.    | June 212.  | July 212.  |
| Mar. 243. | Apr. 243.  | May 242.   | June 243.   | July 242.  | Aug. 243.  |
| Apr. 274. | May 273.   | June 273.  | July 273.   | Aug. 273.  | Sept. 274. |
| May 304.  | June 304.  | July 303.  | Aug. 304.   | Sept. 304. | Oct. 304.  |
| June 335. | July 334.  | Aug. 334.  | Sept. 335.  | Oct. 334.  | Nov. 335.  |
| July 365. | Aug. 365.  | Sept. 365. | Octob. 365. | Nov. 365.  | Dec. 365.  |

The



*The Use of the foregoing Tables.*

1. This Table last above I shall first shew the Use of, because it is naturally in Use before that of Simple Interest.

*Example.]* How many Days is contained between the 19th of November and the 28th of March following?

*Rule.]* This Table shews, that from November 19, &c. to the same day of March, it is 120 Days. To which adding the Days that 28 exceeds 19, viz. 9, gives 129 Days for Answer. But in case the Days you reckon to, are not so many as those you reckon from; then subtract the difference. As, to find the Days between the 28th of November and the 19th of March; November 28, &c. to the same Day of March is 120 Days (as before) from which deducting what 28 exceeds the 19th of November, and the Remainder is 111 Days the Answer.

*The Use of the Table of Simple Interest, at 5 per Cent.*

*Example 1.]* What is the Interest of £. 700 from the 5th of July to the 13th of April following, at 5 per Cent?

The Time is 282 Days, and against £. 700 under } £. 26 : 16 : 11  $\frac{1}{4}$   
280 Days, is \_\_\_\_\_

And under 2 Days is 00 : 3 : 10

*Example 2.]* To find the Interest of £. 470 from the 18th of December to the 5th of April, at 5 per Cent. The Time is 108 Days; and £. 400, under 100 = £. 5 : 9 : 7

*Answer,* £. 27 : 00 : 9  $\frac{1}{4}$   
*Days.* \_\_\_\_\_

£. 70, under 100 = 00 : 19 : 2  
£. 400, under 8 = 00 : 8 : 9  $\frac{1}{4}$   
£. 70, under 8 = 00 : 1 : 6

*Sum or Answer* = £. 6 : 19 : 00  $\frac{1}{4}$

3. Or any more odd or larger Sums in Questions of this kind may be multiplied one in another, and the Product of the Days and Pounds by the

.00013699 at the beginning of the Table of 5 per Cent. for Answer.

4. If you would find the Interest of £. 4700 (or other Sum above 1000) for 108, &c. Days; under 100 Days and 8 Days against ditto you find the marginal Numbers, whose Sum being multiplied by 4.7 (= 4700) gives the Answer £. 69 : 10 : 7  $\frac{1}{2}$ .

Against ditto under 100 is 13.6984  
under 8 is 1.0956

Sum £. 14.7940  
Multiply by 4.7

Product Answer = 69.5318  
Such

Such Persons as are minded to have an Instrument whereby to find the Number of Days between any two in the Year, may see the Form of one, *Plate A. Fig. 25*, and *26*. which being done in Brass, and of a six-inch Radius, will most easily and accurately answer their End.

*The Description and Use of the Circles of Days and Months.*

(*Fig. 25, 26. Plate A.*)

This consists of a Circle (as *Fig. 26.*) divided into 365 Days, which is to turn round concentrically in a Circle, (*Fig. 25.*) divided into the Months in the Year, and each Month into its respective Days: so that the inner Circle naturally measures the Days between any two in any of the Months, in order to find the Interest or Discount for those Days.

*Example.*] How many Days are contained between the 15th of November and the 2d of May following?

For Answer, turn the Hand in the inner Circle to the 15th of November in the outer Circle, and against the 2d of May in that outer, you will find in the inner Circle 168 Days, including the Day you reckon from, and excluding that which you reckon to.

*Rules how by the two Tables of 4 and 5 per Cent. above, to find the Interest at any Rate from 1 to 10 per Cent. inclusive, very easily and briefly.*

For Instance, *l. 500 for 170 Days.*

*Rules.*

*Examples.*

For 1 per Cent. take a fourth }  $l. 9 : 6 : 3 : 3$  } of which  $\frac{1}{2}$  is }  $l. 2 : 6 : 6\frac{1}{2}$  Answer.  
 of the Int. at 4 per Cent. }  
 2 per Cent. take half the }  $l. 9 : 6 : 3 : 3 ; \frac{1}{2}$  is }  $4 : 13 : 1\frac{1}{2}$  Answer.  
 Interest at 4 per Cent. }  
 3 per Cent. is the Inter. }  $l. 2 : 6 : 6 : 3 = 6 : 19 : 9$  Answer.  
 at 4 per Cent. less a 4th }  
 thereof, as less — }  
 6 per Cent. is the Inter- }  $9 : 6 : 3 : 3$  } or  $13 : 19 : 5\frac{1}{2}$  Answ.  
 rest at 4 per Cent. }  
 more half thereof }  
 more  $4 : 13 : 1 : 3$  }  
 7 per C. take }  
 the Int. of 1 p. C. } as 8 p. C. is }  $18 : 12 : 7 : 2$  }  
 from the Int. of } 1 p. C. less }  $2 : 6 : 6 : 3$  gives }  $16 : 6 : 0\frac{1}{2}$  Answ.  
 8 p. C. found thus }

*Rules*

Rules.

Examples.

For 8 per Cent. add the Int. }  
 at 4 per Cent. to itself }  $18:12:7:2 = 18:12:7\frac{1}{2}$  Anf.  
 4 per Cent. is  $9:6:3:3$  }  
 9 per Cent. add the Int. at }  
 4 and 5 per C. together } Sum  $20:19:2\frac{1}{4}$  Anf.  
 5 per Cent. is  $11:12:10:2$  }  
 10 per Cent. add the Int. at }  
 5 per Cent. to itself — }  $11:12:10:2$  }  
 more  $11:12:10:2$  } Sum  $23:5:9$  Answ.

II. The Use of Decimals in calculating Discount.

This matter wants very much to be set in a clear Light, for I have not yet seen it done; tho' I have not perhaps seen all that has been wrote upon it: But I know that a learned Author in Folio has intirely mistaken it.

1. The Discount of Money is the Allowance made by the Creditor out of a Sum of Money due to him at the end of some number of Days, in consideration of the prompt Payment of the Remainder by the Debtor.

2. That Sum paid down instead of the Principal due hereafter, may properly be called *the present Worth*; in regard that if it were put out to Interest for the Days that the Discount is computed, it would amount to the Principal due at the end of those Days.

3. The Interest for any time is more than the Discount for that time; because (suppose of  $l. 1$  for 1 Day at 6 per Cent.) the Quotient must be more when  $.016438$  (= the Interest of  $l. 100$  for one Day) is divided only by 100, than when it is divided by  $l. 100.016438$ .

4. To find *the present Worth* of  $l. 1$  due 1 Day hence at 6 per Cent.

Days.  $l.$  Int. Day.  $l.$  Int.

1<sup>st</sup>, As 365. 6 :: 1.  $.016438$

$l.$   $l.$   $l.$   $l.$  Answer.  
 2<sup>dly</sup>, As 100.  $.016438$ . 100 :: 1.  $.99983565$

5. For the Discount of  $l. 1$  for 1 Day.

1<sup>st</sup>, As 365. 6 :: 1.  $.016438$

2<sup>dly</sup>, As 100.  $.016438$ .  $.016438$  :: 1 to  $.00016435$

Here the Sum of the present Worth and Discount makes up the Principal Sum Proof,  $l. 1.00000000$  payable at time, as a Proof of the Truth of both.





In the last Example, 365 in 100 is 36500, that in 1000 makes 36500000, without farther trouble; then 365 by 100 = 36500; more 90 Days in 6 (the Rate) is 540; and that Sum is = 37040, the Divisor. See the Work.

3704.0) 3650000.0 (985.42117 = the Answer,

31640<sup>0</sup> or present Worth.

2008.0

1560.0

784.0

432.0

616.0

2456.0

2.] For *Discount*; Multiply the Rate in the Principal given, and that in the Days given is your Dividend. And the Product of 365 Days by 100 added to that of the Days, in the Rate given, is the Divisor. Thus the Principal in the Rate is

By the Days = 90

And 365 in 100 = 36500 | 3704.0) 54000.0 = Dividend (14.57883 =  
More 90 in 6 = 540 | 16960<sup>0</sup> Discount, as

2144.0 before.

Divisor = 37040 Sum. |

2920.0

3272.0

3088.0

1248.0

*Note*, By this last Method you use fewer Figures by above 40, and have not a third of the trouble.

9. In Dr Harris's *Lexicon* the Discount for one Day is asserted to be the 365th part of that for a Year: However this Mistake came, I know not; but his two Folio Pages of Table of Discount being made upon the same Principle, are likewise erroneous. And to prove this, it is sufficient from the foregoing due Calculation, That the Discount of *l.* 1 for 1 Day, at 6 per Cent. is .00016435, and not .0001550788, as the *Lexicon* makes it. And to pretend that multiplying the Discount of *l.* 1 for 1 Day, gives the Discount of one Pound for other Days by which you multiply, is a wrong Notion; because every Day's Discount of *l.* 1 differs, being less according as that Day is distant from 1, as appears plain from what follows.

This

This is a full Indication that one Day's Discount at the beginning of the Year is much more than at the middle or end: and therefore were the last Example above of the Discount of *l.* 1000 for 90 Days to be done by the Tables in the *Lexi-*

*con*, it would be } *l.*  
but — — — } 13 : 19 : 1 $\frac{1}{4}$   
Whereas 'tis in }  
truth by the }  
Calculation — } 14 : 11 : 6 $\frac{1}{4}$   
bove — — — }

So that the said *Lexicon*-Tables err in this Instance 12 *s.* 5 *d.* And were the Days fewer, for which the Discount is required, the Error would be proportionably greater.

Hence it may be inferred, That no Tables of Discount can be used with Accuracy, but such as have the Discount for every Day in the Year, because every Day's Discount differeth. And had I time, and room in this Book, I would oblige the Publick with a Table of Discount for every Day, because I know not of any true Table of Discount extant. But in the mean time, the following will be better than any yet published, and will be found accurate enough in Practice.

| Days. | Discount of <i>l.</i> 1.<br>for those Days<br>at 6 per Cent. | Differences or<br>each Day's<br>Discount. |
|-------|--------------------------------------------------------------|-------------------------------------------|
| 1     | .00016435                                                    |                                           |
| 2     | .00032865                                                    | .00016430                                 |
| 3     | .0004929                                                     | .00016425                                 |
| 4     | .0006571                                                     | .0001642                                  |
| 180   | .0287386                                                     |                                           |
| 181   | .0288937                                                     | .0001551                                  |
| 182   | .0290487                                                     | .0001550                                  |
| 183   | .0292036                                                     | .0001549                                  |
| 362   | .05616466                                                    |                                           |
| 363   | .05631106                                                    | .00014642                                 |
| 364   | .05645744                                                    | .00014638                                 |
| 365   | .05660377                                                    | .00014633                                 |

A TABLE



*A TABLE of DISCOUNT at 4 per Cent.  
per Ann. more accurate than any extant.*

|            | 1 Day. |    |       | 2 Days. |    |       | 3 Days. |    |       | 4 Days. |    |       | 5 Days. |    |       | 6 Days. |    |      | 7 Days. |    |       | 8 Days. |    |       |
|------------|--------|----|-------|---------|----|-------|---------|----|-------|---------|----|-------|---------|----|-------|---------|----|------|---------|----|-------|---------|----|-------|
| Principal. | s.     | d. | q.    | s.      | d. | q.    | s.      | d. | q.    | s.      | d. | q.    | s.      | d. | q.    | s.      | d. | q.   | s.      | d. | q.    | s.      | d. | q.    |
| 1          |        |    |       |         |    |       |         |    |       |         |    |       |         |    |       |         |    |      |         |    |       |         |    |       |
| 2          |        |    |       |         |    |       |         |    |       |         |    |       |         |    |       |         |    |      |         |    |       |         |    |       |
| 3          |        |    |       |         |    |       |         |    |       |         |    |       |         |    |       |         |    |      |         |    |       |         |    |       |
| 4          |        |    |       |         |    |       |         |    |       |         |    |       |         |    |       |         |    |      |         |    |       |         |    |       |
| 5          |        |    |       |         |    |       |         |    |       |         |    |       |         |    |       |         |    |      |         |    |       |         |    |       |
| 6          |        |    |       |         |    |       |         |    |       |         |    |       |         |    |       |         |    |      |         |    |       |         |    |       |
| 7          |        |    |       |         |    |       |         |    |       |         |    |       |         |    |       |         |    |      |         |    |       |         |    |       |
| 8          |        |    |       |         |    |       |         |    |       |         |    |       |         |    |       |         |    |      |         |    |       |         |    |       |
| 9          |        |    |       |         |    |       |         |    |       |         |    |       |         |    |       |         |    |      |         |    |       |         |    |       |
| 10         |        |    |       |         |    |       |         |    |       |         |    |       |         |    |       |         |    |      |         |    |       |         |    |       |
| 20         |        |    |       |         |    |       |         |    |       |         |    |       |         |    |       |         |    |      |         |    |       |         |    |       |
| 30         |        |    |       |         |    |       |         |    |       |         |    |       |         |    |       |         |    |      |         |    |       |         |    |       |
| 40         |        |    |       |         |    |       |         |    |       |         |    |       |         |    |       |         |    |      |         |    |       |         |    |       |
| 50         |        |    |       |         |    |       |         |    |       |         |    |       |         |    |       |         |    |      |         |    |       |         |    |       |
| 60         |        |    |       |         |    |       |         |    |       |         |    |       |         |    |       |         |    |      |         |    |       |         |    |       |
| 70         |        |    |       |         |    |       |         |    |       |         |    |       |         |    |       |         |    |      |         |    |       |         |    |       |
| 80         |        |    |       |         |    |       |         |    |       |         |    |       |         |    |       |         |    |      |         |    |       |         |    |       |
| 90         |        |    |       |         |    |       |         |    |       |         |    |       |         |    |       |         |    |      |         |    |       |         |    |       |
| 100        |        |    |       |         |    |       |         |    |       |         |    |       |         |    |       |         |    |      |         |    |       |         |    |       |
| 200        |        |    |       |         |    |       |         |    |       |         |    |       |         |    |       |         |    |      |         |    |       |         |    |       |
| 300        |        |    |       |         |    |       |         |    |       |         |    |       |         |    |       |         |    |      |         |    |       |         |    |       |
| 400        |        |    |       |         |    |       |         |    |       |         |    |       |         |    |       |         |    |      |         |    |       |         |    |       |
| 500        |        |    |       |         |    |       |         |    |       |         |    |       |         |    |       |         |    |      |         |    |       |         |    |       |
| 600        |        |    |       |         |    |       |         |    |       |         |    |       |         |    |       |         |    |      |         |    |       |         |    |       |
| 700        |        |    |       |         |    |       |         |    |       |         |    |       |         |    |       |         |    |      |         |    |       |         |    |       |
| 800        |        |    |       |         |    |       |         |    |       |         |    |       |         |    |       |         |    |      |         |    |       |         |    |       |
| 900        |        |    |       |         |    |       |         |    |       |         |    |       |         |    |       |         |    |      |         |    |       |         |    |       |
| 1000       |        |    |       |         |    |       |         |    |       |         |    |       |         |    |       |         |    |      |         |    |       |         |    |       |
| ditto      | l.     | .  | 10957 | l.      | .  | 21913 | l.      | .  | 32865 | l.      | .  | 43816 | l.      | .  | 54764 | l.      | .  | 6571 | l.      | .  | 76653 | l.      | .  | 87594 |

## A TABLE of DISCOUNT at 4 per Cent.

| Principal.<br>l. | 9 Days. |    |    | 10 Days.  |    |    | 20 Days. |            |    | 30 Days. |    |            | 40 Days. |    |    | 50 Days. |            |    | 60 Days. |    |            |    |    |    |            |  |
|------------------|---------|----|----|-----------|----|----|----------|------------|----|----------|----|------------|----------|----|----|----------|------------|----|----------|----|------------|----|----|----|------------|--|
|                  | s.      | d. | q. | l.        | s. | d. | q.       | l.         | s. | d.       | q. | l.         | s.       | d. | q. | l.       | s.         | d. | q.       | l. | s.         | d. | q. |    |            |  |
| 1                |         |    |    | 1         |    |    |          | 1          |    |          |    | 0          | 2        |    |    | 3        |            |    | 1        | 0  |            |    | 1  | 1  |            |  |
| 2                |         |    |    | 2         |    |    |          | 2          |    |          |    | 1          | 0        |    |    | 1        | 2          |    |          | 2  | 0          |    |    | 2  | 2          |  |
| 3                |         |    |    | 3         |    |    |          | 3          |    |          |    | 1          | 2        |    |    | 2        | 1          |    |          | 3  | 1          |    |    | 4  | 0          |  |
| 4                |         |    |    | 1         | 0  |    |          | 1          | 0  |          |    | 2          | 1        |    |    | 3        | 1          |    |          | 4  | 1          |    |    | 5  | 1          |  |
| 5                |         |    |    | 1         | 1  |    |          | 1          | 1  |          |    | 2          | 3        |    |    | 4        | 0          |    |          | 5  | 1          |    |    | 6  | 2          |  |
| 6                |         |    |    | 1         | 2  |    |          | 1          | 2  |          |    | 3          | 1        |    |    | 4        | 3          |    |          | 6  | 1          |    |    | 7  | 3          |  |
| 7                |         |    |    | 1         | 3  |    |          | 1          | 3  |          |    | 3          | 3        |    |    | 5        | 2          |    |          | 7  | 1          |    |    | 9  | 1          |  |
| 8                |         |    |    | 2         | 0  |    |          | 2          | 1  |          |    | 4          | 1        |    |    | 6        | 1          |    |          | 8  | 1          |    |    | 10 | 2          |  |
| 9                |         |    |    | 2         | 1  |    |          | 2          | 2  |          |    | 4          | 3        |    |    | 7        | 0          |    |          | 9  | 2          |    |    | 11 | 3          |  |
| 10               |         |    |    | 2         | 2  |    |          | 2          | 3  |          |    | 5          | 1        |    |    | 7        | 3          |    |          | 10 | 2          |    |    | 1  | 1          |  |
| 20               |         |    |    | 4         | 3  |    |          | 5          | 1  |          |    | 10         | 2        |    |    | 1        | 3          | 3  |          | 1  | 9          | 0  |    | 2  | 2          |  |
| 30               |         |    |    | 7         | 0  |    |          | 7          | 3  |          |    | 1          | 3        | 3  |    |          | 1          | 1  | 3        |    | 2          | 7  | 2  | 3  | 3          |  |
| 40               |         |    |    | 9         | 2  |    |          | 10         | 2  |          |    | 1          | 9        | 0  |    |          | 2          | 7  | 2        |    | 3          | 6  | 0  | 4  | 4          |  |
| 50               |         |    |    | 1         | 0  | 0  |          | 1          | 1  | 0        |    | 2          | 2        | 1  |    |          | 3          | 3  | 1        |    | 4          | 4  | 2  | 5  | 5          |  |
| 60               |         |    |    | 1         | 2  | 1  |          | 1          | 3  | 3        |    | 2          | 7        | 2  |    |          | 3          | 11 | 1        |    | 5          | 2  | 3  | 6  | 6          |  |
| 70               |         |    |    | 1         | 4  | 2  |          | 1          | 6  | 1        |    | 3          | 0        | 3  |    |          | 4          | 7  | 0        |    | 6          | 1  | 1  | 7  | 7          |  |
| 80               |         |    |    | 1         | 7  | 0  |          | 1          | 9  | 0        |    | 3          | 6        | 0  |    |          | 5          | 3  | 0        |    | 7          | 0  | 0  | 8  | 8          |  |
| 90               |         |    |    | 1         | 9  | 1  |          | 1          | 11 | 3        |    | 3          | 11       | 1  |    |          | 5          | 10 | 3        |    | 7          | 10 | 1  | 9  | 9          |  |
| 100              |         |    |    | 1         | 11 | 3  |          | 2          | 2  | 1        |    | 4          | 4        | 2  |    |          | 6          | 6  | 2        |    | 8          | 8  | 3  | 10 | 10         |  |
| 200              |         |    |    | 3         | 11 | 2  |          | 4          | 4  | 2        |    | 8          | 9        | 0  |    |          | 13         | 1  | 1        |    | 17         | 5  | 2  | 1  | 1          |  |
| 300              |         |    |    | 5         | 11 | 0  |          | 6          | 6  | 3        |    | 13         | 1        | 2  |    |          | 19         | 8  | 0        |    | 6          | 2  | 1  | 12 | 8          |  |
| 400              |         |    |    | 7         | 10 | 3  |          | 8          | 9  | 0        |    | 17         | 6        | 0  |    |          | 1          | 6  | 2        | 2  | 14         | 11 | 0  | 2  | 3          |  |
| 500              |         |    |    | 9         | 10 | 1  |          | 10         | 11 | 2        |    | 1          | 10       | 2  |    |          | 1          | 12 | 9        | 12 | 3          | 7  | 3  | 2  | 14         |  |
| 600              |         |    |    | 11        | 10 | 0  |          | 13         | 1  | 2        |    | 6          | 3        | 0  |    |          | 1          | 19 | 4        | 0  | 2          | 12 | 4  | 2  | 3          |  |
| 700              |         |    |    | 13        | 9  | 2  |          | 15         | 4  | 0        |    | 10         | 7        | 12 |    |          | 5          | 10 | 2        | 3  | 1          | 1  | 1  | 3  | 16         |  |
| 800              |         |    |    | 15        | 9  | 1  |          | 17         | 6  | 0        |    | 15         | 0        | 0  |    |          | 2          | 12 | 5        | 0  | 3          | 9  | 10 | 0  | 4          |  |
| 900              |         |    |    | 17        | 8  | 3  |          | 19         | 8  | 2        |    | 19         | 4        | 22 |    |          | 18         | 11 | 3        | 3  | 18         | 6  | 3  | 4  | 18         |  |
| 1000             |         |    |    | 19        | 8  | 2  |          | 1          | 10 | 32       |    | 3          | 8        | 33 |    |          | 5          | 6  | 24       |    | 7          | 3  | 25 |    | 9          |  |
| ditto            |         |    |    | l. .98532 |    |    |          | l. 1.09469 |    |          |    | l. 2.18699 |          |    |    |          | l. 3.27689 |    |          |    | l. 4.36443 |    |    |    | l. 5.44959 |  |

# A TABLE of DISCOUNT at 4 per Cent. per Ann.

|            | 70 Days. |         |    |    | 80 Days. |         |    |    | 90 Days. |         |    |    | 100 Days. |          |    |    | 110 Days. |         |    |    | 120 Days. |       |    |    |   |
|------------|----------|---------|----|----|----------|---------|----|----|----------|---------|----|----|-----------|----------|----|----|-----------|---------|----|----|-----------|-------|----|----|---|
| Principal. | l.       | s.      | d. | q. | l.       | s.      | d. | q. | l.       | s.      | d. | q. | l.        | s.       | d. | q. | l.        | s.      | d. | q. | l.        | s.    | d. | q. |   |
| 1          |          |         | 1  | 3  |          |         | 2  | 0  |          |         | 2  | 1  |           |          | 2  | 3  |           |         | 3  | 0  |           |       | 3  | 1  |   |
| 2          |          |         | 3  | 3  |          |         | 4  | 1  |          |         | 4  | 3  |           |          | 5  | 1  |           |         | 5  | 3  |           |       | 6  | 1  |   |
| 3          |          |         | 4  | 2  |          |         | 6  | 1  |          |         | 7  | 0  |           |          | 7  | 3  |           |         | 8  | 2  |           |       | 9  | 1  |   |
| 4          |          |         | 7  | 1  |          |         | 8  | 1  |          |         | 9  | 2  |           |          | 10 | 2  |           |         | 11 | 2  |           | 1     | 0  | 2  |   |
| 5          |          |         | 9  | 1  |          |         | 10 | 2  |          |         | 11 | 3  |           | 1        | 1  | 0  |           |         | 1  | 2  | 1         |       | 1  | 3  | 2 |
| 6          |          |         | 11 | 0  |          | 1       | 0  | 2  |          | 1       | 2  | 0  |           | 1        | 3  | 3  |           |         | 1  | 5  | 1         |       | 1  | 6  | 3 |
| 7          | 1        | 0       | 3  |    | 1        | 2       | 2  |    | 1        | 4       | 2  |    | 1         | 6        | 1  |    |           | 1       | 8  | 0  |           | 1     | 9  | 3  |   |
| 8          | 1        | 2       | 2  |    | 1        | 4       | 3  |    | 1        | 6       | 3  |    | 1         | 8        | 3  |    |           | 1       | 11 | 0  |           | 2     | 1  | 0  |   |
| 9          | 1        | 4       | 2  |    | 1        | 6       | 3  |    | 1        | 9       | 0  |    | 1         | 11       | 2  |    |           | 2       | 1  | 3  |           | 2     | 4  | 0  |   |
| 10         | 1        | 6       | 1  |    | 1        | 8       | 3  |    | 1        | 11      | 2  |    | 2         | 2        | 0  |    |           | 2       | 4  | 3  |           | 2     | 7  | 1  |   |
| 20         | 3        | 0       | 2  |    | 3        | 5       | 3  |    | 3        | 11      | 0  |    | 4         | 4        | 1  |    |           | 4       | 9  | 1  |           | 5     | 2  | 1  |   |
| 30         | 4        | 6       | 3  |    | 5        | 2       | 2  |    | 5        | 10      | 1  |    | 6         | 6        | 0  |    |           | 7       | 1  | 3  |           | 7     | 9  | 2  |   |
| 40         | 6        | 1       | 0  |    | 6        | 11      | 2  |    | 7        | 9       | 3  |    | 8         | 8        | 0  |    |           | 9       | 6  | 1  |           | 10    | 4  | 3  |   |
| 50         | 7        | 7       | 1  |    | 8        | 9       | 1  |    | 9        | 9       | 1  |    | 10        | 10       | 0  |    |           | 11      | 11 | 0  |           | 12    | 11 | 3  |   |
| 60         | 9        | 1       | 2  |    | 10       | 5       | 1  |    | 11       | 8       | 3  |    | 13        | 0        | 0  |    |           | 14      | 3  | 2  |           | 15    | 7  | 0  |   |
| 70         | 10       | 7       | 3  |    | 12       | 2       | 0  |    | 13       | 8       | 0  |    | 15        | 0        | 2  |    |           | 16      | 8  | 0  |           | 18    | 2  | 0  |   |
| 80         | 12       | 2       | 1  |    | 13       | 11      | 0  |    | 15       | 7       | 2  |    | 17        | 4        | 1  |    |           | 19      | 0  | 2  |           | 1     | 0  | 9  | 1 |
| 90         | 13       | 8       | 2  |    | 15       | 7       | 3  |    | 17       | 7       | 0  |    | 19        | 6        | 1  |    | 1         | 1       | 5  | 2  |           | 1     | 3  | 4  | 2 |
| 100        | 15       | 2       | 3  |    | 17       | 4       | 3  |    | 19       | 6       | 2  |    | 1         | 1        | 8  | 1  | 1         | 3       | 10 | 0  |           | 1     | 5  | 11 | 2 |
| 200        | 1        | 10      | 5  | 2  | 1        | 14      | 9  | 1  | 1        | 19      | 0  | 3  | 2         | 3        | 4  | 2  | 2         | 7       | 7  | 3  | 2         | 11    | 11 | 1  |   |
| 300        | 2        | 5       | 8  | 0  | 2        | 12      | 1  | 3  | 2        | 18      | 7  | 1  | 3         | 5        | 0  | 2  | 3         | 11      | 5  | 3  | 3         | 17    | 10 | 3  |   |
| 400        | 3        | 0       | 1  | 4  | 0        | 3       | 9  | 6  | 1        | 3       | 18 | 1  | 4         | 6        | 8  | 3  | 4         | 15      | 3  | 2  | 5         | 3     | 10 | 1  |   |
| 500        | 3        | 16      | 1  | 2  | 4        | 6       | 11 | 0  | 4        | 17      | 8  | 1  | 5         | 8        | 5  | 0  | 5         | 19      | 1  | 1  | 6         | 9     | 9  | 3  |   |
| 600        | 4        | 11      | 4  | 1  | 5        | 4       | 3  | 2  | 5        | 17      | 2  | 2  | 6         | 10       | 1  | 0  | 7         | 2       | 11 | 1  | 7         | 15    | 9  | 1  |   |
| 700        | 5        | 6       | 7  | 0  | 6        | 1       | 8  | 0  | 6        | 1       | 8  | 3  | 7         | 11       | 9  | 1  | 8         | 6       | 9  | 0  | 9         | 1     | 8  | 3  |   |
| 800        | 6        | 1       | 9  | 3  | 6        | 19      | 0  | 3  | 7        | 16      | 3  | 1  | 8         | 13       | 5  | 2  | 9         | 10      | 7  | 0  | 10        | 7     | 8  | 1  |   |
| 900        | 6        | 17      | 0  | 1  | 7        | 16      | 5  | 1  | 8        | 15      | 9  | 2  | 9         | 15       | 1  | 2  | 10        | 14      | 5  | 0  | 11        | 13    | 7  | 3  |   |
| 1000       | 7        | 12      | 3  | 0  | 8        | 13      | 9  | 3  | 9        | 15      | 4  | 9  | 10        | 16       | 9  | 3  | 11        | 18      | 2  | 3  | 12        | 19    | 7  | 1  |   |
| ditto      | l.       | 7.61283 |    |    | l.       | 8.69092 |    |    | l.       | 9.76668 |    |    | l.        | 10.84011 |    |    | l.        | 11.9112 |    |    | l.        | 12.98 |    |    |   |



# A TABLE of DISCOUNT at 4 per Cent. per Ann.

| Principal.<br>l. | 130 Days. |     |    |    | 140 Days. |    |     |    | 150 Days. |    |    |     | 160 Days. |     |    |    | 170 Days. |    |     |    | 180 Days. |     |    |    |    |    |     |    |     |
|------------------|-----------|-----|----|----|-----------|----|-----|----|-----------|----|----|-----|-----------|-----|----|----|-----------|----|-----|----|-----------|-----|----|----|----|----|-----|----|-----|
|                  | l.        | s.  | d. | q. | l.        | s. | d.  | q. | l.        | s. | d. | q.  | l.        | s.  | d. | q. | l.        | s. | d.  | q. | l.        | s.  | d. | q. |    |    |     |    |     |
| 1                |           |     | 3  | 2  |           |    | 3   | 3  |           |    |    | 4   | 0         |     |    | 4  | 1         |    |     | 4  | 2         |     |    | 4  | 3  |    |     |    |     |
| 2                |           |     | 6  | 3  |           |    | 7   | 1  |           |    |    | 7   | 3         |     |    | 8  | 1         |    |     | 8  | 3         |     |    | 9  | 1  |    |     |    |     |
| 3                |           |     | 10 | 1  |           |    | 11  | 0  |           |    |    | 11  | 3         |     |    | 1  | 0         | 1  |     | 1  | 1         | 1   |    | 1  | 2  | 0  |     |    |     |
| 4                |           | 1   | 1  | 2  |           | 1  | 2   | 2  |           | 1  | 2  | 2   |           | 1   | 4  | 2  |           | 1  | 5   | 2  |           | 1   | 6  | 2  |    |    |     |    |     |
| 5                |           | 1   | 5  | 0  |           | 1  | 6   | 0  |           | 1  | 7  | 2   |           | 1   | 8  | 3  |           | 1  | 10  | 0  |           | 1   | 11 | 1  |    |    |     |    |     |
| 6                |           | 1   | 8  | 1  |           | 1  | 9   | 3  |           | 1  | 11 | 1   |           | 2   | 0  | 3  |           | 2  | 2   | 2  |           | 2   | 4  | 0  |    |    |     |    |     |
| 7                |           | 1   | 11 | 3  |           | 2  | 1   | 2  |           | 2  | 3  | 1   |           | 2   | 5  | 0  |           | 2  | 6   | 3  |           | 2   | 8  | 2  |    |    |     |    |     |
| 8                |           | 2   | 3  | 0  |           | 2  | 5   | 0  |           | 2  | 7  | 0   |           | 2   | 9  | 0  |           | 2  | 11  | 1  |           | 3   | 1  | 0  |    |    |     |    |     |
| 9                |           | 2   | 6  | 1  |           | 2  | 8   | 3  |           | 2  | 11 | 0   |           | 3   | 1  | 1  |           | 3  | 3   | 2  |           | 3   | 5  | 3  |    |    |     |    |     |
| 10               |           | 2   | 9  | 3  |           | 3  | 0   | 1  |           | 3  | 2  | 3   |           | 3   | 5  | 2  |           | 3  | 7   | 3  |           | 3   | 10 | 2  |    |    |     |    |     |
| 20               |           | 5   | 7  | 2  |           | 6  | 0   | 2  |           | 6  | 5  | 2   |           | 6   | 10 | 3  |           | 7  | 3   | 3  |           | 7   | 8  | 3  |    |    |     |    |     |
| 30               |           | 8   | 5  | 1  |           | 9  | 0   | 3  |           | 9  | 8  | 2   |           | 10  | 4  | 1  |           | 10 | 11  | 3  |           | 11  | 7  | 1  |    |    |     |    |     |
| 40               |           | 11  | 2  | 3  |           | 12 | 1   | 0  |           | 12 | 11 | 1   |           | 13  | 9  | 2  |           | 14 | 7   | 2  |           | 15  | 5  | 3  |    |    |     |    |     |
| 50               |           | 14  | 0  | 2  |           | 15 | 1   | 1  |           | 16 | 2  | 0   |           | 17  | 2  | 3  |           | 18 | 3   | 2  |           | 19  | 4  | 1  |    |    |     |    |     |
| 60               |           | 16  | 10 | 1  |           | 18 | 1   | 2  |           | 19 | 5  | 0   |           | 1   | 0  | 8  | 0         | 1  | 1   | 11 | 2         |     | 1  | 3  | 2  | 2  |     |    |     |
| 70               |           | 19  | 8  | 0  |           | 1  | 1   | 2  | 0         |    | 1  | 2   | 7         | 3   |    | 1  | 4         | 1  | 2   | 1  | 5         | 7   | 1  |    | 1  | 7  | 1   | 0  |     |
| 80               | 1         | 2   | 5  | 3  |           | 1  | 4   | 2  | 1         |    | 1  | 5   | 10        | 2   |    | 1  | 7         | 6  | 3   | 1  | 9         | 3   | 1  |    | 1  | 10 | 11  | 2  |     |
| 90               | 1         | 5   | 3  | 2  |           | 1  | 7   | 2  | 2         |    | 1  | 9   | 1         | 1   |    | 1  | 11        | 0  | 1   | 1  | 12        | 11  | 1  |    | 1  | 14 | 9   | 3  |     |
| 100              | 1         | 8   | 1  | 0  |           | 1  | 10  | 2  | 3         |    | 1  | 12  | 4         | 1   |    | 1  | 14        | 5  | 3   | 1  | 16        | 7   | 0  |    | 1  | 18 | 8   | 1  |     |
| 200              | 2         | 16  | 2  | 1  |           | 3  | 0   | 5  | 2         |    | 3  | 4   | 8         | 1   |    | 3  | 8         | 11 | 1   | 3  | 13        | 1   | 3  |    | 3  | 17 | 4   | 2  |     |
| 300              | 4         | 4   | 3  | 1  |           | 4  | 10  | 8  | 1         |    | 4  | 17  | 0         | 2   |    | 5  | 3         | 4  | 3   | 5  | 9         | 8   | 3  |    | 5  | 16 | 0   | 3  |     |
| 400              | 5         | 12  | 4  | 2  |           | 6  | 0   | 11 | 0         |    | 6  | 9   | 4         | 3   |    | 6  | 17        | 10 | 1   | 7  | 6         | 3   | 3  |    | 7  | 14 | 9   | 0  |     |
| 500              | 7         | 0   | 5  | 2  |           | 7  | 11  | 1  | 2         |    | 8  | 1   | 8         | 3   |    | 8  | 12        | 4  | 0   | 9  | 2         | 10  | 3  |    | 9  | 13 | 5   | 2  |     |
| 600              | 8         | 8   | 6  | 3  |           | 9  | 1   | 4  | 1         |    | 9  | 14  | 0         | 3   |    | 10 | 6         | 9  | 2   | 10 | 19        | 5   | 3  |    | 11 | 12 | 1   | 2  |     |
| 700              | 9         | 16  | 7  | 2  |           | 10 | 11  | 7  | 0         |    | 11 | 6   | 5         | 0   |    | 12 | 1         | 3  | 0   | 12 | 16        | 0   | 2  |    | 13 | 10 | 10  | 0  |     |
| 800              | 11        | 4   | 9  | 0  |           | 12 | 1   | 9  | 3         |    | 12 | 18  | 9         | 1   |    | 13 | 15        | 8  | 2   | 14 | 12        | 7   | 2  |    | 15 | 9  | 6   | 0  |     |
| 900              | 12        | 12  | 10 | 0  |           | 13 | 12  | 0  | 2         |    | 14 | 11  | 1         | 1   |    | 15 | 10        | 2  | 0   | 16 | 9         | 2   | 2  |    | 17 | 8  | 2   | 2  |     |
| 1000             | 14        | 0   | 11 | 1  |           | 15 | 2   | 3  | 1         |    | 16 | 3   | 5         | 2   |    | 17 | 4         | 7  | 3   | 18 | 5         | 9   | 2  |    | 19 | 6  | 10  | 3  |     |
| ditto            | l.        | 14. | 04 | 46 |           | l. | 15. | 11 | 333       |    | l. | 16. | 17        | 251 |    | l. | 17.       | 23 | 209 |    | l.        | 18. | 28 | 94 |    | l. | 19. | 34 | 444 |

# A TABLE of DISCOUNT at 4 per Cent. per Ann.

|            | 190 Days. |       |    |    | 200 Days. |       |    |    | 210 Days. |       |    |    | 220 Days. |       |    |    | 230 Days. |       |    |    | 240 Days. |       |    |    |   |
|------------|-----------|-------|----|----|-----------|-------|----|----|-----------|-------|----|----|-----------|-------|----|----|-----------|-------|----|----|-----------|-------|----|----|---|
| Principal. | l.        | s.    | d. | q. | l.        | s.    | d. | q. | l.        | s.    | d. | q. | l.        | s.    | d. | q. | l.        | s.    | d. | q. | l.        | s.    | d. | q. |   |
| 1          |           |       | 5  | 0  |           |       | 5  | 1  |           |       | 5  | 2  |           |       | 5  | 3  |           |       | 6  | 0  |           |       | 6  | 1  |   |
| 2          |           |       | 9  | 3  |           |       | 10 | 1  |           |       | 10 | 3  |           |       | 11 | 2  |           |       | 1  | 0  | 0         |       | 1  | 0  | 1 |
| 3          |           | 1     | 2  | 3  |           | 1     | 3  | 2  |           | 1     | 4  | 1  |           | 1     | 5  | 0  |           | 1     | 5  | 3  |           | 1     | 6  | 2  |   |
| 4          |           | 1     | 7  | 2  |           | 1     | 8  | 2  |           | 1     | 9  | 2  |           | 1     | 10 | 3  |           | 1     | 11 | 3  |           | 2     | 0  | 2  |   |
| 5          |           | 2     | 0  | 2  |           | 2     | 1  | 3  |           | 2     | 3  | 0  |           | 2     | 4  | 1  |           | 2     | 5  | 2  |           | 2     | 6  | 3  |   |
| 6          |           | 2     | 5  | 2  |           | 2     | 6  | 3  |           | 2     | 8  | 1  |           | 2     | 10 | 0  |           | 2     | 11 | 2  |           | 3     | 1  | 0  |   |
| 7          |           | 2     | 10 | 1  |           | 3     | 0  | 0  |           | 3     | 1  | 3  |           | 3     | 3  | 2  |           | 3     | 5  | 2  |           | 3     | 7  | 0  |   |
| 8          |           | 3     | 3  | 1  |           | 3     | 5  | 1  |           | 3     | 7  | 1  |           | 3     | 9  | 1  |           | 3     | 11 | 1  |           | 4     | 1  | 1  |   |
| 9          |           | 3     | 8  | 0  |           | 3     | 10 | 2  |           | 4     | 0  | 2  |           | 4     | 2  | 3  |           | 4     | 5  | 1  |           | 4     | 7  | 1  |   |
| 10         |           | 4     | 1  | 0  |           | 4     | 3  | 2  |           | 4     | 6  | 0  |           | 4     | 8  | 2  |           | 4     | 11 | 0  |           | 5     | 1  | 2  |   |
| 20         |           | 8     | 2  | 0  |           | 8     | 7  | 0  |           | 9     | 0  | 0  |           | 9     | 5  | 0  |           | 9     | 10 | 0  |           | 10    | 3  | 0  |   |
| 30         |           | 12    | 2  | 3  |           | 12    | 10 | 2  |           | 13    | 6  | 0  |           | 14    | 1  | 2  |           | 14    | 9  | 0  |           | 15    | 4  | 2  |   |
| 40         |           | 16    | 3  | 3  |           | 17    | 2  | 0  |           | 18    | 0  | 0  |           | 18    | 10 | 0  |           | 19    | 8  | 0  |           | 1     | 0  | 6  | 0 |
| 50         | 1         | 0     | 4  | 3  | 1         | 1     | 5  | 1  | 1         | 2     | 6  | 0  | 1         | 3     | 6  | 2  | 1         | 4     | 7  | 0  | 1         | 5     | 7  | 2  |   |
| 60         | 1         | 4     | 5  | 3  | 1         | 5     | 8  | 3  | 1         | 7     | 0  | 0  | 1         | 8     | 3  | 0  | 1         | 9     | 6  | 0  | 1         | 10    | 9  | 0  |   |
| 70         | 1         | 8     | 6  | 3  | 1         | 10    | 0  | 1  | 1         | 11    | 6  | 0  | 1         | 12    | 11 | 2  | 1         | 14    | 5  | 0  | 1         | 15    | 10 | 2  |   |
| 80         | 1         | 12    | 7  | 2  | 1         | 14    | 3  | 3  | 1         | 16    | 0  | 0  | 1         | 17    | 8  | 0  | 1         | 19    | 4  | 0  | 2         | 1     | 0  | 0  |   |
| 90         | 1         | 16    | 8  | 2  | 1         | 18    | 7  | 1  | 2         | 0     | 6  | 0  | 2         | 2     | 4  | 2  | 2         | 4     | 3  | 0  | 2         | 6     | 1  | 2  |   |
| 100        | 2         | 0     | 9  | 2  | 2         | 2     | 10 | 3  | 2         | 5     | 0  | 0  | 2         | 7     | 1  | 0  | 2         | 9     | 2  | 0  | 2         | 11    | 3  | 0  |   |
| 200        | 4         | 1     | 7  | 0  | 4         | 5     | 9  | 2  | 4         | 9     | 11 | 3  | 4         | 14    | 2  | 0  | 4         | 18    | 4  | 1  | 5         | 2     | 6  | 0  |   |
| 300        | 6         | 2     | 4  | 3  | 6         | 8     | 8  | 1  | 6         | 14    | 11 | 3  | 7         | 1     | 3  | 0  | 7         | 7     | 6  | 1  | 7         | 13    | 9  | 1  |   |
| 400        | 8         | 3     | 2  | 0  | 8         | 11    | 7  | 0  | 8         | 19    | 11 | 3  | 9         | 8     | 4  | 0  | 9         | 16    | 8  | 2  | 10        | 5     | 0  | 0  |   |
| 500        | 10        | 3     | 11 | 3  | 10        | 14    | 5  | 3  | 11        | 4     | 11 | 2  | 11        | 15    | 5  | 0  | 12        | 5     | 10 | 1  | 12        | 16    | 3  | 1  |   |
| 600        | 12        | 4     | 9  | 1  | 12        | 17    | 4  | 2  | 13        | 9     | 11 | 2  | 14        | 2     | 6  | 0  | 14        | 15    | 0  | 1  | 15        | 7     | 6  | 1  |   |
| 700        | 14        | 5     | 6  | 3  | 15        | 0     | 3  | 1  | 15        | 14    | 11 | 2  | 16        | 9     | 7  | 0  | 17        | 4     | 2  | 2  | 17        | 18    | 9  | 2  |   |
| 800        | 16        | 6     | 4  | 1  | 17        | 3     | 2  | 0  | 17        | 19    | 11 | 1  | 18        | 16    | 8  | 0  | 19        | 13    | 4  | 2  | 20        | 10    | 0  | 2  |   |
| 900        | 18        | 7     | 1  | 3  | 19        | 6     | 0  | 3  | 20        | 4     | 11 | 1  | 21        | 3     | 9  | 0  | 22        | 2     | 6  | 2  | 23        | 1     | 3  | 2  |   |
| 1000       | 20        | 7     | 11 | 2  | 21        | 8     | 11 | 2  | 22        | 9     | 11 | 0  | 23        | 10    | 10 | 0  | 24        | 11    | 8  | 2  | 25        | 12    | 6  | 2  |   |
| ditto      | l.        | 20.39 | 72 |    | l.        | 21.44 | 72 |    | l.        | 22.49 | 59 | 8  | l.        | 23.54 | 2  |    | l.        | 24.58 | 57 | 8  | l.        | 25.62 | 73 | 3  |   |

# A TABLE of DISCOUNT at 4 per Cent. per Ann.

| Principal.<br>l. | 250 Days. |    |    |    | 260 Days.   |    |    |    | 270 Days.   |    |    |    | 280 Days.   |    |    |    | 290 Days.  |    |    |    | 300 Days.   |    |    |    |
|------------------|-----------|----|----|----|-------------|----|----|----|-------------|----|----|----|-------------|----|----|----|------------|----|----|----|-------------|----|----|----|
|                  | l.        | s. | d. | q. | l.          | s. | d. | q. | l.          | s. | d. | q. | l.          | s. | d. | q. | l.         | s. | d. | q. | l.          | s. | d. | q. |
| 1                |           |    | 6  | 2  |             |    | 6  | 3  |             |    | 7  | 0  |             |    | 7  | 0  |            |    | 7  | 1  |             |    | 7  | 2  |
| 2                |           | 1  | 0  | 3  |             | 1  | 1  | 1  |             | 1  | 1  | 3  |             | 1  | 2  | 1  |            | 1  | 2  | 3  |             | 1  | 3  | 1  |
| 3                |           | 1  | 7  | 1  |             | 1  | 8  | 0  |             | 1  | 8  | 3  |             | 1  | 9  | 2  |            | 1  | 10 | 1  |             | 1  | 11 | 0  |
| 4                |           | 2  | 1  | 2  |             | 2  | 2  | 2  |             | 2  | 3  | 2  |             | 2  | 4  | 3  |            | 2  | 5  | 2  |             | 2  | 6  | 2  |
| 5                |           | 2  | 8  | 0  |             | 2  | 9  | 1  |             | 2  | 10 | 2  |             | 2  | 11 | 3  |            | 3  | 1  | 0  |             | 3  | 2  | 1  |
| 6                |           | 3  | 2  | 2  |             | 3  | 4  | 0  |             | 3  | 5  | 2  |             | 3  | 7  | 0  |            | 3  | 8  | 1  |             | 3  | 9  | 3  |
| 7                |           | 3  | 8  | 3  |             | 3  | 10 | 2  |             | 4  | 0  | 1  |             | 4  | 2  | 0  |            | 4  | 3  | 3  |             | 4  | 5  | 2  |
| 8                |           | 4  | 3  | 1  |             | 4  | 5  | 1  |             | 4  | 7  | 1  |             | 4  | 9  | 1  |            | 4  | 11 | 1  |             | 5  | 1  | 0  |
| 9                |           | 4  | 9  | 2  |             | 5  | 0  | 0  |             | 5  | 2  | 0  |             | 5  | 4  | 1  |            | 5  | 6  | 2  |             | 5  | 8  | 3  |
| 10               |           | 5  | 4  | 0  |             | 5  | 6  | 2  |             | 5  | 9  | 0  |             | 5  | 11 | 2  |            | 6  | 2  | 0  |             | 6  | 4  | 2  |
| 20               |           | 10 | 8  | 0  |             | 11 | 1  | 0  |             | 11 | 6  | 0  |             | 11 | 11 | 0  |            | 12 | 4  | 0  |             | 12 | 8  | 3  |
| 30               |           | 16 | 0  | 0  |             | 16 | 7  | 2  |             | 17 | 3  | 0  |             | 17 | 10 | 2  |            | 18 | 5  | 3  |             | 19 | 1  | 1  |
| 40               | 1         | 1  | 4  | 0  | 1           | 2  | 2  | 0  | 1           | 3  | 0  | 0  | 1           | 3  | 9  | 3  | 1          | 4  | 7  | 3  | 1           | 5  | 5  | 2  |
| 50               | 1         | 6  | 8  | 0  | 1           | 7  | 8  | 2  | 1           | 8  | 8  | 3  | 1           | 9  | 9  | 1  | 1          | 10 | 9  | 2  | 1           | 11 | 10 | 0  |
| 60               | 1         | 12 | 0  | 0  | 1           | 13 | 3  | 0  | 1           | 14 | 5  | 3  | 1           | 15 | 8  | 3  | 1          | 16 | 11 | 3  | 1           | 18 | 2  | 1  |
| 70               | 1         | 17 | 4  | 0  | 1           | 18 | 9  | 2  | 2           | 0  | 2  | 3  | 2           | 1  | 8  | 1  | 2          | 3  | 1  | 2  | 2           | 4  | 6  | 3  |
| 80               | 2         | 2  | 8  | 0  | 2           | 4  | 4  | 0  | 2           | 5  | 11 | 3  | 2           | 7  | 7  | 2  | 2          | 9  | 3  | 2  | 2           | 10 | 11 | 1  |
| 90               | 2         | 8  | 0  | 0  | 2           | 9  | 10 | 2  | 2           | 11 | 8  | 3  | 2           | 13 | 7  | 0  | 2          | 15 | 5  | 1  | 2           | 17 | 3  | 2  |
| 100              | 2         | 13 | 4  | 0  | 2           | 15 | 5  | 0  | 2           | 17 | 5  | 3  | 2           | 19 | 6  | 2  | 2          | 1  | 7  | 1  | 3           | 3  | 8  | 0  |
| 200              | 5         | 6  | 8  | 0  | 5           | 10 | 9  | 3  | 5           | 14 | 11 | 2  | 5           | 19 | 1  | 0  | 6          | 3  | 2  | 2  | 6           | 7  | 4  | 0  |
| 300              | 8         | 0  | 0  | 0  | 8           | 6  | 2  | 3  | 8           | 12 | 5  | 1  | 8           | 18 | 7  | 2  | 9          | 4  | 9  | 3  | 9           | 10 | 11 | 3  |
| 400              | 10        | 13 | 4  | 0  | 11          | 1  | 7  | 2  | 11          | 9  | 11 | 0  | 11          | 18 | 2  | 0  | 12         | 6  | 5  | 0  | 12          | 14 | 7  | 3  |
| 500              | 13        | 6  | 8  | 0  | 13          | 17 | 0  | 2  | 14          | 7  | 4  | 3  | 14          | 17 | 8  | 2  | 15         | 8  | 0  | 1  | 15          | 18 | 3  | 3  |
| 600              | 16        | 0  | 0  | 0  | 16          | 12 | 5  | 2  | 17          | 4  | 10 | 2  | 17          | 17 | 3  | 0  | 18         | 9  | 7  | 2  | 19          | 1  | 11 | 3  |
| 700              | 18        | 13 | 4  | 0  | 19          | 7  | 10 | 1  | 20          | 2  | 4  | 1  | 20          | 16 | 9  | 2  | 21         | 11 | 2  | 3  | 22          | 5  | 7  | 2  |
| 800              | 21        | 6  | 8  | 0  | 22          | 3  | 3  | 1  | 22          | 19 | 9  | 3  | 23          | 16 | 4  | 1  | 24         | 12 | 10 | 0  | 25          | 9  | 3  | 2  |
| 900              | 24        | 0  | 0  | 0  | 24          | 18 | 8  | 0  | 25          | 17 | 3  | 2  | 26          | 15 | 10 | 3  | 27         | 14 | 5  | 1  | 28          | 12 | 11 | 2  |
| 1000             | 26        | 13 | 4  | 0  | 27          | 14 | 1  | 0  | 28          | 14 | 9  | 1  | 29          | 15 | 5  | 1  | 30         | 16 | 0  | 2  | 31          | 16 | 7  | 1  |
| ditto            | l. 26.6 r |    |    |    | l. 27.70378 |    |    |    | l. 28.73853 |    |    |    | l. 29.77139 |    |    |    | l. 30.8019 |    |    |    | l. 31.83024 |    |    |    |



# A TABLE of DISCOUNT at 4 per Cent. per Ann.

|            | 310 Days. |    |       |     | 320 Days. |    |       |     | 330 Days. |    |       |     | 340 Days. |    |       |     | 350 Days. |    |       |     | 360 Days. |    |       |     |   |
|------------|-----------|----|-------|-----|-----------|----|-------|-----|-----------|----|-------|-----|-----------|----|-------|-----|-----------|----|-------|-----|-----------|----|-------|-----|---|
| Principal. | l.        | s. | d.    | q.  | l.        | s. | d.    | q.  | l.        | s. | d.    | q.  | l.        | s. | d.    | q.  | l.        | s. | d.    | q.  | l.        | s. | d.    | q.  |   |
| 1          |           |    | 7     | 3   |           |    | 8     | c   |           |    | 8     | 2   |           |    | 8     | 3   |           |    | 9     | 0   |           |    | 9     | 1   |   |
| 2          |           | 1  | 3     | 3   |           | 1  | 4     | 2   |           | 1  | 4     | 3   |           | 1  | 5     | 1   |           | 1  | 5     | 3   |           | 1  | 6     | 1   |   |
| 3          |           | 1  | 11    | 3   |           | 2  | 0     | 1   |           | 2  | 1     | 1   |           | 2  | 1     | 2   |           | 2  | 2     | 2   |           | 2  | 3     | 1   |   |
| 4          |           | 2  | 7     | 2   |           | 2  | 8     | 2   |           | 2  | 9     | 2   |           | 2  | 10    | 2   |           | 2  | 11    | 2   |           | 3  | 0     | 2   |   |
| 5          |           | 3  | 3     | 2   |           | 3  | 4     | 3   |           | 3  | 6     | 0   |           | 3  | 7     | 0   |           | 3  | 8     | 1   |           | 3  | 9     | 2   |   |
| 6          |           | 3  | 11    | 2   |           | 4  | 0     | 3   |           | 4  | 2     | 1   |           | 4  | 3     | 3   |           | 4  | 5     | 1   |           | 4  | 6     | 3   |   |
| 7          |           | 4  | 7     | 1   |           | 4  | 9     | c   |           | 4  | 10    | 3   |           | 5  | 0     | 1   |           | 5  | 2     | 0   |           | 5  | 3     | 3   |   |
| 8          |           | 5  | 3     | 1   |           | 5  | 5     | 1   |           | 5  | 7     | 0   |           | 5  | 9     | c   |           | 5  | 11    | 0   |           | 6  | 0     | 3   |   |
| 9          |           | 5  | 11    | 0   |           | 6  | 1     | 1   |           | 6  | 3     | 2   |           | 6  | 5     | 2   |           | 6  | 7     | 3   |           | 6  | 10    | 0   |   |
| 10         |           | 6  | 6     | 3   |           | 6  | 9     | 1   |           | 6  | 11    | 3   |           | 7  | 2     | 1   |           | 7  | 4     | 3   |           | 7  | 7     | 0   |   |
| 20         |           | 13 | 1     | 3   |           | 13 | 6     | 3   |           | 13 | 11    | 2   |           | 14 | 4     | 2   |           | 14 | 9     | 1   |           | 15 | 2     | 1   |   |
| 30         |           | 19 | 8     | 2   |           | 1  | 0     | 4   | 0         |    | 1     | 0   | 11        | 1  |       | 1   | 1         | 6  | 3     |     | 1         | 2  | 9     | 1   |   |
| 40         |           | 1  | 6     | 3   | 2         |    | 1     | 7   | 1         | 1  |       | 1   | 7         | 11 | 1     |     | 1         | 8  | 8     | 3   |           | 1  | 10    | 4   | 2 |
| 50         |           | 1  | 12    | 10  | 2         |    | 1     | 13  | 10        | 3  |       | 1   | 14        | 11 | 0     |     | 1         | 15 | 11    | 0   |           | 1  | 16    | 11  | 2 |
| 60         |           | 1  | 19    | 5   | 1         |    | 2     | 0   | 7         | 3  |       | 2   | 1         | 10 | 3     |     | 2         | 3  | 1     | 1   |           | 2  | 4     | 6   | 2 |
| 70         |           | 2  | 6     | 0   | 0         |    | 2     | 7   | 5         | 1  |       | 2   | 8         | 10 | 2     |     | 2         | 10 | 3     | 2   |           | 2  | 11    | 8   | 3 |
| 80         |           | 2  | 12    | 7   | 0         |    | 2     | 14  | 2         | 2  |       | 2   | 15        | 10 | 1     |     | 2         | 17 | 5     | 3   |           | 2  | 19    | 1   | 3 |
| 90         |           | 2  | 19    | 1   | 3         |    | 3     | 0   | 11        | 3  |       | 3   | 2         | 10 | 0     |     | 3         | 4  | 8     | 0   |           | 3  | 6     | 6   | 0 |
| 100        |           | 3  | 5     | 8   | 3         |    | 3     | 7   | 9         | 1  |       | 3   | 9         | 9  | 3     |     | 3         | 11 | 10    | 1   |           | 3  | 13    | 10  | 2 |
| 200        |           | 6  | 11    | 5   | 2         |    | 6     | 15  | 6         | 1  |       | 6   | 19        | 7  | 1     |     | 7         | 3  | 8     | 1   |           | 7  | 7     | 9   | 0 |
| 300        |           | 9  | 17    | 2   | 0         |    | 10    | 3   | 3         | 2  |       | 10  | 9         | 5  | 0     |     | 10        | 15 | 6     | 1   |           | 11 | 1     | 7   | 2 |
| 400        |           | 13 | 2     | 11  | 0         |    | 13    | 11  | 0         | 2  |       | 13  | 19        | 2  | 2     |     | 14        | 7  | 4     | 2   |           | 14 | 15    | 6   | 1 |
| 500        |           | 16 | 8     | 7   | 2         |    | 16    | 18  | 9         | 3  |       | 17  | 9         | 0  | 1     |     | 17        | 19 | 2     | 2   |           | 18 | 9     | 4   | 3 |
| 600        |           | 19 | 14    | 4   | 1         |    | 20    | 6   | 6         | 3  |       | 20  | 18        | 10 | 0     |     | 21        | 11 | 0     | 3   |           | 22 | 3     | 3   | 1 |
| 700        |           | 23 | 0     | 1   | 0         |    | 23    | 14  | 4         | 0  |       | 24  | 8         | 7  | 2     |     | 25        | 2  | 11    | 0   |           | 25 | 17    | 1   | 3 |
| 800        |           | 26 | 5     | 9   | 3         |    | 27    | 2   | 1         | 0  |       | 27  | 18        | 5  | 1     |     | 28        | 14 | 9     | 0   |           | 29 | 11    | 0   | 1 |
| 900        |           | 29 | 11    | 6   | 1         |    | 30    | 9   | 10        | 1  |       | 31  | 8         | 2  | 3     |     | 32        | 6  | 7     | 0   |           | 33 | 4     | 11  | 0 |
| 1000       |           | 32 | 17    | 3   | 1         |    | 33    | 17  | 7         | 1  |       | 34  | 18        | 0  | 2     |     | 35        | 18 | 5     | 1   |           | 36 | 18    | 9   | 2 |
| ditto      |           | l. | 32.86 | 275 |           | l. | 33.88 | 038 |           | l. | 34.90 | 216 |           | l. | 35.92 | 181 |           | l. | 36.93 | 931 |           | l. | 37.95 | 466 |   |



Part II. For Days under 10 in a Medium,  
to add at above 90 Days.

| Principal. | 5 Days. |    |    | 6 Days. |    |    | 7 Days. |    |    | 8 Days. |    |    | 9 Days. |    |    |
|------------|---------|----|----|---------|----|----|---------|----|----|---------|----|----|---------|----|----|
|            | s.      | d. | q. | s.      | d. | q. | s.      | d. | q. | s.      | d. | q. | s.      | d. | q. |
| 1          |         |    |    |         |    |    | 1       |    |    | 1       |    |    | 1       |    | 1  |
| 2          |         |    | 1  |         |    | 1  |         |    | 1  |         |    | 1  |         |    | 2  |
| 3          |         |    | 1  |         |    | 2  |         |    | 2  |         |    | 2  |         |    | 3  |
| 4          |         |    | 2  |         |    | 2  |         |    | 3  |         |    | 3  |         | 1  | 0  |
| 5          |         |    | 2  |         |    | 3  |         |    | 3  |         |    | 1  | 0       | 1  | 1  |
| 6          |         |    | 3  |         |    | 1  | 0       |    | 1  | 0       |    | 1  | 1       | 1  | 1  |
| 7          |         |    | 3  |         |    | 1  | 0       |    | 1  | 1       |    | 1  | 2       | 1  | 2  |
| 8          |         | 1  | 0  |         |    | 1  | 1       |    | 1  | 2       |    | 1  | 2       | 1  | 3  |
| 9          |         | 1  | 1  |         |    | 1  | 1       |    | 1  | 2       |    | 1  | 3       | 2  | 0  |
| 10         |         | 1  | 1  |         |    | 1  | 2       |    | 1  | 3       |    | 2  | 0       | 2  | 1  |
| 20         |         | 2  | 2  |         |    | 3  | 0       |    | 3  | 2       |    | 4  | 0       | 4  | 2  |
| 30         |         | 3  | 3  |         |    | 4  | 2       |    | 5  | 1       |    | 6  | 0       | 6  | 3  |
| 40         |         | 5  | 1  |         |    | 6  | 0       |    | 7  | 0       |    | 8  | 0       | 9  | 0  |
| 50         |         | 6  | 1  |         |    | 7  | 2       |    | 8  | 3       |    | 10 | 0       | 11 | 2  |
| 60         |         | 7  | 2  |         |    | 9  | 0       |    | 10 | 2       | 1  | 0  | 0       | 1  | 1  |
| 70         |         | 8  | 3  |         |    | 10 | 2       | 1  | 0  | 1       | 1  | 2  | 0       | 1  | 3  |
| 80         |         | 10 | 0  | 1       |    | 0  | 0       | 1  | 2  | 0       | 1  | 4  | 0       | 1  | 6  |
| 90         |         | 11 | 2  | 1       |    | 1  | 2       | 1  | 3  | 3       | 1  | 6  | 0       | 1  | 8  |
| 100        | 1       | 0  | 0  | 1       |    | 3  | 0       | 1  | 5  | 2       | 1  | 8  | 0       | 1  | 10 |
| 200        | 2       | 1  | 0  | 2       |    | 6  | 0       | 2  | 11 | 1       | 3  | 4  | 1       | 3  | 9  |
| 300        | 3       | 1  | 3  | 3       |    | 9  | 1       | 4  | 4  | 3       | 5  | 0  | 1       | 5  | 7  |
| 400        | 4       | 2  | 1  | 5       |    | 0  | 1       | 5  | 10 | 2       | 6  | 8  | 1       | 7  | 6  |
| 500        | 5       | 2  | 3  | 6       |    | 3  | 2       | 7  | 4  | 0       | 8  | 4  | 2       | 9  | 5  |
| 600        | 6       | 3  | 2  | 7       |    | 6  | 2       | 8  | 9  | 2       | 10 | 0  | 2       | 11 | 3  |
| 700        | 7       | 4  | 0  | 8       |    | 9  | 2       | 10 | 3  | 0       | 11 | 8  | 3       | 13 | 2  |
| 800        | 8       | 4  | 2  | 10      |    | 0  | 2       | 11 | 8  | 3       | 13 | 4  | 3       | 15 | 0  |
| 900        | 9       | 5  | 0  | 11      |    | 3  | 3       | 13 | 2  | 1       | 15 | 0  | 3       | 16 | 1  |
| 1000       | 10      | 5  | 2  | 12      |    | 6  | 3       | 14 | 7  | 3       | 16 | 9  | 0       | 18 | 10 |
| ditto      | .52335  |    |    | .62802  |    |    | .73269  |    |    | .83737  |    |    | .94204  |    |    |



# A TABLE of DISCOUNT at 5 per Cent. per Ann.

|            | 1 Day. |    |    | 2 Days. |    |    | 3 Days. |    |    | 4 Days. |    |    | 5 Days. |    |    | 6 Days. |    |    | 7 Days. |    |    |   |    |    |    |    |   |    |    |   |   |  |    |    |   |   |
|------------|--------|----|----|---------|----|----|---------|----|----|---------|----|----|---------|----|----|---------|----|----|---------|----|----|---|----|----|----|----|---|----|----|---|---|--|----|----|---|---|
| Principal. | s.     | d. | q. | s.      | d. | q. | s.      | d. | q. | s.      | d. | q. | s.      | d. | q. | s.      | d. | q. | s.      | d. | q. |   |    |    |    |    |   |    |    |   |   |  |    |    |   |   |
| 1          |        |    |    |         |    |    |         |    |    |         |    |    |         |    |    |         |    |    | 1       |    | 1  |   |    |    |    |    |   |    |    |   |   |  |    |    |   |   |
| 2          |        |    |    |         |    |    |         |    | 1  |         |    | 1  |         | 1  |    |         |    | 1  |         | 2  |    |   |    |    |    |    |   |    |    |   |   |  |    |    |   |   |
| 3          |        |    |    |         |    | 1  |         |    | 1  |         |    | 1  |         | 2  |    |         |    | 2  |         | 3  |    |   |    |    |    |    |   |    |    |   |   |  |    |    |   |   |
| 4          |        |    |    |         |    | 1  |         |    | 1  |         |    | 2  |         | 2  |    |         |    | 3  |         | 3  |    |   |    |    |    |    |   |    |    |   |   |  |    |    |   |   |
| 5          |        |    |    |         |    | 1  |         |    | 2  |         |    | 2  |         | 3  |    |         | 1  | 0  |         | 1  | 0  |   |    |    |    |    |   |    |    |   |   |  |    |    |   |   |
| 6          |        |    | 1  |         |    | 1  |         |    | 2  |         |    | 3  |         | 1  | 0  |         | 1  | 1  |         | 1  | 1  |   |    |    |    |    |   |    |    |   |   |  |    |    |   |   |
| 7          |        |    | 1  |         |    | 2  |         |    | 3  |         |    | 3  |         | 1  | 1  |         | 1  | 1  |         | 1  | 2  |   |    |    |    |    |   |    |    |   |   |  |    |    |   |   |
| 8          |        |    | 1  |         |    | 2  |         |    | 3  |         |    | 1  | 0       |    | 1  | 1       |    | 1  | 2       |    | 1  | 3 |    |    |    |    |   |    |    |   |   |  |    |    |   |   |
| 9          |        |    | 1  |         |    | 2  |         |    | 3  |         |    | 1  | 1       |    | 1  | 2       |    | 1  | 3       |    | 2  | 0 |    |    |    |    |   |    |    |   |   |  |    |    |   |   |
| 10         |        |    | 1  |         |    | 3  |         |    | 1  | 0       |    | 1  | 1       |    | 1  | 3       |    | 2  | 0       |    | 2  | 1 |    |    |    |    |   |    |    |   |   |  |    |    |   |   |
| 20         |        |    | 3  |         |    | 1  | 1       |    | 2  | 0       |    | 2  | 3       |    | 3  | 1       |    | 4  | 0       |    | 4  | 2 |    |    |    |    |   |    |    |   |   |  |    |    |   |   |
| 30         |        |    | 1  | 0       |    | 2  | 0       |    | 3  | 0       |    | 4  | 0       |    | 5  | 0       |    | 5  | 2       |    | 6  | 3 |    |    |    |    |   |    |    |   |   |  |    |    |   |   |
| 40         |        |    | 1  | 1       |    | 2  | 2       |    | 4  | 0       |    | 5  | 1       |    | 6  | 2       |    | 7  | 3       |    | 9  | 1 |    |    |    |    |   |    |    |   |   |  |    |    |   |   |
| 50         |        |    | 1  | 3       |    | 3  | 1       |    | 5  | 0       |    | 6  | 2       |    | 8  | 1       |    | 9  | 3       |    | 11 | 2 |    |    |    |    |   |    |    |   |   |  |    |    |   |   |
| 60         |        |    | 2  | 0       |    | 4  | 0       |    | 6  | 0       |    | 7  | 3       |    | 10 | 0       |    | 11 | 3       |    | 1  | 3 |    |    |    |    |   |    |    |   |   |  |    |    |   |   |
| 70         |        |    | 2  | 1       |    | 4  | 3       |    | 6  | 3       |    | 9  | 1       |    | 11 | 2       |    | 1  | 1       | 3  |    | 4 | 1  |    |    |    |   |    |    |   |   |  |    |    |   |   |
| 80         |        |    | 2  | 3       |    | 5  | 1       |    | 7  | 3       |    | 10 | 2       |    | 1  | 1       | 0  |    | 1       | 3  | 3  |   | 1  | 6  | 1  |    |   |    |    |   |   |  |    |    |   |   |
| 90         |        |    | 3  | 0       |    | 6  | 0       |    | 8  | 3       |    | 11 | 3       |    | 1  | 2       | 3  |    | 1       | 5  | 3  |   | 1  | 8  | 3  |    |   |    |    |   |   |  |    |    |   |   |
| 100        |        |    | 3  | 1       |    | 6  | 2       |    | 9  | 3       |    | 1  | 1       | 0  |    | 1       | 4  | 2  |         | 1  | 7  | 3 |    | 1  | 11 | 0  |   |    |    |   |   |  |    |    |   |   |
| 200        |        |    | 6  | 2       |    | 1  | 1       | 0  |    | 1       | 7  | 3  |         | 2  | 2  | 1       |    | 2  | 8       | 3  |    | 3 | 3  | 2  | 3  | 10 | 0 |    |    |   |   |  |    |    |   |   |
| 300        |        |    | 1  | 0       |    | 1  | 7       | 3  |    | 2       | 5  | 3  |         | 3  | 3  | 2       |    | 4  | 1       | 1  |    | 4 | 1  | 0  | 5  | 9  | 0 |    |    |   |   |  |    |    |   |   |
| 400        |        |    | 1  | 1       | 1  |    | 2       | 2  | 1  |         | 3  | 3  | 2       |    | 4  | 4       | 3  |    | 5       | 5  | 3  |   | 6  | 6  | 3  | 7  | 8 | 0  |    |   |   |  |    |    |   |   |
| 500        |        |    | 1  | 4       | 2  |    | 2       | 8  | 3  |         | 4  | 1  | 1       |    | 5  | 5       | 3  |    | 6       | 10 | 0  |   | 8  | 2  | 2  | 9  | 7 | 0  |    |   |   |  |    |    |   |   |
| 600        |        |    | 1  | 7       | 3  |    | 3       | 3  | 2  |         | 4  | 11 | 1       |    | 6  | 6       | 3  |    | 8       | 2  | 2  |   | 9  | 10 | 1  | 11 | 6 | 0  |    |   |   |  |    |    |   |   |
| 700        |        |    | 1  | 11      | 0  |    | 3       | 10 | 0  |         | 5  | 9  | 0       |    | 7  | 8       | 0  |    | 9       | 7  | 0  |   | 11 | 6  | 0  | 13 | 5 | 0  |    |   |   |  |    |    |   |   |
| 800        |        |    | 2  | 2       | 1  |    | 4       | 4  | 3  |         | 6  | 6  | 3       |    | 8  | 9       | 1  |    | 10      | 11 | 2  |   | 13 | 1  | 2  | 15 | 4 | 0  |    |   |   |  |    |    |   |   |
| 900        |        |    | 2  | 5       | 3  |    | 4       | 11 | 1  |         | 7  | 4  | 3       |    | 9  | 10      | 1  |    | 12      | 3  | 3  |   | 14 | 9  | 1  | 17 | 3 | 0  |    |   |   |  |    |    |   |   |
| 1000       |        |    | 2  | 8       | 3  |    | 5       | 5  | 3  |         | 8  | 2  | 2       |    | 10 | 11      | 2  |    | 13      | 8  | 1  |   | 16 | 5  | 1  | 19 | 2 | 0  |    |   |   |  |    |    |   |   |
| ditto      |        |    | l. | 13      | 6  | 9  |         | l. | 27 | 3       | 9  |    | l.      | 41 | 0  | 8       |    | l. | 54      | 7  | 6  |   | l. | 68 | 4  | 8  |   | l. | 82 | 1 | 2 |  | l. | 95 | 7 | 9 |

# A TABLE of DISCOUNT at 5 per Cent. per Ann.

| Principal.<br>l. | 8 Days.   |    |    |    | 9 Days.   |    |    |    | 10 Days.  |    |    |    | 20 Days.  |    |    |    | 30 Days.  |    |    |    | 40 Days.  |    |    |    |    |   |
|------------------|-----------|----|----|----|-----------|----|----|----|-----------|----|----|----|-----------|----|----|----|-----------|----|----|----|-----------|----|----|----|----|---|
|                  | l.        | s. | d. | q. | l.        | s. | d. | q. | l.        | s. | d. | q. | l.        | s. | d. | q. | l.        | s. | d. | q. | l.        | s. | d. | q. |    |   |
| 1                |           |    |    | 1  |           |    |    | 1  |           |    |    | 1  |           |    |    | 3  |           |    |    | 1  | 0         |    |    | 1  | 1  |   |
| 2                |           |    |    | 2  |           |    |    | 2  |           |    |    | 2  |           |    |    | 1  | 1         |    |    | 2  | 0         |    |    | 2  | 2  |   |
| 3                |           |    |    | 3  |           |    |    | 3  |           |    |    | 1  | 0         |    |    | 2  | 0         |    |    | 3  | 0         |    |    | 4  | 0  |   |
| 4                |           |    |    | 1  | 0         |    |    | 1  | 1         |    |    | 1  | 1         |    |    | 2  | 3         |    |    | 4  | 0         |    |    | 5  | 1  |   |
| 5                |           |    |    | 1  | 1         |    |    | 1  | 2         |    |    | 1  | 3         |    |    | 3  | 1         |    |    | 5  | 0         |    |    | 6  | 2  |   |
| 6                |           |    |    | 1  | 2         |    |    | 1  | 3         |    |    | 2  | 0         |    |    | 4  | 0         |    |    | 6  | 0         |    |    | 7  | 3  |   |
| 7                |           |    |    | 1  | 3         |    |    | 2  | 0         |    |    | 2  | 1         |    |    | 4  | 3         |    |    | 6  | 3         |    |    | 9  | 1  |   |
| 8                |           |    |    | 2  | 0         |    |    | 2  | 1         |    |    | 2  | 3         |    |    | 5  | 1         |    |    | 7  | 3         |    |    | 10 | 2  |   |
| 9                |           |    |    | 2  | 1         |    |    | 2  | 3         |    |    | 3  | 0         |    |    | 6  | 0         |    |    | 8  | 3         |    |    | 11 | 3  |   |
| 10               |           |    |    | 2  | 3         |    |    | 3  | 0         |    |    | 3  | 1         |    |    | 6  | 2         |    |    | 9  | 3         |    |    | 1  | 1  | 0 |
| 20               |           |    |    | 5  | 1         |    |    | 5  | 3         |    |    | 6  | 2         |    |    | 1  | 1         | 0  |    | 1  | 7         | 2  |    | 2  | 2  | 0 |
| 30               |           |    |    | 7  | 3         |    |    | 8  | 3         |    |    | 10 | 0         |    |    | 1  | 7         | 2  |    | 2  | 5         | 2  |    | 3  | 3  | 1 |
| 40               |           |    |    | 10 | 2         |    |    | 11 | 3         |    |    | 1  | 1         | 0  |    | 2  | 2         | 1  |    | 3  | 3         | 1  |    | 4  | 4  | 1 |
| 50               |           |    |    | 1  | 1         | 0  |    | 1  | 2         | 3  |    | 1  | 4         | 2  |    | 2  | 8         | 3  |    | 4  | 1         | 0  |    | 5  | 5  | 1 |
| 60               |           |    |    | 1  | 3         | 3  |    | 1  | 5         | 3  |    | 1  | 7         | 3  |    | 3  | 3         | 1  |    | 4  | 1         | 1  | 0  | 6  | 6  | 2 |
| 70               |           |    |    | 1  | 6         | 1  |    | 1  | 8         | 3  |    | 1  | 11        | 0  |    | 3  | 10        | 0  |    | 5  | 8         | 3  |    | 7  | 7  | 2 |
| 80               |           |    |    | 1  | 9         | 0  |    | 1  | 11        | 3  |    | 2  | 2         | 1  |    | 4  | 4         | 2  |    | 6  | 6         | 2  |    | 8  | 8  | 2 |
| 90               |           |    |    | 1  | 11        | 3  |    | 2  | 2         | 2  |    | 2  | 5         | 2  |    | 4  | 10        | 0  |    | 7  | 4         | 2  |    | 9  | 9  | 3 |
| 100              |           |    |    | 2  | 2         | 1  |    | 2  | 5         | 2  |    | 2  | 8         | 3  |    | 5  | 5         | 2  |    | 8  | 2         | 1  |    | 10 | 10 | 3 |
| 200              |           |    |    | 4  | 4         | 2  |    | 4  | 11        | 0  |    | 5  | 5         | 3  |    | 10 | 11        | 1  |    | 16 | 4         | 2  | 1  | 1  | 9  | 2 |
| 300              |           |    |    | 6  | 6         | 3  |    | 7  | 4         | 3  |    | 8  | 2         | 2  |    | 16 | 4         | 3  | 1  | 4  | 6         | 3  | 1  | 12 | 8  | 1 |
| 400              |           |    |    | 8  | 9         | 0  |    | 9  | 10        | 1  |    | 10 | 11        | 2  | 1  | 1  | 10        | 1  | 1  | 12 | 9         | 0  | 2  | 3  | 7  | 0 |
| 500              |           |    |    | 10 | 11        | 2  |    | 12 | 3         | 3  |    | 13 | 8         | 0  | 1  | 7  | 4         | 0  | 2  | 0  | 11        | 0  | 2  | 14 | 6  | 0 |
| 600              |           |    |    | 13 | 1         | 2  |    | 14 | 9         | 1  |    | 16 | 5         | 0  | 1  | 12 | 9         | 2  | 2  | 9  | 1         | 13 |    | 5  | 5  | 0 |
| 700              |           |    |    | 15 | 4         | 0  |    | 17 | 2         | 3  |    | 19 | 1         | 3  | 1  | 18 | 3         | 0  | 2  | 17 | 3         | 2  | 3  | 16 | 3  | 3 |
| 800              |           |    |    | 17 | 6         | 0  |    | 19 | 8         | 2  | 1  | 1  | 10        | 3  | 2  | 3  | 8         | 2  | 3  | 5  | 5         | 3  | 4  | 7  | 2  | 1 |
| 900              |           |    |    | 19 | 8         | 2  | 1  | 2  | 2         | 0  | 1  | 4  | 7         | 2  | 2  | 9  | 2         | 1  | 3  | 13 | 8         | 0  | 4  | 18 | 1  | 0 |
| 1000             |           |    |    | 1  | 10        | 3  | 1  | 4  | 7         | 2  | 1  | 7  | 4         | 1  | 2  | 14 | 7         | 3  | 4  | 1  | 10        | 1  | 5  | 9  | 0  | 0 |
| ditto            | l. 1.0947 |    |    |    | l. 1.2313 |    |    |    | l. 1.3679 |    |    |    | l. 2.7322 |    |    |    | l. 4.0925 |    |    |    | l. 5.4496 |    |    |    |    |   |

# A TABLE of DISCOUNT at 5 per Cent. per Ann.

| Principal.<br>l. | 50 Days.  |    |    |    | 60 Days.  |    |    |    | 70 Days.  |    |    |    | 80 Days.   |    |    |    | 90 Days.   |    |    |    | 100 Days. |    |    |    |   |
|------------------|-----------|----|----|----|-----------|----|----|----|-----------|----|----|----|------------|----|----|----|------------|----|----|----|-----------|----|----|----|---|
|                  | l.        | s. | d. | q. | l.        | s. | d. | q. | l.        | s. | d. | q. | l.         | s. | d. | q. | l.         | s. | d. | q. | l.        | s. | d. | q. |   |
| 1                |           |    | 1  | 3  |           |    | 2  | 0  |           |    | 2  | 1  |            |    | 2  | 2  |            |    | 3  | 0  |           |    | 3  | 1  |   |
| 2                |           |    | 3  | 1  |           |    | 4  | 0  |           |    | 4  | 3  |            |    | 5  | 1  |            |    | 5  | 3  |           |    | 6  | 2  |   |
| 3                |           |    | 5  | 0  |           |    | 6  | 0  |           |    | 6  | 3  |            |    | 7  | 3  |            |    | 9  | 3  |           |    | 9  | 3  |   |
| 4                |           |    | 6  | 2  |           |    | 7  | 3  |           |    | 9  | 0  |            |    | 10 | 2  |            |    | 11 | 3  |           | 1  | 1  | 0  |   |
| 5                |           |    | 8  | 1  |           |    | 9  | 3  |           |    | 11 | 2  |            | 1  | 1  | 0  |            | 1  | 2  | 2  |           | 1  | 4  | 1  |   |
| 6                |           |    | 9  | 3  |           |    | 11 | 3  |           | 1  | 1  | 3  |            | 1  | 3  | 3  |            | 1  | 5  | 2  |           | 1  | 7  | 2  |   |
| 7                |           | 1  | 1  | 2  |           | 1  | 1  | 3  |           | 1  | 4  | 0  |            | 1  | 6  | 1  |            | 1  | 8  | 2  |           | 1  | 10 | 3  |   |
| 8                |           | 1  | 1  | 0  |           | 1  | 3  | 3  |           | 1  | 6  | 1  |            | 1  | 8  | 3  |            | 1  | 11 | 2  |           | 2  | 2  | 0  |   |
| 9                |           | 1  | 2  | 3  |           | 1  | 5  | 2  |           | 1  | 8  | 2  |            | 1  | 11 | 2  |            | 2  | 2  | 1  |           | 2  | 5  | 1  |   |
| 10               |           | 1  | 4  | 2  |           | 1  | 7  | 2  |           | 1  | 10 | 3  |            | 2  | 2  | 0  |            | 2  | 5  | 1  |           | 2  | 8  | 2  |   |
| 20               |           | 2  | 8  | 3  |           | 3  | 3  | 1  |           | 3  | 9  | 2  |            | 4  | 4  | 0  |            | 4  | 10 | 2  |           | 5  | 5  | 0  |   |
| 30               |           | 4  | 1  | 0  |           | 4  | 10 | 3  |           | 5  | 8  | 1  |            | 6  | 6  | 0  |            | 7  | 3  | 3  |           | 8  | 1  | 2  |   |
| 40               |           | 5  | 5  | 1  |           | 6  | 6  | 1  |           | 7  | 7  | 1  |            | 8  | 8  | 0  |            | 9  | 9  | 0  |           | 10 | 10 | 1  |   |
| 50               |           | 6  | 9  | 3  |           | 8  | 1  | 3  |           | 9  | 6  | 0  |            | 10 | 10 | 0  |            | 12 | 2  | 1  |           | 13 | 6  | 2  |   |
| 60               |           | 8  | 2  | 0  |           | 9  | 9  | 2  |           | 11 | 5  | 0  |            | 13 | 0  | 0  |            | 14 | 7  | 1  |           | 16 | 3  | 1  |   |
| 70               |           | 9  | 6  | 1  |           | 11 | 5  | 0  |           | 13 | 3  | 2  |            | 15 | 2  | 0  |            | 17 | 0  | 2  |           | 18 | 11 | 3  |   |
| 80               |           | 10 | 10 | 3  |           | 13 | 0  | 2  |           | 15 | 2  | 1  |            | 17 | 4  | 1  |            | 19 | 5  | 3  |           | 1  | 1  | 8  | 1 |
| 90               |           | 12 | 3  | 0  |           | 14 | 8  | 0  |           | 17 | 1  | 1  |            | 19 | 6  | 1  |            | 1  | 1  | 11 | 0         | 1  | 4  | 5  | 0 |
| 100              |           | 13 | 7  | 1  |           | 16 | 3  | 3  |           | 19 | 0  | 0  |            | 1  | 1  | 8  | 1          | 1  | 4  | 4  | 1         | 1  | 7  | 1  | 1 |
| 200              | 1         | 7  | 2  | 2  | 1         | 12 | 7  | 1  | 1         | 18 | 0  | 0  | 2          | 3  | 4  | 2  | 2          | 8  | 8  | 2  | 2         | 14 | 2  | 3  |   |
| 300              | 2         | 0  | 9  | 3  | 2         | 8  | 11 | 0  | 2         | 16 | 11 | 3  | 3          | 5  | 0  | 2  | 3          | 13 | 0  | 3  | 4         | 1  | 4  | 0  |   |
| 400              | 2         | 14 | 5  | 13 | 5         | 2  | 2  | 3  | 15        | 11 | 3  | 4  | 6          | 8  | 3  | 4  | 17         | 5  | 1  | 5  | 8         | 5  | 2  |    |   |
| 500              | 3         | 8  | 0  | 14 | 1         | 6  | 14 | 14 | 11        | 3  | 5  | 8  | 4          | 3  | 6  | 1  | 9          | 2  | 6  | 15 | 6         | 2  |    |    |   |
| 600              | 4         | 1  | 7  | 24 | 17        | 10 | 0  | 5  | 13        | 11 | 3  | 6  | 10         | 1  | 0  | 7  | 6          | 1  | 3  | 8  | 2         | 8  | 0  |    |   |
| 700              | 4         | 15 | 2  | 35 | 14        | 1  | 2  | 6  | 12        | 11 | 3  | 7  | 11         | 9  | 1  | 8  | 10         | 6  | 0  | 9  | 9         | 9  | 1  |    |   |
| 800              | 5         | 8  | 10 | 16 | 10        | 5  | 17 | 11 | 11        | 2  | 8  | 13 | 5          | 1  | 9  | 14 | 10         | 1  | 10 | 16 | 10        | 3  |    |    |   |
| 900              | 6         | 2  | 5  | 27 | 6         | 8  | 3  | 8  | 10        | 11 | 2  | 9  | 15         | 1  | 2  | 10 | 19         | 2  | 2  | 12 | 4         | 0  | 0  |    |   |
| 1000             | 6         | 16 | 0  | 38 | 3         | 0  | 2  | 9  | 9         | 11 | 2  | 10 | 16         | 9  | 3  | 12 | 3          | 7  | 0  | 13 | 10        | 1  | 1  |    |   |
| ditto            | l. 6.8027 |    |    |    | l. 8.1521 |    |    |    | l. 9.4976 |    |    |    | l. 10.8401 |    |    |    | l. 12.1786 |    |    |    | l. 13.5 r |    |    |    |   |



# A TABLE of DISCOUNT at 5 per Cent. per Ann.

|            | 110 Days. |    |      |     | 120 Days. |    |      |     | 130 Days. |    |      |     | 140 Days. |    |      |     | 150 Days. |    |      |     | 160 Days. |    |      |     |   |    |    |    |   |
|------------|-----------|----|------|-----|-----------|----|------|-----|-----------|----|------|-----|-----------|----|------|-----|-----------|----|------|-----|-----------|----|------|-----|---|----|----|----|---|
| Principal. | l.        | s. | d.   | q.  | l.        | s. | d.   | q.  | l.        | s. | d.   | q.  | l.        | s. | d.   | q.  | l.        | s. | d.   | q.  | l.        | s. | d.   | q.  |   |    |    |    |   |
| 1          |           |    | 3    | 2   |           |    | 4    | 0   |           |    | 4    | 1   |           |    | 4    | 2   |           |    | 5    | 0   |           |    | 5    | 1   |   |    |    |    |   |
| 2          |           |    | 7    | 0   |           |    | 7    | 3   |           |    | 8    | 2   |           |    | 9    | 0   |           |    | 9    | 3   |           |    | 10   | 1   |   |    |    |    |   |
| 3          |           |    | 10   | 3   |           |    | 11   | 3   |           | 1  | 0    | 2   |           | 1  | 1    | 2   |           | 1  | 2    | 2   |           | 1  | 3    | 2   |   |    |    |    |   |
| 4          |           | 1  | 2    | 1   |           | 1  | 2    | 2   |           | 1  | 4    | 3   |           | 1  | 6    | 0   |           | 1  | 7    | 1   |           | 1  | 8    | 2   |   |    |    |    |   |
| 5          |           | 1  | 5    | 3   |           | 1  | 7    | 2   |           | 1  | 9    | 0   |           | 1  | 10   | 2   |           | 2  | 0    | 0   |           | 2  | 1    | 3   |   |    |    |    |   |
| 6          |           | 1  | 9    | 1   |           | 1  | 11   | 2   |           | 2  | 1    | 1   |           | 2  | 3    | 1   |           | 2  | 5    | 0   |           | 2  | 6    | 3   |   |    |    |    |   |
| 7          |           | 2  | 0    | 3   |           | 2  | 3    | 1   |           | 2  | 5    | 2   |           | 2  | 7    | 2   |           | 2  | 9    | 3   |           | 3  | 0    | 0   |   |    |    |    |   |
| 8          |           | 2  | 4    | 2   |           | 2  | 7    | 0   |           | 2  | 9    | 2   |           | 3  | 0    | 0   |           | 3  | 2    | 3   |           | 3  | 5    | 1   |   |    |    |    |   |
| 9          |           | 2  | 8    | 0   |           | 2  | 11   | 0   |           | 3  | 1    | 3   |           | 3  | 4    | 2   |           | 3  | 7    | 2   |           | 3  | 10   | 1   |   |    |    |    |   |
| 10         |           | 2  | 11   | 3   |           | 3  | 2    | 3   |           | 3  | 6    | 0   |           | 3  | 9    | 1   |           | 4  | 0    | 1   |           | 4  | 3    | 2   |   |    |    |    |   |
| 20         |           | 5  | 11   | 1   |           | 6  | 5    | 2   |           | 7  | 0    | 0   |           | 7  | 6    | 1   |           | 8  | 0    | 2   |           | 8  | 7    | 0   |   |    |    |    |   |
| 30         |           | 8  | 11   | 0   |           | 9  | 8    | 2   |           | 10 | 6    | 0   |           | 11 | 3    | 2   |           | 12 | 1    | 0   |           | 12 | 10   | 2   |   |    |    |    |   |
| 40         |           | 11 | 10   | 2   |           | 12 | 11   | 1   |           | 14 | 0    | 0   |           | 15 | 0    | 2   |           | 16 | 1    | 1   |           | 17 | 2    | 0   |   |    |    |    |   |
| 50         |           | 14 | 10   | 1   |           | 16 | 2    | 0   |           | 17 | 6    | 0   |           | 18 | 9    | 3   |           | 1  | 0    | 1   | 2         | 1  | 1    | 5   | 2 |    |    |    |   |
| 60         |           | 17 | 9    | 3   |           | 19 | 5    | 0   |           | 1  | 1    | 0   | 0         | 1  | 2    | 7   | 0         |    | 1    | 4   | 2         | 0  | 1    | 5   | 8 | 3  |    |    |   |
| 70         |           | 1  | 0    | 9   | 2         |    | 1    | 2   | 7         | 3  |      | 1   | 4         | 6  | 0    |     | 1         | 6  | 4    | 1   |           | 1  | 8    | 2   | 1 | 1  | 10 | 0  | 1 |
| 80         |           | 1  | 3    | 9   | 0         |    | 1    | 5   | 10        | 2  |      | 1   | 8         | 0  | 0    |     | 1         | 10 | 1    | 1   |           | 1  | 12   | 2   | 2 | 1  | 14 | 3  | 3 |
| 90         |           | 1  | 6    | 8   | 3         |    | 1    | 9   | 1         | 1  |      | 1   | 11        | 6  | 0    |     | 1         | 13 | 10   | 2   |           | 1  | 16   | 3   | 0 | 1  | 18 | 7  | 1 |
| 100        |           | 1  | 9    | 8   | 1         |    | 1    | 12  | 4         | 1  |      | 1   | 15        | 0  | 0    |     | 1         | 17 | 7    | 2   |           | 2  | 0    | 3   | 1 | 2  | 2  | 10 | 3 |
| 200        |           | 2  | 19   | 4   | 3         |    | 3    | 4   | 8         | 1  |      | 3   | 9         | 11 | 3    |     | 3         | 15 | 3    | 1   |           | 4  | 0    | 6   | 2 | 4  | 5  | 9  | 2 |
| 300        |           | 4  | 9    | 0   | 3         |    | 4    | 17  | 0         | 1  |      | 5   | 4         | 11 | 3    |     | 5         | 12 | 11   | 0   |           | 6  | 0    | 9   | 3 | 6  | 8  | 8  | 1 |
| 400        |           | 5  | 18   | 9   | 0         |    | 6    | 9   | 4         | 3  |      | 6   | 19        | 11 | 3    |     | 7         | 10 | 6    | 2   |           | 8  | 1    | 0   | 3 | 8  | 11 | 7  | 0 |
| 500        |           | 7  | 8    | 5   | 1         |    | 8    | 1   | 8         | 3  |      | 8   | 14        | 11 | 2    |     | 9         | 8  | 2    | 0   |           | 10 | 1    | 4   | 1 | 10 | 14 | 5  | 3 |
| 600        |           | 8  | 18   | 1   | 2         |    | 9    | 14  | 0         | 3  |      | 10  | 9         | 11 | 2    |     | 11        | 5  | 9    | 3   |           | 12 | 1    | 7   | 1 | 12 | 17 | 4  | 2 |
| 700        |           | 10 | 7    | 10  | 0         |    | 11   | 6   | 5         | 0  |      | 12  | 4         | 11 | 2    |     | 13        | 3  | 5    | 1   |           | 14 | 1    | 10  | 2 | 15 | 0  | 3  | 1 |
| 800        |           | 11 | 17   | 6   | 1         |    | 12   | 18  | 9         | 1  |      | 13  | 19        | 11 | 2    |     | 15        | 1  | 1    | 0   |           | 16 | 2    | 1   | 3 | 17 | 3  | 2  | 0 |
| 900        |           | 13 | 7    | 2   | 2         |    | 14   | 11  | 1         | 1  |      | 15  | 14        | 11 | 1    |     | 16        | 18 | 8    | 2   |           | 18 | 2    | 5   | 0 | 19 | 6  | 0  | 3 |
| 1000       |           | 14 | 16   | 10  | 3         |    | 16   | 3   | 5         | 2  |      | 17  | 9         | 11 | 1    |     | 18        | 16 | 4    | 1   |           | 20 | 2    | 8   | 1 | 21 | 8  | 11 | 2 |
| ditto      |           | l. | 14.8 | 448 |           | l. | 16.1 | 725 |           | l. | 17.4 | 966 |           | l. | 18.8 | 172 |           | l. | 20.1 | 342 |           | l. | 21.4 | 477 |   |    |    |    |   |

# A TABLE of DISCOUNT at 5 per Cent. per Ann.

| Principal.<br>l. | 170 Days.  |    |    |    | 180 Days.   |    |    |    | 190 Days.   |    |    |    | 200 Days.   |    |    |    | 210 Days.  |    |    |    | 220 Days.  |    |    |    |   |
|------------------|------------|----|----|----|-------------|----|----|----|-------------|----|----|----|-------------|----|----|----|------------|----|----|----|------------|----|----|----|---|
|                  | l.         | s. | d. | q. | l.          | s. | d. | q. | l.          | s. | d. | q. | l.          | s. | d. | q. | l.         | s. | d. | q. | l.         | s. | d. | q. |   |
| 1                |            |    | 5  | 2  |             |    | 5  | 3  |             |    | 6  | 0  |             |    | 6  | 2  |            |    | 6  | 3  |            |    | 7  | 0  |   |
| 2                |            |    | 11 | 0  |             |    | 11 | 2  |             | 1  | 0  | 0  |             | 1  | 0  | 3  |            | 1  | 1  | 2  |            | 1  | 2  | 0  |   |
| 3                |            | 1  | 4  | 2  |             | 1  | 5  | 1  |             | 1  | 6  | 1  |             | 1  | 7  | 1  |            | 1  | 8  | 1  |            | 1  | 9  | 0  |   |
| 4                |            | 1  | 10 | 0  |             | 1  | 11 | 0  |             | 2  | 0  | 1  |             | 2  | 1  | 2  |            | 1  | 2  | 3  |            | 2  | 4  | 1  |   |
| 5                |            | 2  | 3  | 1  |             | 2  | 5  | 0  |             | 2  | 6  | 2  |             | 2  | 8  | 1  |            | 1  | 9  | 2  |            | 2  | 11 | 0  |   |
| 6                |            | 2  | 8  | 3  |             | 2  | 10 | 3  |             | 3  | 0  | 2  |             | 3  | 2  | 2  |            | 3  | 4  | 2  |            | 3  | 6  | 1  |   |
| 7                |            | 3  | 2  | 1  |             | 3  | 4  | 2  |             | 3  | 6  | 3  |             | 3  | 8  | 3  |            | 3  | 11 | 0  |            | 4  | 1  | 1  |   |
| 8                |            | 3  | 7  | 3  |             | 3  | 10 | 1  |             | 4  | 0  | 3  |             | 4  | 3  | 1  |            | 4  | 5  | 3  |            | 4  | 8  | 1  |   |
| 9                |            | 4  | 1  | 0  |             | 4  | 4  | 0  |             | 4  | 6  | 3  |             | 4  | 9  | 3  |            | 5  | 0  | 1  |            | 5  | 3  | 1  |   |
| 10               |            | 4  | 6  | 3  |             | 4  | 9  | 3  |             | 5  | 1  | 0  |             | 5  | 4  | 0  |            | 5  | 7  | 0  |            | 5  | 10 | 1  |   |
| 20               |            | 9  | 1  | 1  |             | 9  | 7  | 2  |             | 10 | 1  | 3  |             | 10 | 8  | 0  |            | 11 | 2  | 1  |            | 11 | 8  | 2  |   |
| 30               |            | 13 | 7  | 3  |             | 14 | 5  | 1  |             | 15 | 2  | 3  |             | 16 | 0  | 0  |            | 16 | 9  | 1  |            | 17 | 6  | 2  |   |
| 40               |            | 18 | 2  | 2  |             | 19 | 3  | 0  |             | 1  | 0  | 3  | 2           | 1  | 1  | 4  | 0          | 1  | 2  | 4  | 2          | 1  | 3  | 5  | 0 |
| 50               | 1          | 2  | 9  | 0  | 1           | 4  | 0  | 3  | 1           | 5  | 4  | 2  | 1           | 6  | 8  | 0  | 1          | 7  | 11 | 3  | 1          | 9  | 3  | 0  |   |
| 60               | 1          | 7  | 3  | 3  | 1           | 8  | 10 | 2  | 1           | 10 | 5  | 2  | 1           | 12 | 0  | 0  | 1          | 13 | 6  | 3  | 1          | 15 | 1  | 1  |   |
| 70               | 1          | 11 | 10 | 0  | 1           | 13 | 8  | 1  | 1           | 15 | 6  | 1  | 1           | 17 | 4  | 0  | 1          | 19 | 1  | 3  | 2          | 0  | 11 | 2  |   |
| 80               | 1          | 16 | 5  | 0  | 1           | 18 | 6  | 0  | 2           | 0  | 7  | 0  | 2           | 2  | 8  | 0  | 2          | 4  | 9  | 0  | 2          | 6  | 9  | 3  |   |
| 90               | 2          | 0  | 11 | 2  | 2           | 3  | 3  | 3  | 2           | 5  | 8  | 0  | 2           | 8  | 0  | 0  | 2          | 10 | 4  | 0  | 2          | 12 | 8  | 0  |   |
| 100              | 2          | 5  | 6  | 1  | 2           | 8  | 1  | 2  | 2           | 10 | 8  | 3  | 2           | 13 | 4  | 0  | 2          | 15 | 11 | 1  | 2          | 18 | 6  | 0  |   |
| 200              | 4          | 11 | 0  | 1  | 4           | 16 | 3  | 0  | 5           | 1  | 5  | 2  | 5           | 6  | 8  | 0  | 5          | 11 | 10 | 1  | 5          | 17 | 0  | 1  |   |
| 300              | 6          | 16 | 6  | 2  | 7           | 4  | 4  | 3  | 7           | 12 | 2  | 2  | 8           | 0  | 0  | 0  | 8          | 7  | 9  | 1  | 8          | 15 | 6  | 2  |   |
| 400              | 9          | 2  | 0  | 3  | 9           | 12 | 6  | 1  | 10          | 2  | 11 | 1  | 10          | 13 | 4  | 0  | 11         | 3  | 8  | 2  | 11         | 14 | 0  | 2  |   |
| 500              | 11         | 7  | 7  | 0  | 12          | 0  | 7  | 3  | 12          | 13 | 8  | 0  | 13          | 6  | 8  | 0  | 13         | 19 | 7  | 2  | 14         | 12 | 6  | 2  |   |
| 600              | 13         | 13 | 1  | 0  | 14          | 8  | 9  | 1  | 15          | 4  | 5  | 0  | 16          | 0  | 0  | 0  | 16         | 15 | 6  | 3  | 17         | 11 | 0  | 3  |   |
| 700              | 15         | 18 | 7  | 1  | 16          | 16 | 10 | 3  | 17          | 15 | 1  | 3  | 18          | 13 | 4  | 0  | 19         | 11 | 5  | 3  | 20         | 9  | 7  | 0  |   |
| 800              | 18         | 4  | 1  | 2  | 19          | 5  | 0  | 1  | 20          | 5  | 10 | 2  | 21          | 6  | 8  | 0  | 22         | 7  | 2  | 2  | 23         | 8  | 1  | 0  |   |
| 900              | 20         | 9  | 7  | 3  | 21          | 13 | 1  | 3  | 22          | 16 | 7  | 1  | 24          | 0  | 0  | 0  | 25         | 3  | 4  | 0  | 26         | 6  | 7  | 0  |   |
| 1000             | 22         | 15 | 1  | 3  | 24          | 1  | 3  | 2  | 25          | 7  | 4  | 1  | 26          | 13 | 4  | 0  | 27         | 19 | 3  | 1  | 29         | 5  | 1  | 1  |   |
| ditto            | l. 22.7577 |    |    |    | l. 24.06420 |    |    |    | l. 25.36710 |    |    |    | l. 26.6 r 1 |    |    |    | l. 27.9627 |    |    |    | l. 29.2553 |    |    |    |   |

# A TABLE of DISCOUNT at 5 per Cent. per Ann.

|            | 230 Days. |         |    |    | 240 Days. |         |    |    | 250 Days. |         |    |    | 260 Days. |         |    |    | 270 Days. |         |    |    | 280 Days. |         |    |    |   |
|------------|-----------|---------|----|----|-----------|---------|----|----|-----------|---------|----|----|-----------|---------|----|----|-----------|---------|----|----|-----------|---------|----|----|---|
| Principal. | l.        | s.      | d. | q. | l.        | s.      | d. | q. | l.        | s.      | d. | q. | l.        | s.      | d. | q. | l.        | s.      | d. | q. | l.        | s.      | d. | q. |   |
| 1          |           |         | 7  | 1  |           |         | 7  | 2  |           |         | 8  | 0  |           |         | 8  | 1  |           |         | 8  | 2  |           |         | 8  | 3  |   |
| 2          |           | 1       | 2  | 3  |           | 1       | 3  | 1  |           | 1       | 4  | 0  |           | 1       | 4  | 2  |           | 1       | 5  | 1  |           | 1       | 5  | 3  |   |
| 3          |           | 1       | 10 | 0  |           | 1       | 11 | 0  |           | 1       | 11 | 3  |           | 2       | 0  | 3  |           | 2       | 1  | 3  |           | 2       | 2  | 2  |   |
| 4          |           | 2       | 5  | 1  |           | 2       | 6  | 2  |           | 2       | 7  | 3  |           | 2       | 9  | 0  |           | 2       | 10 | 1  |           | 2       | 11 | 2  |   |
| 5          |           | 3       | 0  | 3  |           | 3       | 2  | 1  |           | 3       | 3  | 3  |           | 3       | 5  | 1  |           | 3       | 6  | 3  |           | 3       | 8  | 1  |   |
| 6          |           | 3       | 8  | 0  |           | 3       | 9  | 3  |           | 3       | 11 | 3  |           | 4       | 1  | 2  |           | 4       | 3  | 2  |           | 4       | 5  | 1  |   |
| 7          |           | 4       | 3  | 1  |           | 4       | 5  | 2  |           | 4       | 7  | 2  |           | 4       | 9  | 3  |           | 5       | 0  | 0  |           | 5       | 2  | 0  |   |
| 8          |           | 4       | 10 | 2  |           | 5       | 1  | 0  |           | 5       | 3  | 2  |           | 5       | 6  | 0  |           | 5       | 8  | 2  |           | 5       | 11 | 0  |   |
| 9          |           | 5       | 6  | 0  |           | 5       | 8  | 3  |           | 5       | 11 | 2  |           | 6       | 2  | 1  |           | 6       | 5  | 0  |           | 6       | 7  | 3  |   |
| 10         |           | 6       | 1  | 1  |           | 6       | 4  | 2  |           | 6       | 7  | 2  |           | 6       | 10 | 2  |           | 7       | 1  | 2  |           | 7       | 4  | 3  |   |
| 20         |           | 12      | 2  | 2  |           | 12      | 8  | 3  |           | 13      | 3  | 0  |           | 13      | 9  | 0  |           | 14      | 3  | 1  |           | 14      | 9  | 1  |   |
| 30         |           | 18      | 4  | 0  |           | 19      | 1  | 1  |           | 19      | 10 | 2  |           | 1       | 0  | 7  | 2         | 1       | 1  | 5  | 0         | 1       | 2  | 2  | 0 |
| 40         | 1         | 4       | 5  | 1  | 1         | 5       | 5  | 2  | 1         | 6       | 6  | 0  | 1         | 7       | 6  | 0  | 1         | 8       | 6  | 2  | 1         | 9       | 6  | 3  |   |
| 50         | 1         | 10      | 6  | 2  | 1         | 11      | 10 | 0  | 1         | 13      | 1  | 1  | 1         | 14      | 4  | 3  | 1         | 15      | 8  | 0  | 1         | 16      | 11 | 1  |   |
| 60         | 1         | 16      | 7  | 3  | 1         | 18      | 2  | 1  | 1         | 19      | 8  | 3  | 2         | 1       | 3  | 1  | 2         | 2       | 9  | 2  | 2         | 4       | 4  | 0  |   |
| 70         | 2         | 2       | 9  | 1  | 2         | 4       | 6  | 3  | 2         | 6       | 4  | 1  | 2         | 8       | 1  | 3  | 2         | 9       | 11 | 1  | 2         | 11      | 8  | 2  |   |
| 80         | 2         | 8       | 10 | 2  | 2         | 10      | 11 | 1  | 2         | 12      | 11 | 3  | 2         | 15      | 0  | 1  | 2         | 17      | 0  | 3  | 2         | 19      | 1  | 1  |   |
| 90         | 2         | 14      | 11 | 3  | 2         | 17      | 3  | 2  | 2         | 19      | 7  | 1  | 3         | 1       | 11 | 0  | 3         | 4       | 2  | 2  | 3         | 6       | 6  | 0  |   |
| 100        | 3         | 1       | 1  | 0  | 3         | 3       | 8  | 0  | 3         | 6       | 2  | 3  | 3         | 8       | 9  | 2  | 3         | 11      | 4  | 0  | 3         | 13      | 10 | 2  |   |
| 200        | 6         | 2       | 2  | 0  | 6         | 7       | 4  | 0  | 6         | 12      | 5  | 2  | 6         | 17      | 6  | 3  | 7         | 2       | 8  | 0  | 7         | 7       | 9  | 0  |   |
| 300        | 9         | 3       | 3  | 1  | 9         | 10      | 11 | 3  | 9         | 18      | 8  | 0  | 10        | 6       | 4  | 1  | 10        | 14      | 0  | 0  | 11        | 1       | 7  | 2  |   |
| 400        | 12        | 4       | 4  | 2  | 12        | 14      | 7  | 3  | 13        | 4       | 10 | 3  | 13        | 15      | 1  | 2  | 14        | 5       | 4  | 0  | 14        | 15      | 6  | 0  |   |
| 500        | 15        | 5       | 5  | 1  | 15        | 18      | 3  | 3  | 16        | 11      | 1  | 2  | 17        | 3       | 6  | 1  | 17        | 16      | 8  | 0  | 18        | 9       | 4  | 3  |   |
| 600        | 18        | 6       | 6  | 2  | 19        | 1       | 11 | 2  | 19        | 17      | 4  | 1  | 20        | 12      | 8  | 1  | 21        | 8       | 0  | 0  | 22        | 3       | 3  | 1  |   |
| 700        | 21        | 7       | 7  | 2  | 22        | 5       | 7  | 2  | 23        | 3       | 7  | 0  | 24        | 1       | 5  | 3  | 24        | 19      | 4  | 0  | 25        | 17      | 1  | 3  |   |
| 800        | 24        | 8       | 8  | 2  | 25        | 9       | 3  | 2  | 26        | 9       | 9  | 2  | 27        | 10      | 3  | 1  | 28        | 10      | 8  | 0  | 29        | 11      | 0  | 1  |   |
| 900        | 27        | 9       | 9  | 2  | 28        | 12      | 11 | 1  | 29        | 16      | 0  | 1  | 30        | 19      | 0  | 2  | 32        | 2       | 0  | 0  | 33        | 4       | 11 | 0  |   |
| 1000       | 30        | 10      | 10 | 3  | 31        | 16      | 7  | 1  | 33        | 2       | 3  | 0  | 34        | 7       | 10 | 0  | 35        | 13      | 4  | 0  | 36        | 18      | 9  | 2  |   |
| ditto      | l.        | 30.5444 |    |    | l.        | 31.8302 |    |    | l.        | 33.1125 |    |    | l.        | 34.3915 |    |    | l.        | 35.6671 |    |    | l.        | 36.9393 |    |    |   |



# A TABLE of DISCOUNT at 5 per Cent. per Ann.

|            | 290 Days. |     |    |    | 300 Days. |     |    |    | 310 Days. |     |    |    | 320 Days. |     |    |    | 330 Days. |     |    |    | 340 Days. |     |    |    |
|------------|-----------|-----|----|----|-----------|-----|----|----|-----------|-----|----|----|-----------|-----|----|----|-----------|-----|----|----|-----------|-----|----|----|
| Principal. | l.        | s.  | d. | q. | l.        | s.  | d. | q. | l.        | s.  | d. | q. | l.        | s.  | d. | q. | l.        | s.  | d. | q. | l.        | s.  | d. | q. |
| 1          |           |     | 9  | 1  |           |     | 9  | 2  |           |     | 9  | 3  |           |     | 10 | 0  |           |     | 10 | 2  |           |     | 10 | 3  |
| 2          |           | 1   | 6  | 1  |           | 1   | 7  | 0  |           | 1   | 7  | 2  |           | 1   | 8  | 0  |           | 1   | 8  | 3  |           | 1   | 9  | 1  |
| 3          |           | 2   | 3  | 2  |           | 2   | 4  | 2  |           | 2   | 5  | 1  |           | 2   | 6  | 1  |           | 2   | 7  | 1  |           | 2   | 8  | 0  |
| 4          |           | 3   | 0  | 3  |           | 3   | 1  | 3  |           | 3   | 3  | 0  |           | 3   | 4  | 2  |           | 3   | 5  | 2  |           | 3   | 6  | 3  |
| 5          |           | 3   | 10 | 0  |           | 3   | 11 | 2  |           | 4   | 0  | 3  |           | 4   | 2  | 2  |           | 4   | 4  | 0  |           | 4   | 5  | 2  |
| 6          |           | 4   | 7  | 0  |           | 4   | 8  | 3  |           | 4   | 10 | 3  |           | 5   | 0  | 2  |           | 5   | 2  | 1  |           | 5   | 4  | 1  |
| 7          |           | 5   | 4  | 1  |           | 5   | 6  | 1  |           | 5   | 8  | 2  |           | 5   | 10 | 2  |           | 6   | 0  | 3  |           | 6   | 2  | 3  |
| 8          |           | 6   | 1  | 1  |           | 6   | 3  | 3  |           | 6   | 6  | 1  |           | 6   | 8  | 2  |           | 6   | 11 | 0  |           | 7   | 1  | 2  |
| 9          |           | 6   | 10 | 2  |           | 7   | 1  | 1  |           | 7   | 4  | 0  |           | 7   | 6  | 3  |           | 7   | 9  | 2  |           | 8   | 0  | 0  |
| 10         |           | 7   | 7  | 3  |           | 7   | 10 | 3  |           | 8   | 1  | 3  |           | 8   | 5  | 0  |           | 8   | 7  | 3  |           | 8   | 11 | 0  |
| 20         |           | 15  | 3  | 2  |           | 15  | 9  | 2  |           | 16  | 3  | 2  |           | 16  | 9  | 2  |           | 17  | 3  | 3  |           | 17  | 9  | 3  |
| 30         | 1         | 2   | 11 | 0  | 1         | 3   | 8  | 1  | 1         | 4   | 5  | 1  | 1         | 5   | 2  | 1  | 1         | 5   | 11 | 2  | 1         | 6   | 8  | 2  |
| 40         | 1         | 10  | 6  | 3  | 1         | 11  | 7  | 0  | 1         | 12  | 7  | 0  | 1         | 13  | 7  | 0  | 1         | 14  | 7  | 1  | 1         | 15  | 7  | 1  |
| 50         | 1         | 18  | 2  | 2  | 1         | 19  | 5  | 3  | 2         | 0   | 8  | 3  | 2         | 2   | 0  | 0  | 2         | 3   | 3  | 0  | 2         | 4   | 6  | 0  |
| 60         | 2         | 5   | 10 | 1  | 2         | 7   | 4  | 2  | 2         | 8   | 10 | 3  | 2         | 10  | 4  | 3  | 2         | 11  | 10 | 3  | 2         | 13  | 4  | 3  |
| 70         | 2         | 13  | 6  | 0  | 2         | 15  | 3  | 1  | 2         | 17  | 0  | 1  | 2         | 18  | 9  | 2  | 3         | 0   | 6  | 2  | 3         | 2   | 3  | 3  |
| 80         | 3         | 1   | 1  | 2  | 3         | 3   | 2  | 0  | 3         | 5   | 2  | 0  | 3         | 7   | 2  | 1  | 3         | 9   | 2  | 2  | 3         | 11  | 2  | 2  |
| 90         | 3         | 8   | 9  | 1  | 3         | 11  | 0  | 2  | 3         | 13  | 4  | 0  | 3         | 15  | 7  | 0  | 3         | 17  | 10 | 1  | 4         | 0   | 1  | 1  |
| 100        | 3         | 16  | 5  | 0  | 3         | 18  | 11 | 2  | 4         | 1   | 5  | 3  | 4         | 3   | 11 | 3  | 4         | 6   | 6  | 0  | 4         | 9   | 0  | 0  |
| 200        | 7         | 12  | 10 | 0  | 7         | 17  | 10 | 3  | 8         | 2   | 11 | 2  | 8         | 7   | 11 | 3  | 8         | 13  | 0  | 0  | 8         | 18  | 0  | 0  |
| 300        | 11        | 9   | 3  | 0  | 11        | 16  | 10 | 1  | 12        | 4   | 5  | 0  | 12        | 11  | 11 | 3  | 12        | 19  | 6  | 0  | 13        | 7   | 0  | 0  |
| 400        | 15        | 5   | 8  | 0  | 15        | 15  | 9  | 2  | 16        | 5   | 10 | 3  | 16        | 15  | 11 | 2  | 17        | 6   | 0  | 0  | 17        | 16  | 0  | 1  |
| 500        | 19        | 2   | 1  | 0  | 19        | 14  | 8  | 3  | 20        | 7   | 4  | 1  | 20        | 19  | 11 | 2  | 21        | 12  | 6  | 0  | 22        | 5   | 0  | 1  |
| 600        | 22        | 18  | 6  | 0  | 23        | 13  | 8  | 1  | 24        | 8   | 10 | 0  | 25        | 3   | 11 | 1  | 25        | 19  | 0  | 0  | 26        | 14  | 0  | 1  |
| 700        | 26        | 14  | 11 | 0  | 27        | 12  | 7  | 2  | 28        | 10  | 3  | 3  | 29        | 7   | 11 | 0  | 30        | 5   | 6  | 0  | 31        | 3   | 0  | 2  |
| 800        | 30        | 11  | 4  | 0  | 31        | 11  | 7  | 0  | 32        | 11  | 9  | 1  | 33        | 11  | 11 | 0  | 34        | 12  | 0  | 0  | 35        | 12  | 0  | 2  |
| 900        | 34        | 7   | 9  | 0  | 35        | 10  | 6  | 1  | 36        | 13  | 3  | 0  | 37        | 15  | 11 | 0  | 38        | 18  | 6  | 0  | 40        | 1   | 0  | 2  |
| 1000       | 38        | 4   | 2  | 0  | 39        | 9   | 5  | 3  | 40        | 14  | 8  | 2  | 41        | 19  | 10 | 3  | 43        | 5   | 0  | 0  | 44        | 10  | 0  | 2  |
| ditto      | l.        | 38. | 20 | 81 | l.        | 39. | 47 | 36 | l.        | 40. | 73 | 58 | l.        | 41. | 99 | 47 | l.        | 43. | 25 | 03 | l.        | 44. | 50 | 26 |

Discount at 5 per Cent.

Part II. For Days under 10 in  
a Medium, to add at above  
90 Days.

| Prin-<br>cipal. | 350 Days. |       |    |    | 360 Days. |       |    |    | 365 Days. |       |    |    | 1 Day. |    |    | 2 Days. |    |    | 3 Days. |    |    |
|-----------------|-----------|-------|----|----|-----------|-------|----|----|-----------|-------|----|----|--------|----|----|---------|----|----|---------|----|----|
|                 | l.        | s.    | d. | q. | l.        | s.    | d. | q. | l.        | s.    | d. | q. | s.     | d. | q. | s.      | d. | q. | s.      | d. | q. |
| 1               | 1         |       | 11 | 0  |           |       | 11 | 1  |           |       | 11 | 1  |        |    |    |         |    |    |         |    |    |
| 2               |           | 1     | 10 | 0  |           | 1     | 10 | 2  |           | 1     | 11 | 0  |        |    |    |         |    |    |         | 1  |    |
| 3               |           | 2     | 9  | 0  |           | 2     | 9  | 3  |           | 2     | 10 | 1  |        |    |    |         |    |    |         | 1  |    |
| 4               |           | 3     | 8  | 0  |           | 3     | 9  | 1  |           | 3     | 9  | 2  |        |    |    |         |    |    |         | 1  |    |
| 5               |           | 4     | 7  | 0  |           | 4     | 8  | 2  |           | 4     | 9  | 0  |        |    |    |         |    |    |         | 1  |    |
| 6               |           | 5     | 6  | 0  |           | 5     | 7  | 3  |           | 5     | 8  | 2  |        |    |    |         |    |    |         | 1  |    |
| 7               |           | 6     | 5  | 0  |           | 6     | 7  | 0  |           | 6     | 8  | 0  |        |    |    |         |    |    |         | 1  |    |
| 8               |           | 7     | 4  | 0  |           | 7     | 6  | 1  |           | 7     | 7  | 2  |        |    |    |         |    |    |         | 1  |    |
| 9               |           | 8     | 3  | 0  |           | 8     | 5  | 2  |           | 8     | 6  | 3  |        |    |    |         |    |    |         | 1  |    |
| 10              |           | 9     | 2  | 0  |           | 9     | 4  | 3  |           | 9     | 6  | 1  |        |    |    |         |    |    |         | 1  |    |
| 20              |           | 18    | 3  | 3  |           | 18    | 9  | 2  |           | 19    | 0  | 2  |        |    |    |         |    |    |         | 1  |    |
| 30              | 1         | 7     | 5  | 2  | 1         | 8     | 2  | 2  | 1         | 8     | 6  | 3  |        |    |    |         |    |    |         | 1  |    |
| 40              | 1         | 16    | 7  | 1  | 1         | 17    | 7  | 1  | 1         | 18    | 1  | 1  |        |    |    |         |    |    |         | 1  |    |
| 50              | 2         | 5     | 9  | 0  | 2         | 7     | 0  | 0  | 2         | 7     | 7  | 2  |        |    |    |         |    |    |         | 1  |    |
| 60              | 2         | 14    | 10 | 3  | 2         | 16    | 4  | 3  | 2         | 17    | 1  | 3  |        |    |    |         |    |    |         | 1  |    |
| 70              | 3         | 4     | 0  | 2  | 3         | 5     | 9  | 2  | 3         | 6     | 8  | 0  |        |    |    |         |    |    |         | 1  |    |
| 80              | 3         | 13    | 2  | 2  | 3         | 15    | 2  | 2  | 3         | 16    | 2  | 1  |        |    |    |         |    |    |         | 1  |    |
| 90              | 4         | 2     | 4  | 1  | 4         | 4     | 7  | 1  | 4         | 5     | 8  | 2  |        |    |    |         |    |    |         | 1  |    |
| 100             | 4         | 11    | 6  | 0  | 4         | 14    | 0  | 0  | 4         | 15    | 2  | 3  |        |    |    |         |    |    |         | 1  |    |
| 200             | 9         | 3     | 0  | 0  | 9         | 7     | 11 | 3  | 9         | 10    | 5  | 3  |        |    |    |         |    |    |         | 1  |    |
| 300             | 13        | 14    | 6  | 0  | 14        | 1     | 11 | 3  | 14        | 5     | 8  | 2  |        |    |    |         |    |    |         | 1  |    |
| 400             | 18        | 6     | 0  | 0  | 18        | 15    | 11 | 3  | 19        | 0     | 11 | 2  |        |    |    |         |    |    |         | 1  |    |
| 500             | 22        | 17    | 6  | 1  | 23        | 9     | 11 | 3  | 23        | 16    | 2  | 1  |        |    |    |         |    |    |         | 1  |    |
| 600             | 27        | 9     | 0  | 1  | 28        | 3     | 11 | 2  | 28        | 11    | 5  | 1  |        |    |    |         |    |    |         | 1  |    |
| 700             | 32        | 0     | 6  | 1  | 32        | 17    | 11 | 2  | 33        | 6     | 8  | 0  |        |    |    |         |    |    |         | 1  |    |
| 800             | 36        | 12    | 0  | 1  | 37        | 11    | 11 | 2  | 38        | 1     | 11 | 0  |        |    |    |         |    |    |         | 1  |    |
| 900             | 41        | 3     | 6  | 1  | 42        | 5     | 11 | 2  | 42        | 17    | 1  | 3  |        |    |    |         |    |    |         | 1  |    |
| 1000            | 45        | 15    | 0  | 1  | 46        | 19    | 11 | 1  | 47        | 12    | 4  | 3  |        |    |    |         |    |    |         | 1  |    |
| ditto           | l.        | 45.75 | 16 |    | l.        | 46.29 | 73 |    | l.        | 47.61 | 9  |    |        |    |    |         |    |    |         |    |    |

l. 0.1304 l. 0.2608 l. 0.3912

Part II. For Days under 10 in a Medium, to add at  
above 90 Days.

| Principal.<br>l. | 4 Days. |    |      | 5 Days. |    |        | 6 Days. |     |         | 7 Days. |    |         | 8 Days. |    |          | 9 Days. |    |      |    |        |    |      |  |  |    |    |      |
|------------------|---------|----|------|---------|----|--------|---------|-----|---------|---------|----|---------|---------|----|----------|---------|----|------|----|--------|----|------|--|--|----|----|------|
|                  | s.      | d. | q.   | s.      | d. | q.     | s.      | d.  | q.      | s.      | d. | q.      | l.      | s. | d.       | q.      | l. | s.   | d. | q.     |    |      |  |  |    |    |      |
| 1                |         |    |      |         |    | 0      |         |     | 1       |         |    | 1       |         |    | 1        |         |    |      |    | 1      |    |      |  |  |    |    |      |
| 2                |         |    | 1    |         |    | 1      |         |     | 1       |         |    | 2       |         |    | 2        |         |    |      |    | 2      |    |      |  |  |    |    |      |
| 3                |         |    | 1    |         |    | 2      |         |     | 2       |         |    | 3       |         |    | 3        |         |    |      |    | 3      |    |      |  |  |    |    |      |
| 4                |         |    | 2    |         |    | 2      |         |     | 3       |         |    | 3       |         |    | 1 0      |         |    |      |    | 1 0    |    |      |  |  |    |    |      |
| 5                |         |    | 2    |         |    | 3      |         |     | 1 0     |         |    | 1 0     |         |    | 1 1      |         |    |      |    | 1 2    |    |      |  |  |    |    |      |
| 6                |         |    | 3    |         |    | 1 0    |         |     | 1 0     |         |    | 1 1     |         |    | 1 2      |         |    |      |    | 1 3    |    |      |  |  |    |    |      |
| 7                |         |    | 3    |         |    | 1 0    |         |     | 1 1     |         |    | 1 2     |         |    | 1 3      |         |    |      |    | 2 0    |    |      |  |  |    |    |      |
| 8                |         |    | 1 0  |         |    | 1 1    |         |     | 1 2     |         |    | 1 3     |         |    | 2 0      |         |    |      |    | 2 1    |    |      |  |  |    |    |      |
| 9                |         |    | 1 0  |         |    | 1 2    |         |     | 1 3     |         |    | 2 0     |         |    | 2 1      |         |    |      |    | 2 2    |    |      |  |  |    |    |      |
| 10               |         |    | 1 1  |         |    | 1 2    |         |     | 2 0     |         |    | 2 1     |         |    | 2 2      |         |    |      |    | 3 0    |    |      |  |  |    |    |      |
| 20               |         |    | 2 2  |         |    | 3 1    |         |     | 3 3     |         |    | 4 2     |         |    | 5 0      |         |    |      |    | 5 3    |    |      |  |  |    |    |      |
| 30               |         |    | 3 3  |         |    | 4 3    |         |     | 5 3     |         |    | 6 2     |         |    | 7 2      |         |    |      |    | 8 2    |    |      |  |  |    |    |      |
| 40               |         |    | 5 0  |         |    | 6 1    |         |     | 7 2     |         |    | 8 3     |         |    | 10 0     |         |    |      |    | 11 0   |    |      |  |  |    |    |      |
| 50               |         |    | 6 1  |         |    | 7 3    |         |     | 9 2     |         |    | 11 0    |         |    | 1 0 2    |         |    |      |    | 1 2 1  |    |      |  |  |    |    |      |
| 60               |         |    | 7 2  |         |    | 9 2    |         |     | 11 1    |         |    | 1 1 1   |         |    | 1 3 0    |         |    |      |    | 1 5 0  |    |      |  |  |    |    |      |
| 70               |         |    | 8 3  |         |    | 11 0   |         |     | 1 1 1   |         |    | 1 3 1   |         |    | 1 5 2    |         |    |      |    | 1 7 3  |    |      |  |  |    |    |      |
| 80               |         |    | 10 0 |         |    | 1 0 2  |         |     | 1 3 0   |         |    | 1 5 2   |         |    | 1 8 0    |         |    |      |    | 1 10 2 |    |      |  |  |    |    |      |
| 90               |         |    | 11 1 |         |    | 1 2 0  |         |     | 1 5 0   |         |    | 1 7 3   |         |    | 1 10 2   |         |    |      |    | 2 1 1  |    |      |  |  |    |    |      |
| 100              | 1       |    | 0 2  |         |    | 1 3 3  |         |     | 1 6 3   |         |    | 1 10 0  |         |    | 2 1 0    |         |    |      |    | 2 4 1  |    |      |  |  |    |    |      |
| 200              | 2       |    | 1 0  |         |    | 2 7 1  |         |     | 3 1 2   |         |    | 3 7 3   |         |    | 4 2 0    |         |    |      |    | 4 8 1  |    |      |  |  |    |    |      |
| 300              | 3       |    | 1 2  |         |    | 3 11 0 |         |     | 4 8 1   |         |    | 5 5 3   |         |    | 6 3 0    |         |    |      |    | 7 0 2  |    |      |  |  |    |    |      |
| 400              | 4       |    | 2 0  |         |    | 5 2 2  |         |     | 6 3 1   |         |    | 7 3 3   |         |    | 8 4 0    |         |    |      |    | 9 4 3  |    |      |  |  |    |    |      |
| 500              | 5       |    | 2 2  |         |    | 6 6 1  |         |     | 7 10 0  |         |    | 9 1 2   |         |    | 10 5 1   |         |    |      |    | 11 8 3 |    |      |  |  |    |    |      |
| 600              | 6       |    | 3 1  |         |    | 7 10 0 |         |     | 9 4 3   |         |    | 10 11 2 |         |    | 12 6 1   |         |    |      |    | 14 1 0 |    |      |  |  |    |    |      |
| 700              | 7       |    | 3 3  |         |    | 9 1 2  |         |     | 10 11 2 |         |    | 12 9 1  |         |    | 14 7 1   |         |    |      |    | 16 5 1 |    |      |  |  |    |    |      |
| 800              | 8       |    | 4 1  |         |    | 10 5 1 |         |     | 12 6 1  |         |    | 14 7 1  |         |    | 16 8 1   |         |    |      |    | 18 9 1 |    |      |  |  |    |    |      |
| 900              | 9       |    | 4 3  |         |    | 11 8 3 |         |     | 14 1 0  |         |    | 16 5 1  |         |    | 18 9 1   |         |    |      |    | 1 1 2  |    |      |  |  |    |    |      |
| 1000             | 10      |    | 5 1  |         |    | 13 0 0 |         |     | 15 7 2  |         |    | 18 3 0  |         |    | 1 0 10 1 |         |    |      |    | 3 5 3  |    |      |  |  |    |    |      |
| ditto            | l.      | 0. | 5216 |         |    | l.     | 0.      | 652 |         |         | l. | 0.      | 7824    |    |          | l.      | 0. | 9128 |    | l.     | 1. | 0432 |  |  | l. | 1. | 1736 |



The Use of the Tables of Discount foregoing.

*Quest. 1.* What is the Discount of *l.* 700 (if paid 137 days before due) at 4 per Cent?

In the 1st Table against *l.* 700, under 130 days is *l.* 9 : 16 : 7½  
 700 under 7 days (part the 2d, } 00 : 10 : 3  
 at the end of the Table) —

Sum, or Answer = *l.* 10 : 6 : 10½

*Quest. 2.* What is the Discount of *l.* 1500 for 57 Days at 5 per Cent?

By the Method in the Margin,  
 the Answer is found

*l.* 11 : 12 : 4½.

I shall give another Example, which takes in all the Varieties of the Use of the Table.

*Example 2.]* What Discount at 5 per Cent. is to be allowed, to satisfy a Debt of *l.* 5000, 91 Days before the same is due?

*l.* Days.

1000 for 50 = *l.* 6 : 16 : 0½

500 — — 3 : 8 : 0½

1000 for 7 = 0 : 19 : 2

500 — — 0 : 9 : 1½

*l.* 11 : 12 : 4½ = Sum

*Note,* That for 1, 2, 3, 4, 5, 6, 7, 8, and 9 Days, from and above 90, &c. to 360; I take those Tabular Numbers against 1, 2, &c. to 9, at the end of the Table of Discount.

*l.*

Days.

Against 1000 (or ditto) under 90 stands 12.1786 } add  
 And under 1 (part 2.) = 0.1304 }

Sum = 12.3090

Which multiply by the Thousands — = 5

The Answer or Product = 61.545 = *l.* 61 : 10 : 11

But had this Question been done by the Tables in Dr Harris's *Lexicon*, which he says are accurate; it will be found *l.* 2 : 3 : 8½ too little. For 91 Days multiplied in *l.* 5000 gives 455000. And by those Tables you have against 400000 = 52.1853

50000 = 6.5232

5000 = 0.6523

Sum = 59.3608 = *l.* 59 : 7 : 2½

E c Which

Which taken from my Discount above, is too little by  $l. 2 : 3 : 8\frac{1}{2}$  as I have fully proved before the Tables above.

### III. Of Compound Interest: The Use of Decimals.

I have shewed above, what Compound Interest is, and now proceed to give the Reader the Use of Decimals in computing thereof.

*Case 1.* To find the Amount of any Sum of Money for any Number of Years, as of  $l. 650$ , for 5 Years at 5 per Cent. per Ann. Compound Interest.

*Rule.]* First find the Amount of  $l. 1$  for 5 Years, and multiply the 5th Year by  $l. 650$ , the Principal given, thus:

*Example.]* 1st,  $100. 105 :: 1. 1.05 =$  at the end of 1 Year.

2dly,  $100. 105 :: 1.05$

$1.05$

3dly,  $100. 105 :: 1.1025 =$  of the 2d Year.

$1.05$

4thly,  $100. 105 :: 1.157625 =$  of the 3d Year.

$1.05$

5thly,  $100. 105 :: 1.21550625 =$  the 4th Year.

$1.05$

Answer  $= 1.2762815625 =$  5th Year.

$650$  multiply.

The Amount of  $l. 650$  }  $= 829.5830156 =$  Answer.  
for 5 Years. — —

Or it will be the same thing to multiply the 3d Number by  $1.05$  continually; *i. e.* to multiply the Surfolid of  $1.05$  by  $650$ .

In this Example you see that every Year requires a Multiplication performed as *per Compend. 2.* of Multiplication of Decimals, and the last by the  $650$ , as *per Compend. 6.* whereby 'tis found, that if  $l. 650$  be put out and forborn 5 Years, the Interest of that Principal, and of the Interest accruing, will amount to  $l. 829 : 11 : 8$ .

*Case 2.* To find the present Worth of any Sum due any Number of Years hence, as of  $l. 829 : 11 : 8$  (or  $l. 829.583015625$ ) due at the end of 5 Years. See the Operation at 5 per Cent. Compound Interest, or divide the Number given, &c. by  $1.05$ . continually.

$1.05.$

# SECT. VIII. Use of Decimals in Compound Interest.

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1.05. 100 :: 829.5, &c.

1.05) 82958.3015625 (790.076200595 = 1 Year hence.

945

800

651

210

625

1000

550

25

105) 79007.620059500 (752.453524376

..... Worth 2 Years hence.

550

257

476

562

370

550

255

459

395

800

650

3dly, 105. 100 :: 752.453524376. 716.622423 = 3 Years hence.

4thly, 105. 100 :: 716.622423. 682.497164 = present Worth 4 (Years hence.

5thly, 105. 100 :: 682.497164. 650. = the present Worth of the Sum given, viz. l. 829 : 11 : 8 due at the end of 5 Years.

Here you may observe, that the 4th Proportional is always the 3d Number in the Proportion the next stating : And having for every Year multiplied by 100, and divided by 105, and the proposed Rate of Interest, (directly contrary to what you did in the last

E e 2

Case)



Casè) the 4th Proportional in the 5th Operation is *l.* 650, which is the present Worth of *l.* 829 : 11 : 8, due at the end of 5 Years; which proves the Truth of the last Casè.

And after the same manner, had you desired to have found the present Worth of *l.* 1 due 5 Years hence, the several Years Decrease would be as in the following Table, Column 2. from that of Years.

| <i>Years.</i> | <i>The Amount of l. 1.</i> | <i>The present Worth of l. 1.</i> | <i>The Amount of l. 1. Annuity.</i> | <i>The present Worth of l. 1. Annuity.</i> |
|---------------|----------------------------|-----------------------------------|-------------------------------------|--------------------------------------------|
| 1             | 1.05                       | .95238095                         | 1                                   | .95238095                                  |
| 2             | 1.1025                     | .90702947                         | 2.05                                | 1.85941042                                 |
| 3             | 1.157625                   | .86383759                         | 3.1525                              | 2.72324801                                 |
| 4             | 1.21550625                 | .82270247                         | 4.310125                            | 3.54595048                                 |
| 5             | 1.27628156                 | .78352616                         | 5.52563125                          | 4.32947664                                 |

*Casè 3.* To find the Amount of Annuities forborn any Number of Years, you will easily consider, that at the end of 1 Year there is 1 Year's Income due without any Interest; for the 2d Year, at the end of that there is 2 Years Income due, and the Interest of 1 Year; at the end of the 3d Year, there is due 3 Years Income; 2 Years Interest of the 1st Year's Income, 1 Year's Interest of the 2d Year's Income, and 1 Year's Interest of the Interest of the 1st Year's Income, &c.

Thus suppose the Rent 1 *l.* per Ann. the 3 Years income is *l.* 3  
 2 Years Interest of the 1st Years Income = 0.1  
 1 Years Interest of the 2d Years Income = 0.05

And 1 Year's Interest of the Interest of the 1st Year's Income. — — — — — } = 0.0025

The Sum is the utmost Improvement of this } = 3.1525  
 Annuity for 3 Years, viz. — — —

Hence it follows, that a Table of the Amount of 1 *l.* Annuity, as the 3d Column above, is made from the Column of the Amount of *l.* 1, by making the Tabular Number for the 1st Year 1; for the 2d in the 3d Column the Sum of the 2 first Numbers in the 1st and 3d Columns; the 3d Number in the 3d Column is the Sum of the 2d Number in the 1st and 3d Columns, &c.

*Casè*

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*Case 4.* To find the present Worth of an Annuity due any Number of Years to come, it must be considered that the present Worth of  $l. 1$  Annuity to continue 1 Year is the same with the present Worth of  $l. 1$  due a Year hence. So that the present Worth of an Annuity to continue 2 Years, is the Sum of the present Worth thereof for 1 Year, and the present Worth of the like Sum with the Annuity to continue 2 Years; and thus the 4th Column in the above Table is made: For

N<sup>o</sup>. Col.

|                   |            |                            |
|-------------------|------------|----------------------------|
| The 1 in the 4 is | .95238095  | } Sum = the 2d in Column 4 |
| 2 ——— 2 —         | .90702947  |                            |
| 2 ——— 4 —         | 1.85941042 | } Sum = the 3d in Column 4 |
| 3 ——— 2 —         | .86383759  |                            |

viz. 1.85941042.  
viz. 2.72324801, &c.

*Case 5.* To find what Annuity, to continue any Number of Years, any Sum will purchase. Divide a Unit by any of the Numbers which are the present Worth of  $l. 1$  Annuity for any Years, and the Quotient shews the Annuity that  $l. 1$  will purchase to continue those Years. Thus if I would know what  $l. 1$  will purchase *per Ann.* to continue 5 Years.

As 4.32947664. 1 : : 1.23097480 = Answ. = 4 s. 7 d.  $\frac{1}{2}$  per Ann.

*Farther Rules concerning the five Cases above.*

1. To enlarge the Numbers tabulated above, and what is under the 5th Case, to any Number of Years, two ways.

2. How to use them for any other Sum, above  $l. 1$ .

1. You may augment the first and second Columns by several Operations, as taught above, from 5 Years to 50, &c. and from them may make the third and fourth Columns, and another shewing what  $l. 1$  will purchase to continue those Years.

Or, Because the Numbers in the 1st Column of Compound Interest are in proportion as 100 to 105, or as 1 to 1.05, therefore that Column is a Series in Geometrical Proportion, whose Ratio is 1.05, so that (as in *Progression*, Sect. 2. Chap. II.) if you have no occasion for the intermediate Numbers, but would know the Amount, present Worth of any Sum or Annuity, &c. for 41 Years; for the Amount of  $l. 1$ . multiply the 1.27628156 by itself, and it produceth the Amount for 10 Years; which if you multiply by itself, produceth the Amount for 20 Years; and that multiplied by itself, produceth the Amount for 40 Years. And having done the like with

with the 2d Column (of the present Worth of *l. 1*) and found the present Worth for 40 Years; you may easily by the Rules under the first and second Case find the Amount or present Worth for one Year more. And having those two for 40 and 41 Years, you may easily from thence calculate the Amount, present Worth, or Purchase of Annuities, as is shewed under the 3d, 4th and 5th Cases.

For Sums of Money or Annuities above *l. 1*, you have nothing to do but to multiply respectively by such Pounds: Thus for *l. 500* 5 Years.

*l.*

If you multiply 1.27628156 by 500 you have the Amount of  
(*l. 500*, 5 Years.

.78352616 by 500, you have the present  
(Worth of *l. 500* due 5 Years hence.

5.52563125 by 500, you have the Amount  
(of *l. 500 per Ann.* for 5 Years.

4.32947664 by 500, you have the present  
Worth of *l. 500 per Ann.*  
to continue 5 Years.

And as in Case 5. .2309748 by 500, you have the Annuity  
that *l. 500* will purchase  
to continue 5 Years.

And the like for any other Sums or Term of Years.

But all these things (if you are without Tables of Interest) are most easily and concisely done by the Logarithms, as appears in that Part of Arithmetic hereafter following.

*Compound*





## COMPOUND INTEREST at 3 per Cent.

| Years. | TABLE I.<br><i>The Amount<br/>of l. 1.</i> | TABLE II.<br><i>The pre-<br/>sent Worth<br/>of l. 1.</i> | TABLE III.<br><i>The Amount<br/>of l. 1 An-<br/>nuity.</i> | TABLE IV.<br><i>The present<br/>Worth of l. 1.<br/>Annuity.</i> |
|--------|--------------------------------------------|----------------------------------------------------------|------------------------------------------------------------|-----------------------------------------------------------------|
| 1      | 1.03                                       | .9708738                                                 | 1                                                          | .9708738                                                        |
| 2      | 1.0609                                     | .9425959                                                 | 2.03                                                       | 1.9134697                                                       |
| 3      | 1.092727                                   | .9151417                                                 | 3.0909                                                     | 2.8286114                                                       |
| 4      | 1.1255088                                  | .8884870                                                 | 4.183627                                                   | 3.7170984                                                       |
| 5      | 1.1592741                                  | .8626088                                                 | 5.3091358                                                  | 4.5797072                                                       |
| 6      | 1.1940523                                  | .8374843                                                 | 6.4684099                                                  | 5.4171914                                                       |
| 7      | 1.2298739                                  | .8130915                                                 | 7.6624622                                                  | 6.2302829                                                       |
| 8      | 1.2667701                                  | .7894092                                                 | 8.8923360                                                  | 7.0196922                                                       |
| 9      | 1.3047732                                  | .7664167                                                 | 10.1591061                                                 | 7.7861089                                                       |
| 10     | 1.3439164                                  | .7440939                                                 | 11.4638793                                                 | 8.5302028                                                       |
| 11     | 1.3842339                                  | .7224213                                                 | 12.8077957                                                 | 9.2526241                                                       |
| 12     | 1.4257609                                  | .7013799                                                 | 14.1920296                                                 | 9.9540040                                                       |
| 13     | 1.4685337                                  | .6809513                                                 | 15.6177904                                                 | 10.6349553                                                      |
| 14     | 1.5125897                                  | .6611178                                                 | 17.0863242                                                 | 11.2960731                                                      |
| 15     | 1.5579674                                  | .6418619                                                 | 18.5989139                                                 | 11.9379351                                                      |
| 16     | 1.6047064                                  | .6231669                                                 | 20.1568813                                                 | 12.5611020                                                      |
| 17     | 1.6528476                                  | .6050164                                                 | 21.7615877                                                 | 13.1661185                                                      |
| 18     | 1.7024331                                  | .5873946                                                 | 23.4144354                                                 | 13.7535131                                                      |
| 19     | 1.7535061                                  | .5702860                                                 | 25.1168684                                                 | 14.3237991                                                      |
| 20     | 1.8061112                                  | .5536758                                                 | 26.8703745                                                 | 14.8774748                                                      |
| 21     | 1.8602946                                  | .5375493                                                 | 28.6764857                                                 | 15.4150241                                                      |
| 22     | 1.9161034                                  | .5218925                                                 | 30.5367803                                                 | 15.9369166                                                      |
| 23     | 1.9735865                                  | .5066917                                                 | 32.4528837                                                 | 16.4436084                                                      |
| 24     | 2.0327941                                  | .4919337                                                 | 34.4264702                                                 | 16.9355421                                                      |
| 25     | 2.0937779                                  | .4776056                                                 | 36.4592643                                                 | 17.4131477                                                      |
| 26     | 2.1565913                                  | .4636947                                                 | 38.5530422                                                 | 17.8768420                                                      |
| 27     | 2.2212890                                  | .4501891                                                 | 40.7096335                                                 | 18.3270315                                                      |
| 28     | 2.2879277                                  | .4370768                                                 | 42.9309225                                                 | 18.7641082                                                      |
| 29     | 2.3565655                                  | .4243464                                                 | 45.2188502                                                 | 19.1884546                                                      |
| 30     | 2.4272625                                  | .4119868                                                 | 47.5754157                                                 | 19.6004413                                                      |
| 31     | 2.5000803                                  | .3999871                                                 | 50.0026782                                                 | 20.0004285                                                      |

## COMPOUND INTEREST at 3 per Cent.

| <i>Years.</i> | TABLE I.<br>Continu'd.<br><i>The Amount<br/>of l. 1.</i> | TABLE II.<br>Continu'd.<br><i>The present<br/>Worth of<br/>l. 1.</i> | TABLE III.<br>Continu'd.<br><i>The Amount of<br/>l. 1. Annuity.</i> | TABLE IV.<br>Continu'd.<br><i>The present<br/>Worth of l. 1.<br/>Annuity.</i> |
|---------------|----------------------------------------------------------|----------------------------------------------------------------------|---------------------------------------------------------------------|-------------------------------------------------------------------------------|
| 32            | 2.5750828                                                | .3883370                                                             | 52.5027585                                                          | 20.3887655                                                                    |
| 33            | 2.6523352                                                | .3770262                                                             | 55.0778413                                                          | 20.7657918                                                                    |
| 34            | 2.7319053                                                | .3660449                                                             | 57.7301765                                                          | 21.1318367                                                                    |
| 35            | 2.8138624                                                | .3553834                                                             | 60.4620818                                                          | 21.4872200                                                                    |
| 36            | 2.8982783                                                | .3450324                                                             | 63.2759443                                                          | 21.8322525                                                                    |
| 37            | 2.9852267                                                | .3349829                                                             | 66.1742226                                                          | 22.1672354                                                                    |
| 38            | 3.0747835                                                | .3252262                                                             | 69.1594493                                                          | 22.4924616                                                                    |
| 39            | 3.1670270                                                | .3157535                                                             | 72.2342327                                                          | 22.8082151                                                                    |
| 40            | 3.2620378                                                | .3065568                                                             | 75.4012597                                                          | 23.1147719                                                                    |
| 41            | 3.3598989                                                | .2976280                                                             | 78.6632975                                                          | 23.4123999                                                                    |
| 42            | 3.4606959                                                | .2889592                                                             | 82.0231964                                                          | 23.7013592                                                                    |
| 43            | 3.5645168                                                | .2805429                                                             | 85.4838923                                                          | 23.9819021                                                                    |
| 44            | 3.6714523                                                | .2723718                                                             | 89.0484191                                                          | 24.2542739                                                                    |
| 45            | 3.7815959                                                | .2644386                                                             | 92.7198614                                                          | 24.5187125                                                                    |
| 46            | 3.8950437                                                | .2567365                                                             | 96.5014572                                                          | 24.7754490                                                                    |
| 47            | 4.0118950                                                | .2492588                                                             | 100.3965009                                                         | 25.0247078                                                                    |
| 48            | 4.1322519                                                | .2419988                                                             | 104.4083960                                                         | 25.2667066                                                                    |
| 49            | 4.2562194                                                | .2349503                                                             | 108.5406479                                                         | 25.5016569                                                                    |
| 50            | 4.3839060                                                | .2281071                                                             | 112.7968673                                                         | 25.7297640                                                                    |
| 51            | 4.5154232                                                | .2214632                                                             | 117.1807733                                                         | 25.9512271                                                                    |
| 52            | 4.6508859                                                | .2150128                                                             | 121.6961966                                                         | 26.1662399                                                                    |
| 53            | 4.7904125                                                | .2087503                                                             | 126.3470825                                                         | 26.3749902                                                                    |
| 54            | 4.9341249                                                | .2026702                                                             | 131.1374949                                                         | 26.5776040                                                                    |
| 55            | 5.0821486                                                | .1967672                                                             | 136.0716198                                                         | 26.7744276                                                                    |
| 56            | 5.2346131                                                | .1910361                                                             | 141.1537684                                                         | 26.9654637                                                                    |
| 57            | 5.3916515                                                | .1854719                                                             | 146.3883815                                                         | 27.1509356                                                                    |
| 58            | 5.5534010                                                | .1800698                                                             | 151.7800329                                                         | 27.3310054                                                                    |
| 59            | 5.7200030                                                | .1748251                                                             | 157.3334339                                                         | 27.5058305                                                                    |
| 60            | 5.8916031                                                | .1697331                                                             | 163.0534369                                                         | 27.6755636                                                                    |
| 61            | 6.0683512                                                | .1647894                                                             | 168.9450400                                                         | 27.8403530                                                                    |



## COMPOUND INTEREST at 4 per Cent.

| <i>Years.</i> | TABLE V.                       | TAB. VI.                                  | TABLE VII.                             | TAB. VIII.                                         |
|---------------|--------------------------------|-------------------------------------------|----------------------------------------|----------------------------------------------------|
|               | <i>The Amount<br/>of l. 1.</i> | <i>The present<br/>Worth of<br/>l. 1.</i> | <i>The Amount of<br/>l. 1 Annuity.</i> | <i>The present<br/>Worth of l. 1.<br/>Annuity.</i> |
| 1             | 1.04                           | .9615385                                  | 1.0                                    | .9615385                                           |
| 2             | 1.0816                         | .9245562                                  | 2.04                                   | 1.8860947                                          |
| 3             | 1.124864                       | .8889964                                  | 3.1216                                 | 2.7750910                                          |
| 4             | 1.1698586                      | .8548042                                  | 4.246464                               | 3.6298952                                          |
| 5             | 1.2166529                      | .8219271                                  | 5.4163226                              | 4.4518223                                          |
| 6             | 1.2653190                      | .7903145                                  | 6.6329755                              | 5.2421369                                          |
| 7             | 1.3159318                      | .7599178                                  | 7.8982945                              | 6.0020547                                          |
| 8             | 1.3685691                      | .7306901                                  | 9.2142263                              | 6.7327448                                          |
| 9             | 1.4233118                      | .7025866                                  | 10.5827953                             | 7.4353314                                          |
| 10            | 1.4802443                      | .6755641                                  | 12.0061071                             | 8.1108955                                          |
| 11            | 1.5394541                      | .6495808                                  | 13.4863514                             | 8.7604763                                          |
| 12            | 1.6010322                      | .6245970                                  | 15.0258055                             | 9.3850733                                          |
| 13            | 1.6650735                      | .6005740                                  | 16.6268377                             | 9.9856473                                          |
| 14            | 1.7316764                      | .5774750                                  | 18.2919112                             | 10.5631223                                         |
| 15            | 1.8009435                      | .5552644                                  | 20.0235876                             | 11.1183868                                         |
| 16            | 1.8729812                      | .5339081                                  | 21.8245311                             | 11.6522949                                         |
| 17            | 1.9479005                      | .5133732                                  | 23.6975124                             | 12.1656680                                         |
| 18            | 2.0258165                      | .4936281                                  | 25.6454129                             | 12.6592961                                         |
| 19            | 2.1068492                      | .4746424                                  | 27.6712294                             | 13.1339385                                         |
| 20            | 2.1911231                      | .4563869                                  | 29.7780786                             | 13.5903253                                         |
| 21            | 2.2787681                      | .4388335                                  | 31.9592017                             | 14.0291589                                         |
| 22            | 2.3699188                      | .4219553                                  | 34.2479698                             | 14.4511142                                         |
| 23            | 2.4647155                      | .4057263                                  | 36.6178886                             | 14.8568405                                         |
| 24            | 2.5633042                      | .3901214                                  | 39.0826041                             | 15.2469619                                         |
| 25            | 2.6658363                      | .3751168                                  | 41.6459083                             | 15.6220787                                         |
| 26            | 2.7724698                      | .3606892                                  | 44.3117446                             | 15.9827678                                         |
| 27            | 2.8833686                      | .3468165                                  | 47.0842144                             | 16.3295844                                         |
| 28            | 2.9987033                      | .3334774                                  | 49.9675830                             | 16.6630618                                         |
| 29            | 3.1186515                      | .3206514                                  | 52.9662863                             | 16.9837132                                         |
| 30            | 3.2433975                      | .3083186                                  | 56.0849377                             | 17.2920318                                         |
| 31            | 3.3731334                      | .2964602                                  | 59.3283352                             | 17.5884921                                         |

COMPOUND INTEREST at 4 per Cent.

| <i>Years.</i> | TABLE V.<br>Continu'd.<br><i>The Amount<br/>of l. 1.</i> | TABLE VI.<br>Continu'd.<br><i>The present<br/>Worth of<br/>l. 1.</i> | TABLE VII.<br>Continu'd.<br><i>The Amount of<br/>l. 1. Annuity.</i> | TABLE VIII.<br>Continu'd.<br><i>The present<br/>Worth of l. 1.<br/>Annuity.</i> |
|---------------|----------------------------------------------------------|----------------------------------------------------------------------|---------------------------------------------------------------------|---------------------------------------------------------------------------------|
| 32            | 3.5080587                                                | .2850579                                                             | 62.7014687                                                          | 17.8735500                                                                      |
| 33            | 3.6483811                                                | .2740941                                                             | 66.2095274                                                          | 18.1476441                                                                      |
| 34            | 3.7943163                                                | .2635521                                                             | 69.8579045                                                          | 18.4111962                                                                      |
| 35            | 3.9460890                                                | .2534154                                                             | 73.6522248                                                          | 18.6646116                                                                      |
| 36            | 4.1039326                                                | .2436687                                                             | 77.5983138                                                          | 18.9082803                                                                      |
| 37            | 4.2680899                                                | .2342968                                                             | 81.7022464                                                          | 19.1425771                                                                      |
| 38            | 4.4388134                                                | .2252854                                                             | 85.9703362                                                          | 19.3678625                                                                      |
| 39            | 4.6163660                                                | .2166206                                                             | 90.4091497                                                          | 19.5844831                                                                      |
| 40            | 4.8010206                                                | .2082890                                                             | 95.0255157                                                          | 19.7927721                                                                      |
| 41            | 4.9930615                                                | .2002779                                                             | 99.8265363                                                          | 19.9930500                                                                      |
| 42            | 5.1927839                                                | .1925749                                                             | 104.8195978                                                         | 20.1856250                                                                      |
| 43            | 5.4004953                                                | .1851682                                                             | 110.0123817                                                         | 20.3707931                                                                      |
| 44            | 5.6165151                                                | .1780463                                                             | 115.4128769                                                         | 20.5488395                                                                      |
| 45            | 5.8411757                                                | .1711984                                                             | 121.0293920                                                         | 20.7200378                                                                      |
| 46            | 6.0748227                                                | .1646138                                                             | 126.8705677                                                         | 20.8846517                                                                      |
| 47            | 6.3178156                                                | .1582825                                                             | 132.9453904                                                         | 21.0429342                                                                      |
| 48            | 6.5705282                                                | .1521947                                                             | 139.2632060                                                         | 21.1951289                                                                      |
| 49            | 6.8333494                                                | .1463411                                                             | 145.8337342                                                         | 21.3414700                                                                      |
| 50            | 7.1066833                                                | .1407126                                                             | 152.6670836                                                         | 21.4821826                                                                      |
| 51            | 7.3909507                                                | .1353006                                                             | 159.7737670                                                         | 21.6174832                                                                      |
| 52            | 7.6865887                                                | .1300967                                                             | 167.1647176                                                         | 21.7475800                                                                      |
| 53            | 7.9940522                                                | .1250930                                                             | 174.8513063                                                         | 21.8726729                                                                      |
| 54            | 8.3138143                                                | .1202817                                                             | 182.8453586                                                         | 21.9929546                                                                      |
| 55            | 8.6463669                                                | .1156555                                                             | 191.1591729                                                         | 22.1086100                                                                      |
| 56            | 8.9922216                                                | .1112072                                                             | 199.8055398                                                         | 22.2198172                                                                      |
| 57            | 9.3519105                                                | .1069300                                                             | 208.7977614                                                         | 22.3267472                                                                      |
| 58            | 9.7259869                                                | .1028173                                                             | 218.1496719                                                         | 22.4295645                                                                      |
| 59            | 10.1150263                                               | .09888628                                                            | 227.8756588                                                         | 22.5284273                                                                      |
| 60            | 10.5196274                                               | .0950604                                                             | 237.9906851                                                         | 22.6234877                                                                      |
| 61            | 10.9404125                                               | .0914042                                                             | 248.5103125                                                         | 22.7148919                                                                      |

## COMPOUND INTEREST at 5 per Cent.

| Years. | TABLE IX.                      | TAB. X.                                   | TABLE XI.                               | TAB. XII.                                          |
|--------|--------------------------------|-------------------------------------------|-----------------------------------------|----------------------------------------------------|
|        | <i>The Amount<br/>of l. 1.</i> | <i>The present<br/>Worth of<br/>l. 1.</i> | <i>The Amount of<br/>l. 1. Annuity.</i> | <i>The present<br/>Worth of l. 1.<br/>Annuity.</i> |
| 1      | 1.05                           | .9523809                                  | 1.0                                     | .9523809                                           |
| 2      | 1.1025                         | .9070294                                  | 2.05                                    | 1.8594103                                          |
| 3      | 1.157625                       | .8638376                                  | 3.1525                                  | 2.7232480                                          |
| 4      | 1.2155063                      | .8227025                                  | 4.310125                                | 3.5459505                                          |
| 5      | 1.2762816                      | .7835262                                  | 5.5256312                               | 4.3294767                                          |
| 6      | 1.3400956                      | .7462154                                  | 6.8019128                               | 5.0756921                                          |
| 7      | 1.4071064                      | .7106813                                  | 8.1420084                               | 5.7863734                                          |
| 8      | 1.4774554                      | .6768394                                  | 9.5491089                               | 6.4632128                                          |
| 9      | 1.5513282                      | .6446089                                  | 11.0265643                              | 7.1078217                                          |
| 10     | 1.6288946                      | .6139133                                  | 12.5778925                              | 7.7217349                                          |
| 11     | 1.7103393                      | .5846929                                  | 14.2067871                              | 8.3064142                                          |
| 12     | 1.7958563                      | .5568374                                  | 15.9171265                              | 8.8632516                                          |
| 13     | 1.8856491                      | .5303213                                  | 17.7129828                              | 9.3935730                                          |
| 14     | 1.9799316                      | .5050680                                  | 19.5986320                              | 9.8986409                                          |
| 15     | 2.0789282                      | .4810171                                  | 21.5785636                              | 10.3796580                                         |
| 16     | 2.1828746                      | .4581115                                  | 23.6574918                              | 10.8377695                                         |
| 17     | 2.2920183                      | .4362967                                  | 25.8403664                              | 11.2740662                                         |
| 18     | 2.4066192                      | .4155207                                  | 28.1323847                              | 11.6895869                                         |
| 19     | 2.5269502                      | .3957340                                  | 30.5390039                              | 12.0853208                                         |
| 20     | 2.6532977                      | .3768895                                  | 33.0659541                              | 12.4622103                                         |
| 21     | 2.7859626                      | .3589424                                  | 35.7192518                              | 12.8211527                                         |
| 22     | 2.9252607                      | .3418499                                  | 38.5052144                              | 13.1630026                                         |
| 23     | 3.0715238                      | .3255713                                  | 41.4304751                              | 13.4885739                                         |
| 24     | 3.2251000                      | .3100679                                  | 44.5019989                              | 13.7986418                                         |
| 25     | 3.3863549                      | .2953028                                  | 47.7270988                              | 14.0939445                                         |
| 26     | 3.5556727                      | .2812407                                  | 51.1134537                              | 14.3751853                                         |
| 27     | 3.7334563                      | .2678483                                  | 54.6691264                              | 14.6430336                                         |
| 28     | 3.9201291                      | .2550936                                  | 58.4025827                              | 14.8981272                                         |
| 29     | 4.1161356                      | .2429463                                  | 62.3227119                              | 15.1410735                                         |
| 30     | 4.3219424                      | .2313774                                  | 66.4388474                              | 15.3724510                                         |
| 31     | 4.5380395                      | .2203595                                  | 70.7607898                              | 15.5928104                                         |



# COMPOUND INTEREST at 5 per Cent.

| <i>Years.</i> | TABLE IX.<br>Continu'd.<br><i>The Amount<br/>of l. 1.</i> | TABLE X.<br>Continu'd.<br><i>The present<br/>Worth of<br/>l. 1.</i> | TABLE XI.<br>Continu'd.<br><i>The Amount of<br/>l. 1. Annuity.</i> | TAB. XII.<br>Continu'd.<br><i>The present<br/>Worth of l. 1.<br/>Annuity.</i> |
|---------------|-----------------------------------------------------------|---------------------------------------------------------------------|--------------------------------------------------------------------|-------------------------------------------------------------------------------|
| 32            | 4.7649415                                                 | .2098562                                                            | 75.2988293                                                         | 15.8026766                                                                    |
| 33            | 5.0031885                                                 | .1998725                                                            | 80.0637708                                                         | 16.0025491                                                                    |
| 34            | 5.2533480                                                 | .1903548                                                            | 85.0669593                                                         | 16.1929039                                                                    |
| 35            | 5.5160154                                                 | .1812903                                                            | 90.3203073                                                         | 16.3741942                                                                    |
| 36            | 5.7918161                                                 | .1726574                                                            | 95.8363226                                                         | 16.5468516                                                                    |
| 37            | 6.0814069                                                 | .1644356                                                            | 101.6281387                                                        | 16.7112872                                                                    |
| 38            | 6.3854773                                                 | .1566054                                                            | 107.7095457                                                        | 16.8678926                                                                    |
| 39            | 6.7047511                                                 | .1491480                                                            | 114.0950229                                                        | 17.0170406                                                                    |
| 40            | 7.0399887                                                 | .1420457                                                            | 120.7997741                                                        | 17.1590862                                                                    |
| 41            | 7.3919881                                                 | .1352816                                                            | 127.8397628                                                        | 17.2943678                                                                    |
| 42            | 7.7615875                                                 | .1288396                                                            | 135.2317509                                                        | 17.4232074                                                                    |
| 43            | 8.1496669                                                 | .1227044                                                            | 142.9933385                                                        | 17.5459118                                                                    |
| 44            | 8.5571503                                                 | .1168613                                                            | 151.1430054                                                        | 17.6627732                                                                    |
| 45            | 8.9850078                                                 | .1112965                                                            | 159.7001556                                                        | 17.7740697                                                                    |
| 46            | 9.4342582                                                 | .1059967                                                            | 168.6851634                                                        | 17.8800663                                                                    |
| 47            | 9.9059711                                                 | .1009492                                                            | 178.1194216                                                        | 17.9810155                                                                    |
| 48            | 10.4012696                                                | .0961421                                                            | 188.0253927                                                        | 18.0771576                                                                    |
| 49            | 10.9213331                                                | .0915639                                                            | 198.4266623                                                        | 18.1687215                                                                    |
| 50            | 11.4674000                                                | .0872037                                                            | 209.3479954                                                        | 18.2559253                                                                    |
| 51            | 12.0407698                                                | .0830512                                                            | 220.8153952                                                        | 18.3389764                                                                    |
| 52            | 12.6428082                                                | .0790963                                                            | 232.8561649                                                        | 18.4180728                                                                    |
| 53            | 13.2749486                                                | .0753298                                                            | 245.4989731                                                        | 18.4934026                                                                    |
| 54            | 13.9386961                                                | .0717427                                                            | 258.7739218                                                        | 18.5651453                                                                    |
| 55            | 14.6356309                                                | .0683264                                                            | 272.7126179                                                        | 18.6334720                                                                    |
| 56            | 15.3674124                                                | .0650727                                                            | 287.3482488                                                        | 18.6985444                                                                    |
| 57            | 16.1357831                                                | .0619740                                                            | 302.7156612                                                        | 18.7605185                                                                    |
| 58            | 16.9425722                                                | .0590229                                                            | 318.8514442                                                        | 18.8195414                                                                    |
| 59            | 17.7897008                                                | .0562123                                                            | 335.7940164                                                        | 18.8757536                                                                    |
| 60            | 18.6791858                                                | .0535355                                                            | 353.5837172                                                        | 18.9292882                                                                    |
| 61            | 19.6131451                                                | .0509862                                                            | 372.2629031                                                        | 18.9802743                                                                    |

## COMPOUND INTEREST at 6 per Cent.

| <i>Years.</i> | TAB. XIII.<br><i>The Amount<br/>of l. i.</i> | TAB. XIV.<br><i>The present<br/>Worth of<br/>l. i.</i> | TABLE XV.<br><i>The Amount of<br/>l. i. Annuity.</i> | TABLE XVI.<br><i>The present<br/>Worth of l. i.<br/>Annuity.</i> |
|---------------|----------------------------------------------|--------------------------------------------------------|------------------------------------------------------|------------------------------------------------------------------|
| 1             | 1.06                                         | .9433962                                               | 1.0                                                  | .9433962                                                         |
| 2             | 1.1236                                       | .8899964                                               | 2.06                                                 | 1.8333926                                                        |
| 3             | 1.191016                                     | .8396193                                               | 3.1836                                               | 2.6730119                                                        |
| 4             | 1.2624769                                    | .7920937                                               | 4.3746016                                            | 3.4651056                                                        |
| 5             | 1.3382256                                    | .7472582                                               | 5.6370930                                            | 4.2123638                                                        |
| 6             | 1.4185191                                    | .7049605                                               | 6.9753187                                            | 4.9173244                                                        |
| 7             | 1.5036303                                    | .6650571                                               | 8.3938378                                            | 5.5823815                                                        |
| 8             | 1.5938481                                    | .6274124                                               | 9.8974681                                            | 6.2097939                                                        |
| 9             | 1.6894790                                    | .5918985                                               | 11.4913162                                           | 6.8016923                                                        |
| 10            | 1.7908477                                    | .5583948                                               | 13.1807958                                           | 7.3600871                                                        |
| 11            | 1.8982980                                    | .5267875                                               | 14.9716435                                           | 7.8868747                                                        |
| 12            | 2.0121965                                    | .4969694                                               | 16.8699420                                           | 8.3838440                                                        |
| 13            | 2.1329283                                    | .4688390                                               | 18.8821385                                           | 8.8526831                                                        |
| 14            | 2.2609039                                    | .4423010                                               | 21.0150667                                           | 9.2949840                                                        |
| 15            | 2.3965582                                    | .4172651                                               | 23.2759707                                           | 9.7122491                                                        |
| 16            | 2.5402517                                    | .3936463                                               | 25.6725189                                           | 10.1058953                                                       |
| 17            | 2.6927728                                    | .3713644                                               | 28.2128806                                           | 10.4772597                                                       |
| 18            | 2.8543392                                    | .3503438                                               | 30.9056534                                           | 10.8276035                                                       |
| 19            | 3.0255995                                    | .3305130                                               | 33.7599925                                           | 11.1581165                                                       |
| 20            | 3.2071355                                    | .3118047                                               | 36.7855920                                           | 11.4699213                                                       |
| 21            | 3.3995636                                    | .2941554                                               | 39.9927275                                           | 11.7640767                                                       |
| 22            | 3.6035374                                    | .2775051                                               | 43.3922911                                           | 12.0415818                                                       |
| 23            | 3.8197497                                    | .2617973                                               | 46.9958285                                           | 12.3033790                                                       |
| 24            | 4.0489346                                    | .2469786                                               | 50.8155782                                           | 12.5503576                                                       |
| 25            | 4.2918707                                    | .2329986                                               | 54.8645128                                           | 12.7833562                                                       |
| 26            | 4.5493829                                    | .2198100                                               | 59.1563835                                           | 13.0031663                                                       |
| 27            | 4.8223459                                    | .2073680                                               | 63.7057664                                           | 13.2105342                                                       |
| 28            | 5.1116866                                    | .1956301                                               | 68.5281123                                           | 13.4061644                                                       |
| 29            | 5.4183878                                    | .1845567                                               | 73.6397990                                           | 13.5907211                                                       |
| 30            | 6.7434911                                    | .1741101                                               | 79.0581868                                           | 13.7648312                                                       |
| 31            | 6.0881006                                    | .1642548                                               | 84.8016779                                           | 13.9290861                                                       |

# COMPOUND INTEREST at 6 per Cent.

| <i>Years.</i> | TAB. XIII.<br>Continu'd.<br><i>The Amount<br/>of l. 1.</i> | TAB. XIV.<br>Continu'd.<br><i>The present<br/>Worth of<br/>l. 1.</i> | TAB. XV.<br>Continu'd.<br><i>The Amount of<br/>l. 1. Annuity.</i> | TAB. XVI.<br>Continu'd.<br><i>The present<br/>Worth of l. 1.<br/>Annuity.</i> |
|---------------|------------------------------------------------------------|----------------------------------------------------------------------|-------------------------------------------------------------------|-------------------------------------------------------------------------------|
| 32            | 6.4533866                                                  | .1549574                                                             | 90.8897785                                                        | 14.0840435                                                                    |
| 33            | 6.8405898                                                  | .1461862                                                             | 97.3431652                                                        | 14.2302297                                                                    |
| 34            | 7.2510252                                                  | .1379115                                                             | 104.1837550                                                       | 14.3681412                                                                    |
| 35            | 7.6860867                                                  | .1301052                                                             | 111.4347802                                                       | 14.4982465                                                                    |
| 36            | 8.1472519                                                  | .1227407                                                             | 119.1208669                                                       | 14.6209872                                                                    |
| 37            | 8.6360870                                                  | .1157932                                                             | 127.2681188                                                       | 14.7367804                                                                    |
| 38            | 9.1542523                                                  | .1092388                                                             | 135.9042059                                                       | 14.8460192                                                                    |
| 39            | 9.7035074                                                  | .1030555                                                             | 145.0584581                                                       | 14.9490747                                                                    |
| 40            | 10.2857178                                                 | .0972222                                                             | 154.7619655                                                       | 15.0462969                                                                    |
| 41            | 10.9028609                                                 | .0917190                                                             | 165.0476833                                                       | 15.1380160                                                                    |
| 42            | 11.5570326                                                 | .0865274                                                             | 175.9505442                                                       | 15.2245434                                                                    |
| 43            | 12.2504545                                                 | .0816296                                                             | 187.5075769                                                       | 15.3061730                                                                    |
| 44            | 12.9854818                                                 | .0770091                                                             | 199.7580314                                                       | 15.3831821                                                                    |
| 45            | 13.7646107                                                 | .0726500                                                             | 212.7435132                                                       | 15.4558321                                                                    |
| 46            | 14.5904873                                                 | .0685378                                                             | 226.5081239                                                       | 15.5243699                                                                    |
| 47            | 15.4659166                                                 | .0646583                                                             | 241.0986112                                                       | 15.5890282                                                                    |
| 48            | 16.3938716                                                 | .0609984                                                             | 256.5645278                                                       | 15.6500266                                                                    |
| 49            | 17.3775039                                                 | .0575457                                                             | 272.9583994                                                       | 15.7075723                                                                    |
| 50            | 18.4201541                                                 | .0542884                                                             | 290.3359032                                                       | 15.7618610                                                                    |
| 51            | 19.5253634                                                 | .0512154                                                             | 308.7560573                                                       | 15.8130761                                                                    |
| 52            | 20.6968852                                                 | .0483164                                                             | 328.2814207                                                       | 15.8613925                                                                    |
| 53            | 21.9386983                                                 | .0455816                                                             | 348.9783059                                                       | 15.9069741                                                                    |
| 54            | 23.2550202                                                 | .0430015                                                             | 370.9170041                                                       | 15.9499760                                                                    |
| 55            | 24.6503214                                                 | .0405674                                                             | 394.1720243                                                       | 15.9905430                                                                    |
| 56            | 26.1293406                                                 | .0382712                                                             | 418.8223456                                                       | 16.0288141                                                                    |
| 57            | 27.6971011                                                 | .0361049                                                             | 444.9516863                                                       | 16.0649190                                                                    |
| 58            | 29.3589272                                                 | .0340612                                                             | 472.6487874                                                       | 16.0989802                                                                    |
| 59            | 31.1204628                                                 | .0321332                                                             | 502.0077145                                                       | 16.1311134                                                                    |
| 60            | 32.9876905                                                 | .0303143                                                             | 533.1281773                                                       | 16.1614277                                                                    |
| 61            | 34.9669520                                                 | .0285984                                                             | 566.1158679                                                       | 16.1900261                                                                    |



## COMPOUND INTEREST at 8 per Cent.

| <i>Years.</i> | TAB. XVII.                     | TAB. XVIII.                               | TABLE XIX.                              | TABLE XX.                                          |
|---------------|--------------------------------|-------------------------------------------|-----------------------------------------|----------------------------------------------------|
|               | <i>The Amount<br/>of l. i.</i> | <i>The present<br/>Worth of<br/>l. i.</i> | <i>The Amount of<br/>l. i. Annuity.</i> | <i>The present<br/>Worth of l. i.<br/>Annuity.</i> |
| 1             | 1.08                           | .9259259                                  | 1.0                                     | .9259259                                           |
| 2             | 1.1664                         | .8573388                                  | 2.08                                    | 1.7832648                                          |
| 3             | 1.259712                       | .7938224                                  | 3.2464                                  | 2.5770979                                          |
| 4             | 1.3604890                      | .7350299                                  | 4.506112                                | 3.3121268                                          |
| 5             | 1.4693281                      | .6805832                                  | 5.866010                                | 3.9927100                                          |
| 6             | 1.5868743                      | .6301696                                  | 7.3359290                               | 4.6228797                                          |
| 7             | 1.7138243                      | .5834904                                  | 8.9228034                               | 5.2063701                                          |
| 8             | 1.8509302                      | .5402689                                  | 10.6366276                              | 5.7466389                                          |
| 9             | 1.9990046                      | .5002490                                  | 12.4875579                              | 6.2468879                                          |
| 10            | 2.1589250                      | .4631935                                  | 14.4865626                              | 6.7100814                                          |
| 11            | 2.3316390                      | .4288829                                  | 16.6454876                              | 7.1389643                                          |
| 12            | 2.5181701                      | .3971138                                  | 18.9771266                              | 7.5360780                                          |
| 13            | 2.7196237                      | .3676979                                  | 21.4952967                              | 7.9037759                                          |
| 14            | 2.9371936                      | .3404610                                  | 24.2149204                              | 8.2442370                                          |
| 15            | 3.1721691                      | .3152417                                  | 27.1521140                              | 8.5594790                                          |
| 16            | 3.4259426                      | .2918905                                  | 30.3242831                              | 8.8513691                                          |
| 17            | 3.7000181                      | .2702689                                  | 33.7502258                              | 9.1216381                                          |
| 18            | 3.9960195                      | .2502490                                  | 37.4502438                              | 9.3718871                                          |
| 19            | 4.3157011                      | .2317121                                  | 41.4462633                              | 9.6035992                                          |
| 20            | 4.6609571                      | .2145482                                  | 45.7619644                              | 9.8181474                                          |
| 21            | 5.0338337                      | .1986557                                  | 50.4229215                              | 10.0168031                                         |
| 22            | 5.4365404                      | .1839405                                  | 55.4567552                              | 10.2007436                                         |
| 23            | 5.8714636                      | .1703153                                  | 60.8932956                              | 10.3710589                                         |
| 24            | 6.3411807                      | .1576993                                  | 66.7647593                              | 10.5287582                                         |
| 25            | 6.8484752                      | .1460179                                  | 73.1059400                              | 10.6747761                                         |
| 26            | 7.3963532                      | .1352018                                  | 79.9544152                              | 10.8099779                                         |
| 27            | 7.9880615                      | .1251868                                  | 87.3507684                              | 10.9351647                                         |
| 28            | 8.6271064                      | .1159137                                  | 95.3388299                              | 11.0510784                                         |
| 29            | 9.3172749                      | .1073275                                  | 103.9659363                             | 11.1584059                                         |
| 30            | 10.0626569                     | .0993773                                  | 113.2832112                             | 11.2577833                                         |
| 31            | 10.8676694                     | .0920160                                  | 123.3458680                             | 11.3497993                                         |

## COMPOUND INTEREST at 8 per Cent.

| <i>Years.</i> | TAB. XVII.<br>Continu'd.<br><i>The Amount<br/>of l. i.</i> | TAB. XVIII.<br>Continu'd.<br><i>The present<br/>Worth of<br/>l. i.</i> | TAB. XIX.<br>Continu'd.<br><i>The Amount of<br/>l. i. Annuity.</i> | TAB. XX.<br>Continu'd.<br><i>The present<br/>Worth of l. i.<br/>Annuity.</i> |
|---------------|------------------------------------------------------------|------------------------------------------------------------------------|--------------------------------------------------------------------|------------------------------------------------------------------------------|
| 32            | 11.7370830                                                 | .0852000                                                               | 134.2135375                                                        | 11.4349993                                                                   |
| 33            | 12.6760496                                                 | .0788889                                                               | 145.9506205                                                        | 11.5138883                                                                   |
| 34            | 13.6901336                                                 | .0730453                                                               | 158.6266701                                                        | 11.5869326                                                                   |
| 35            | 14.7853443                                                 | .0676345                                                               | 172.3168037                                                        | 11.6545681                                                                   |
| 36            | 15.9681718                                                 | .0626246                                                               | 187.1021480                                                        | 11.7171927                                                                   |
| 37            | 17.2456255                                                 | .0579857                                                               | 203.0703198                                                        | 11.7751784                                                                   |
| 38            | 18.6252756                                                 | .0536905                                                               | 220.3159454                                                        | 11.8288689                                                                   |
| 39            | 20.1152976                                                 | .0497134                                                               | 238.9412209                                                        | 11.8785823                                                                   |
| 40            | 21.7245214                                                 | .0460309                                                               | 259.0565186                                                        | 11.9246132                                                                   |
| 41            | 23.4624832                                                 | .0426212                                                               | 280.7810400                                                        | 11.9672344                                                                   |
| 42            | 25.3394818                                                 | .0394641                                                               | 304.2435232                                                        | 12.0066985                                                                   |
| 43            | 27.3666404                                                 | .0365408                                                               | 329.5830050                                                        | 12.0432394                                                                   |
| 44            | 29.5559716                                                 | .0338341                                                               | 356.9496454                                                        | 12.0770735                                                                   |
| 45            | 31.9204493                                                 | .0313279                                                               | 386.5056169                                                        | 12.1084014                                                                   |
| 46            | 34.4740853                                                 | .0290073                                                               | 418.4260663                                                        | 12.1374087                                                                   |
| 47            | 37.2320121                                                 | .0268586                                                               | 452.9001515                                                        | 12.1642673                                                                   |
| 48            | 40.2105730                                                 | .0248691                                                               | 490.1321636                                                        | 12.1891363                                                                   |
| 49            | 43.4274189                                                 | .0230269                                                               | 530.3427367                                                        | 12.2121633                                                                   |
| 50            | 46.9016124                                                 | .0213212                                                               | 573.7701556                                                        | 12.2334845                                                                   |
| 51            | 50.6537414                                                 | .0197419                                                               | 620.6717680                                                        | 12.2532264                                                                   |
| 52            | 54.7060407                                                 | .0182795                                                               | 671.3255094                                                        | 12.2715059                                                                   |
| 53            | 59.0825240                                                 | .0169255                                                               | 726.0315501                                                        | 12.2884314                                                                   |
| 54            | 63.8091259                                                 | .0156717                                                               | 785.1140741                                                        | 12.3041031                                                                   |
| 55            | 68.9138560                                                 | .0145109                                                               | 848.9232000                                                        | 12.3186140                                                                   |
| 56            | 74.4269644                                                 | .0134360                                                               | 917.8370559                                                        | 12.3320500                                                                   |
| 57            | 80.3811216                                                 | .0124408                                                               | 992.2640203                                                        | 12.3444908                                                                   |
| 58            | 86.8116113                                                 | .0115192                                                               | 1072.6451419                                                       | 12.3560100                                                                   |
| 59            | 93.7565402                                                 | .0106660                                                               | 1159.4567532                                                       | 12.3666760                                                                   |
| 60            | 101.2570634                                                | .0098759                                                               | 1253.2132934                                                       | 12.3765518                                                                   |
| 61            | 109.3576285                                                | .0091443                                                               | 1354.4703569                                                       | 12.3856962                                                                   |

## COMPOUND INTEREST at 10 per Cent.

| Years. | TAB. XXI.<br>The Amount<br>of l. 1. | TAB. XXII.<br>The present<br>Worth of<br>l. 1. | TAB. XXIII.<br>The Amount of<br>l. 1 Annuity. | TAB. XXIV.<br>The present<br>Worth of l. 1.<br>Annuity. |
|--------|-------------------------------------|------------------------------------------------|-----------------------------------------------|---------------------------------------------------------|
|        |                                     |                                                |                                               |                                                         |
| 1      | 1.1                                 | .9090909                                       | 1.                                            | .9090909                                                |
| 2      | 1.21                                | .8264463                                       | 2.1                                           | 1.7355372                                               |
| 3      | 1.331                               | .7513148                                       | 3.31                                          | 2.4868520                                               |
| 4      | 1.4641                              | .6830135                                       | 4.641                                         | 3.1698654                                               |
| 5      | 1.61051                             | .6209213                                       | 6.1051                                        | 3.7907868                                               |
| 6      | 1.771561                            | .5644739                                       | 7.71561                                       | 4.3552607                                               |
| 7      | 1.9487171                           | .5131581                                       | 9.487171                                      | 4.8684188                                               |
| 8      | 2.1435888                           | .4665074                                       | 11.4358881                                    | 5.3349262                                               |
| 9      | 2.3579477                           | .4240976                                       | 13.5794769                                    | 5.7590238                                               |
| 10     | 2.5937425                           | .3855433                                       | 15.9374246                                    | 6.1445671                                               |
| 11     | 2.8531167                           | .3504939                                       | 18.5311671                                    | 6.4950610                                               |
| 12     | 3.1384234                           | .3186308                                       | 21.3842838                                    | 6.8136918                                               |
| 13     | 3.4522712                           | .2896644                                       | 24.5227121                                    | 7.1033562                                               |
| 14     | 3.7974983                           | .2633313                                       | 27.9749834                                    | 7.3666875                                               |
| 15     | 4.1772482                           | .2393920                                       | 31.7724817                                    | 7.6060795                                               |
| 16     | 4.5949730                           | .2176291                                       | 35.9497299                                    | 7.8237086                                               |
| 17     | 5.0544703                           | .1978446                                       | 40.5447028                                    | 8.0215533                                               |
| 18     | 5.5599173                           | .1798588                                       | 45.5991731                                    | 8.2014121                                               |
| 19     | 6.1159090                           | .1635080                                       | 51.1590904                                    | 8.3649201                                               |
| 20     | 6.7274999                           | .1486436                                       | 57.2749995                                    | 8.5135637                                               |
| 21     | 7.4002499                           | .1351306                                       | 64.0924994                                    | 8.6486943                                               |
| 22     | 8.1402749                           | .1228460                                       | 71.4027494                                    | 8.7715403                                               |
| 23     | 8.9543024                           | .1116782                                       | 79.5430243                                    | 8.8832184                                               |
| 24     | 9.8497327                           | .1015256                                       | 88.4973268                                    | 8.9847440                                               |
| 25     | 10.8347059                          | .0922960                                       | 98.3470594                                    | 9.0770400                                               |
| 26     | 11.9181765                          | .0839055                                       | 109.1817654                                   | 9.1609455                                               |
| 27     | 13.1099942                          | .0762777                                       | 121.0999419                                   | 9.2372232                                               |
| 28     | 14.4209936                          | .0693433                                       | 134.2099361                                   | 9.3065665                                               |
| 29     | 15.8630930                          | .0630394                                       | 148.6409297                                   | 9.3696059                                               |
| 30     | 17.449423                           | .0573086                                       | 164.4940227                                   | 9.4269145                                               |
| 31     | 19.1943425                          | .0520987                                       | 181.9434250                                   | 9.4790132                                               |



## COMPOUND INTEREST at 10 per Cent.

| Years. | TABLE XXI.<br>Continu'd.       | TAB. XXII.<br>Continu'd.                  | TAB. XXIII.<br>Continu'd.               | TAB. XXIV.<br>Continu'd.                           |
|--------|--------------------------------|-------------------------------------------|-----------------------------------------|----------------------------------------------------|
|        | <i>The Amount<br/>of l. 1.</i> | <i>The present<br/>Worth of<br/>l. 1.</i> | <i>The Amount of<br/>l. 1. Annuity.</i> | <i>The present<br/>Worth of l. 1.<br/>Annuity.</i> |
| 32     | 21.1137757                     | .0473624                                  | 201.1377674                             | 9.5263755                                          |
| 33     | 23.2251544                     | .0430568                                  | 222.2515442                             | 9.5694324                                          |
| 34     | 25.5476699                     | .0391425                                  | 245.4766986                             | 9.6085749                                          |
| 35     | 28.1024368                     | .0355841                                  | 271.0243685                             | 9.6441590                                          |
| 36     | 30.9126805                     | .0323492                                  | 299.1268053                             | 9.6765082                                          |
| 37     | 34.0039486                     | .0294083                                  | 330.0394858                             | 9.7059165                                          |
| 38     | 37.4043434                     | .0267349                                  | 364.0434344                             | 9.7326514                                          |
| 39     | 41.1447778                     | .0243044                                  | 401.4477779                             | 9.7569558                                          |
| 40     | 45.2592556                     | .0220949                                  | 442.5925557                             | 9.7790507                                          |
| 41     | 49.7851811                     | .0200863                                  | 487.8518112                             | 9.7991370                                          |
| 42     | 54.7636992                     | .0182603                                  | 537.6369924                             | 9.8173973                                          |
| 43     | 60.2400692                     | .0166002                                  | 592.4006916                             | 9.8339975                                          |
| 44     | 66.2640761                     | .0150911                                  | 652.6407608                             | 9.8490887                                          |
| 45     | 72.8904837                     | .0137192                                  | 718.9048368                             | 9.8628079                                          |
| 46     | 80.1795321                     | .0124720                                  | 791.7953205                             | 9.8752799                                          |
| 47     | 88.1974853                     | .0113382                                  | 871.9748526                             | 9.8866181                                          |
| 48     | 97.0172338                     | .0103074                                  | 960.1723378                             | 9.8969255                                          |
| 49     | 106.7189572                    | .0093704                                  | 1057.1895716                            | 9.9062959                                          |
| 50     | 117.3908529                    | .0085186                                  | 1163.9085288                            | 9.9148145                                          |
| 51     | 129.1299382                    | .0077441                                  | 1281.2993810                            | 9.9225586                                          |
| 52     | 142.0429320                    | .0070401                                  | 1410.4293198                            | 9.9295987                                          |
| 53     | 156.2472252                    | .0064001                                  | 1552.4722518                            | 9.9359989                                          |
| 54     | 171.8719477                    | .0058183                                  | 1708.7194769                            | 9.9418171                                          |
| 55     | 189.0591425                    | .0052893                                  | 1880.5914247                            | 9.9471065                                          |
| 56     | 207.9650567                    | .0048085                                  | 2069.6505671                            | 9.9519150                                          |
| 57     | 228.7615624                    | .0043714                                  | 2277.6156238                            | 9.9562864                                          |
| 58     | 251.6377186                    | .0039740                                  | 2506.3771862                            | 9.9602603                                          |
| 59     | 276.8014905                    | .0036127                                  | 2758.0149048                            | 9.9638730                                          |
| 60     | 304.4816395                    | .0032843                                  | 3034.8163953                            | 9.9671573                                          |
| 61     | 334.9298035                    | .0029857                                  | 3339.2980349                            | 9.9701430                                          |

I have designed these Rates thus: The Columns of 3, 4 or 5 *per Cent.* for Incomes of Lands, &c. that are certain; That Table at 6 *per Cent.* for Houses new or in good Repair, Brick; That of 8 *per Cent.* for the Purchase, &c. of Houses that are pretty old; That of 10 *per Cent.* for very old Houses, or not in very good Repair. And they may be used as every one's Discretion directs, according to the foregoing Rules. And as to this Table of Lives, if the Years Purchase be multiplied by the Annuity, the Product shews the Value of such Estate for such Life. Examples follow.

| <i>The Value of a Life at the several Ages thereof.</i> |                        |             |                        |             |                        |
|---------------------------------------------------------|------------------------|-------------|------------------------|-------------|------------------------|
| <i>Age.</i>                                             | <i>Years Purchase.</i> | <i>Age.</i> | <i>Years Purchase.</i> | <i>Age.</i> | <i>Years Purchase.</i> |
| 1                                                       | 10.28                  | 25          | 12.27                  | 50          | 9.21                   |
| 5                                                       | 13.4                   | 30          | 11.72                  | 55          | 8.51                   |
| 10                                                      | 13.44                  | 35          | 11.12                  | 60          | 7.60                   |
| 15                                                      | 13.33                  | 40          | 10.57                  | 65          | 6.54                   |
| 20                                                      | 12.78                  | 45          | 9.91                   | 70          | 5.32                   |

*The Use of these Tables of Compound Interest.*

*Prop. 1.]* To find the Amount of any Sum of Money for any Number of Years. This is done by the first Column towards the left hand; so the Amount of *l. 50* in 50 Years Compound Interest allow'd at 5 *per Cent.* is *l. 573:7:5*; found by multiplying the Tabular Number against 50 Years by *l. 50* thus.

$$\begin{array}{r} 11.4674091 \\ 50 \\ \hline \text{L. } 573.3704550 \end{array}$$

*Prop. 2.]* An Office is worth *l. 500*: What may be paid in present Money to enter upon it 15 Years hence? This is done by the 2d Column at 5 *per Cent.* by multiplying the present Worth of *l. 1.* due 15 Years hence by 500, as in the Margin, where the Answer is *l. 240:10:2*.

$$\begin{array}{r} .4810171 \\ 500 \\ \hline \text{L. } 240.5085500 \end{array}$$

*Prop. 3.]* Annuity of 70 *l. per Ann.* is forborn 33 Years, what is the Improvement, Compound Interest being

being allow'd the Owner at 5 per Cent? By multiplying the Amount of *l. 1 per Ann.* in 33 Years by the Annuity *l. 70*, you have the Answer *l. 5604 : 9 : 4 ½*.

*Prop. 4.]* There are 13 Years to come in the Lease of an House that is pretty old, but in good Repair, and is *l. 50 per Ann.* clear; what is a farther Lease for 31 Years worth in present Money, at the Rate of 8 per Cent?

I add the 31 Years Reversion to the 13 *in esse* makes 44.

The present Worth of *l. 1* due 44 Years hence, is *l. 12.0770735*; from which deduct the present Worth for 13 Years, and the Rest is multiplied by 50 for Answer: *All by the Table of 8 per Cent.*

*Years.*  
*l. 1. per Ann. 33 = 80.0638328*  
*70*

*l. 5604.4682960*

*Years.*  
*l. 1 per Ann. 44 = l. 12.0770735*  
*l. 1 — — 13 = l. 7.9037759*

*Rests = l. 4.1732976*  
*50*

*Answ. l. 208.66488*

And if you multiply any of the Numbers in the 4th Column (according as the Estate is in Land or Houses, as above said) by the Annuity you would know the present Worth of, the Product shews the present Value of such Annuity for any Years required under 62.

*To find the Present Worth of Estates for Lives.*

*Prop. 5.]* To find the Value of an Estate of *l. 35 per Ann.* in Land for a Life of 45 Years. By the little Table above, as computed by the Learned Dr Halley, that Life is Years Purchase—9.91 Which being multiplied by the Annuity 35, the Product is *l. 346 : 17 : 00 = the Answer.*

*Prop. 6.]* It many times happens, that a Life and so many Years certain to come, are proposed in the same Question; therefore it becomes necessary, as in Computations for two Lives also, to reduce the Years Purchase of any Life into Years certain to come, or Years in a Lease: which is done thus.

I find a Life of 50 worth 9.21 Years Purchase. Now if you turn to any of the four Columns in the Tables above, which shew the present Worth of Annuities according to the several Rates, you will find at 5 per Cent. the next to the said 9.21 is 9.393573, against which in the Column under Years is 13 Years to come. At 6 per Cent. the next to 9.21 is 9.294984, against which under Years are 14 Years. At 8 per Cent. are 9.1216381, against which under Years are

*Prod. l. 346.85*



are 17 Years Lease. And at 10 per Cent. stands 9.2372232, (as next to the said 9.21) and against that are 27 Years Lease.

*Example.* What is the Value of an House of l. 96 per Ann. clear Rent for 23 Years after the Death of a Person 60 Yearsold?

The Age being found = 12 Years to come; therefore by the Table of 8 per Cent. Column 4. the Answer, as under Prop. 4. is found to be as per Margin, l. 395.37504 = l. 395 : 7 : 6.

|               |                     |                         |                 |
|---------------|---------------------|-------------------------|-----------------|
|               |                     |                         | <i>Years to</i> |
| <i>Years.</i> | <i>Years Purch.</i> | <i>Col. 4. 8 per C.</i> | <i>come.</i>    |
| 60 —          | 7.60 —              | or 7.536078             | or = 12         |

|                                              |   |               |           |
|----------------------------------------------|---|---------------|-----------|
| <i>The pres. Worth of</i><br>l. 1 per Ann. — | } | <i>Years.</i> | <i>l.</i> |
|                                              |   | for 35 =      | 11.654568 |
|                                              |   | for 12 =      | 7.536078  |

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of l. 1 Reversion = 4.11849  
96

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Anfw. = 395.37504

*Prop. 7.] In Computations of the Value of Estates for two Lives; as suppose 50 and 55. First, find by the little Table of the Value of Lives the Years Purchase that the elder Life is worth; then find in the fourth Column at 5 per Cent. (supposing the Purchase to be a clear certain Income) the 8.51 found in the said little Table, or the next to it, which is = 8.3064142; right against which, in the Column of Years, is 11 Years: to which add the Difference in the Ages (5) and the Sum is 16 Years; against which, in the 4th Column from the Years, is 10.8377695, the Value of l. 1 per Ann. for the said two Lives: which multiplied by the Annuity, gives the Answer or Value thereof for the said two Lives, near enough the Truth. Farther Directions might be given on this head; but I have not room to enlarge on things so uncertain, and which at best depend on many casual Circumstances.*

A T A B L E

A TABLE shewing by Inspection the Fines payable for any Number of Years lapsed or expired in a Church or College Lease of their Lands, to make up such Lease 21 Years.

| Annual<br>Rent<br>of Lands. | 1 Year lapsed  |    |       | 2 Years lapsed |       |       | 3 Years lapsed |        |       | 4 Years lapsed |        |       | 5 Years lapsed |        |       | 6 Years lapsed |        |       |
|-----------------------------|----------------|----|-------|----------------|-------|-------|----------------|--------|-------|----------------|--------|-------|----------------|--------|-------|----------------|--------|-------|
|                             | Fines Payable. |    |       | Fines Payable. |       |       | Fines Payable. |        |       | Fines Payable. |        |       | Fines Payable. |        |       | Fines Payable. |        |       |
|                             | l.             | s. | d. q. | l.             | s.    | d. q. | l.             | s.     | d. q. | l.             | s.     | d. q. | l.             | s.     | d. q. | l.             | s.     | d. q. |
| 1                           |                | 2  | 0 0   |                | 4     | 3 0   |                | 6      | 9 0   |                | 9      | 6 2   |                | 12     | 7 3   |                | 16     | 1 2   |
| 2                           |                | 4  | 0 1   |                | 8     | 6 0   |                | 13     | 6 0   |                | 19     | 1 0   |                | 1 5    | 3 3   |                | 1 12   | 3 0   |
| 3                           |                | 6  | 0 1   |                | 12    | 9 0   |                | 1 0    | 3 1   |                | 1 8    | 7 2   |                | 1 17   | 11 2  |                | 2 8    | 4 2   |
| 4                           |                | 8  | 0 2   |                | 17    | 0 0   |                | 1 7    | 9 1   |                | 1 18   | 2 0   |                | 2 10   | 7 2   |                | 3 4    | 6 0   |
| 5                           |                | 10 | 0 2   |                | 1 1   | 3 1   |                | 1 13   | 9 1   |                | 2 7    | 8 3   |                | 3 3    | 3 1   |                | 4 0    | 7 2   |
| 6                           |                | 12 | 0 3   |                | 1 5   | 6 1   |                | 2 0    | 6 1   |                | 2 17   | 3 1   |                | 3 15   | 11 1  |                | 4 16   | 9 1   |
| 7                           |                | 14 | 1 0   |                | 1 9   | 9 1   |                | 2 7    | 3 1   |                | 3 6    | 9 3   |                | 4 8    | 7 0   |                | 5 12   | 10 3  |
| 8                           |                | 16 | 1 0   |                | 1 14  | 0 1   |                | 2 14   | 0 1   |                | 3 16   | 4 1   |                | 5 1    | 3 0   |                | 6 9    | 0 1   |
| 9                           |                | 18 | 1 1   |                | 1 18  | 3 1   |                | 3 0    | 9 2   |                | 4 5    | 10 3  |                | 5 13   | 10 3  |                | 7 5    | 1 3   |
| 10                          | 1              | 0  | 1 1   |                | 2 2   | 6 1   |                | 3 7    | 6 2   |                | 4 15   | 1 1   |                | 6 6    | 6 3   |                | 8 1    | 3 1   |
| 20                          | 2              | 0  | 2 2   |                | 4 5   | 0 3   |                | 6 15   | 1 0   |                | 9 10   | 10 2  |                | 12 13  | 1 2   |                | 16 2   | 6 3   |
| 30                          | 3              | 0  | 3 3   |                | 6 7   | 7 0   |                | 10 2   | 7 2   |                | 14 6   | 4 0   |                | 18 19  | 8 1   |                | 24 3   | 10 0  |
| 40                          | 4              | 0  | 5 0   |                | 8 10  | 1 1   |                | 13 10  | 2 0   |                | 19 1   | 9 1   |                | 25 6   | 3 0   |                | 32 5   | 1 1   |
| 50                          | 5              | 0  | 6 1   |                | 10 12 | 7 3   |                | 16 17  | 8 2   |                | 23 17  | 2 2   |                | 31 12  | 9 3   |                | 40 6   | 4 3   |
| 60                          | 6              | 0  | 7 2   |                | 12 15 | 2 0   |                | 20 5   | 3 0   |                | 28 12  | 7 3   |                | 37 19  | 4 2   |                | 48 7   | 8 0   |
| 70                          | 7              | 0  | 8 3   |                | 14 17 | 8 1   |                | 23 12  | 9 1   |                | 33 8   | 1 1   |                | 44 5   | 11 1  |                | 56 8   | 11 1  |
| 80                          | 8              | 0  | 10 0  |                | 17 0  | 2 3   |                | 27 0   | 3 3   |                | 38 3   | 6 2   |                | 50 12  | 6 0   |                | 64 10  | 2 3   |
| 90                          | 9              | 0  | 11 1  |                | 19 2  | 9 0   |                | 30 7   | 10 1  |                | 42 18  | 11 3  |                | 56 19  | 0 3   |                | 72 11  | 6 0   |
| 100                         | 10             | 1  | 0 2   |                | 21 5  | 3 2   |                | 33 15  | 4 3   |                | 47 14  | 5 1   |                | 63 5   | 7 2   |                | 80 12  | 9 1   |
| 200                         | 20             | 2  | 0 1   |                | 42 10 | 7 0   |                | 67 10  | 9 3   |                | 95 8   | 10 1  |                | 126 11 | 3 0   |                | 161 5  | 7 0   |
| 300                         | 30             | 3  | 1 2   |                | 63 15 | 10 2  |                | 101 6  | 2 2   |                | 143 3  | 3 1   |                | 189 16 | 10 2  |                | 241 18 | 4 1   |
| 400                         | 40             | 4  | 2 0   |                | 85 1  | 1 2   |                | 135 1  | 7 2   |                | 190 17 | 8 2   |                | 253 2  | 6 0   |                | 322 11 | 1 2   |
| 500                         | 50             | 5  | 2 2   |                | 106 6 | 5 0   |                | 168 17 | 0 0   |                | 238 12 | 1 1   |                | 316 8  | 1 1   |                | 403 3  | 11 0  |

*A TABLE shewing by Inspection the Fines payable for any Number of Years lapsed or expired in a Church or College Lease, &c.*

| Annual<br>Rent<br>of Lands. | 7 Years laps.      | 8 Years laps.      | 9 Years laps.      | 10 Years laps.     | 11 Years laps.     |
|-----------------------------|--------------------|--------------------|--------------------|--------------------|--------------------|
|                             | Fines<br>Payable.  | Fines<br>Payable.  | Fines<br>Payable.  | Fines<br>Payable.  | Fines<br>Payable.  |
| <i>l.</i>                   | <i>l. s. d. q.</i> | <i>l. s. d. q.</i> | <i>l. s. d. q.</i> | <i>l. s. d. q.</i> | <i>l. s. d. q.</i> |
| 1                           | 1                  | 1 4 4 0            | 1 9 1 3            | 1 14 6 1           | 2 0 6 0            |
| 2                           | 2                  | 2 8 7 3            | 2 18 3 1           | 3 9 1 0            | 4 1 0 1            |
| 3                           | 3                  | 3 12 11 2          | 4 7 5 0            | 5 3 6 2            | 6 1 6 2            |
| 4                           | 4                  | 4 17 3 2           | 5 16 6 2           | 6 18 0 3           | 8 2 0 2            |
| 5                           | 5                  | 6 1 7 1            | 7 5 8 1            | 8 12 6 3           | 10 2 6 3           |
| 6                           | 6                  | 7 5 11 0           | 8 14 10 0          | 10 7 1 0           | 12 3 0 3           |
| 7                           | 7                  | 8 10 3 0           | 10 3 11 2          | 12 1 7 1           | 14 3 7 0           |
| 8                           | 8                  | 9 14 7 0           | 11 13 1 1          | 13 16 1 2          | 16 4 1 1           |
| 9                           | 9                  | 10 18 10 2         | 13 2 2 3           | 15 10 7 2          | 18 4 7 1           |
| 10                          | 10                 | 12 3 2 2           | 14 11 4 2          | 17 5 1 3           | 20 5 1 2           |
| 20                          | 20                 | 24 6 5 0           | 29 2 9 0           | 34 10 3 2          | 40 10 2 3          |
| 30                          | 30                 | 36 9 7 1           | 43 14 1 2          | 51 15 5 1          | 60 15 4 1          |
| 40                          | 40                 | 48 12 9 3          | 58 5 6 0           | 69 0 7 0           | 81 0 5 3           |
| 50                          | 50                 | 60 16 0 0          | 72 16 10 2         | 86 5 8 3           | 101 5 7 0          |
| 60                          | 60                 | 72 19 2 2          | 87 8 3 0           | 103 10 10 2        | 121 10 8 2         |
| 70                          | 70                 | 85 2 5 0           | 101 19 7 2         | 120 16 0 1         | 141 15 10 0        |
| 80                          | 80                 | 97 5 7 1           | 116 11 0 0         | 138 1 2 0          | 162 0 11 1         |
| 90                          | 90                 | 109 8 9 3          | 131 2 4 3          | 155 6 3 3          | 182 6 0 3          |
| 100                         | 100                | 121 12 0 0         | 145 13 9 0         | 172 11 5 2         | 202 11 2 1         |
| 200                         | 200                | 243 4 0 0          | 291 7 6 2          | 345 2 11 1         | 405 2 4 2          |
| 300                         | 300                | 364 16 0 0         | 437 1 3 2          | 517 14 4 3         | 607 13 6 3         |
| 400                         | 400                | 486 8 0 0          | 582 15 0 2         | 690 5 10 1         | 810 4 9 0          |
| 500                         | 500                | 608 0 0 0          | 728 8 10 0         | 862 17 3 3         | 1012 15 11 1       |



*A TABLE shewing by Inspection the Fines payable for any Number of Years lapsed or expired in a Church or College Lease, &c.*

| <i>Annual Rent of Lands.</i> | <i>12 Years lapsed.</i> |           |           |           | <i>13 Years lapsed.</i> |           |           |           | <i>14 Years lapsed.</i> |           |           |           | <i>15 Years lapsed.</i> |           |           |           | <i>16 Years lapsed.</i> |           |           |           |
|------------------------------|-------------------------|-----------|-----------|-----------|-------------------------|-----------|-----------|-----------|-------------------------|-----------|-----------|-----------|-------------------------|-----------|-----------|-----------|-------------------------|-----------|-----------|-----------|
|                              | <i>Fines Payable.</i>   |           |           |           | <i>Fines Payable.</i>   |           |           |           | <i>Fines Payable.</i>   |           |           |           | <i>Fines Payable.</i>   |           |           |           | <i>Fines Payable.</i>   |           |           |           |
| <i>l.</i>                    | <i>l.</i>               | <i>s.</i> | <i>d.</i> | <i>q.</i> | <i>l.</i>               | <i>s.</i> | <i>d.</i> | <i>q.</i> | <i>l.</i>               | <i>s.</i> | <i>d.</i> | <i>q.</i> | <i>l.</i>               | <i>s.</i> | <i>d.</i> | <i>q.</i> | <i>l.</i>               | <i>s.</i> | <i>d.</i> | <i>q.</i> |
| 1                            | 2                       | 7         | 2         | 2         | 2                       | 14        | 8         | 0         | 3                       | 3         | 0         | 0         | 3                       | 12        | 3         | 2         | 4                       | 2         | 7         | 3         |
| 2                            | 4                       | 14        | 4         | 3         | 5                       | 9         | 4         | 0         | 6                       | 5         | 11        | 3         | 7                       | 4         | 6         | 3         | 8                       | 5         | 3         | 2         |
| 3                            | 7                       | 1         | 7         | 1         | 8                       | 4         | 0         | 0         | 9                       | 8         | 11        | 3         | 10                      | 16        | 10        | 1         | 12                      | 7         | 11        | 1         |
| 4                            | 9                       | 8         | 9         | 3         | 10                      | 18        | 8         | 0         | 12                      | 11        | 11        | 3         | 14                      | 9         | 1         | 3         | 16                      | 10        | 7         | 1         |
| 5                            | 11                      | 16        | 0         | 0         | 13                      | 13        | 4         | 0         | 15                      | 14        | 11        | 2         | 18                      | 1         | 5         | 0         | 20                      | 13        | 3         | 0         |
| 6                            | 14                      | 3         | 2         | 2         | 16                      | 8         | 0         | 0         | 18                      | 17        | 11        | 2         | 21                      | 13        | 8         | 2         | 24                      | 15        | 10        | 3         |
| 7                            | 16                      | 10        | 5         | 0         | 19                      | 2         | 8         | 0         | 22                      | 0         | 11        | 2         | 25                      | 5         | 11        | 3         | 28                      | 18        | 6         | 2         |
| 8                            | 18                      | 17        | 7         | 1         | 21                      | 17        | 4         | 0         | 25                      | 3         | 11        | 1         | 28                      | 18        | 0         | 3         | 33                      | 1         | 2         | 1         |
| 9                            | 21                      | 4         | 9         | 3         | 24                      | 12        | 0         | 0         | 28                      | 6         | 11        | 1         | 32                      | 10        | 4         | 1         | 37                      | 3         | 10        | 0         |
| 10                           | 23                      | 12        | 0         | 1         | 27                      | 6         | 8         | 0         | 31                      | 9         | 11        | 1         | 36                      | 2         | 10        | 0         | 41                      | 6         | 5         | 3         |
| 20                           | 47                      | 4         | 0         | 2         | 54                      | 13        | 4         | 0         | 62                      | 19        | 10        | 1         | 72                      | 5         | 8         | 0         | 82                      | 12        | 11        | 3         |
| 30                           | 70                      | 16        | 0         | 3         | 82                      | 0         | 0         | 0         | 94                      | 9         | 9         | 2         | 108                     | 8         | 6         | 1         | 123                     | 19        | 5         | 3         |
| 40                           | 94                      | 8         | 1         | 0         | 109                     | 6         | 8         | 0         | 125                     | 19        | 8         | 3         | 144                     | 11        | 4         | 1         | 165                     | 5         | 11        | 2         |
| 50                           | 118                     | 0         | 1         | 1         | 136                     | 13        | 4         | 0         | 157                     | 9         | 8         | 0         | 180                     | 14        | 2         | 1         | 206                     | 12        | 5         | 2         |
| 60                           | 141                     | 12        | 1         | 2         | 164                     | 0         | 0         | 0         | 188                     | 19        | 7         | 1         | 216                     | 17        | 0         | 1         | 247                     | 18        | 11        | 1         |
| 70                           | 165                     | 4         | 1         | 3         | 191                     | 6         | 8         | 0         | 220                     | 9         | 6         | 2         | 252                     | 19        | 10        | 1         | 289                     | 5         | 5         | 1         |
| 80                           | 188                     | 16        | 2         | 0         | 218                     | 13        | 4         | 0         | 251                     | 19        | 5         | 3         | 289                     | 0         | 8         | 2         | 330                     | 11        | 11        | 0         |
| 90                           | 212                     | 8         | 2         | 1         | 246                     | 0         | 0         | 0         | 283                     | 9         | 4         | 3         | 325                     | 3         | 6         | 2         | 371                     | 18        | 5         | 0         |
| 100                          | 236                     | 0         | 2         | 3         | 273                     | 6         | 8         | 0         | 314                     | 19        | 4         | 1         | 361                     | 8         | 4         | 2         | 413                     | 4         | 10        | 3         |
| 200                          | 472                     | 0         | 5         | 2         | 546                     | 13        | 3         | 0         | 629                     | 18        | 8         | 1         | 722                     | 16        | 9         | 1         | 826                     | 9         | 10        | 0         |
| 300                          | 708                     | 0         | 8         | 1         | 819                     | 19        | 10        | 2         | 944                     | 18        | 0         | 1         | 1084                    | 5         | 1         | 3         | 1239                    | 14        | 8         | 2         |
| 400                          | 944                     | 0         | 11        | 0         | 1093                    | 6         | 6         | 0         | 1259                    | 17        | 4         | 2         | 1445                    | 13        | 6         | 2         | 1652                    | 19        | 7         | 1         |
| 500                          | 1180                    | 1         | 1         | 1         | 1366                    | 13        | 1         | 1         | 1574                    | 16        | 8         | 2         | 1807                    | 1         | 11        | 0         | 2066                    | 4         | 6         | 1         |

*A TABLE shewing by Inspection the Fines payable for any Number of Years lapsed or expired in a Church or College Lease, &c.*

| <i>Annual<br/>Rent of<br/>Lands.</i> | <i>17 Years lapsed.</i> | <i>18 Years lapsed.</i> | <i>19 Years lapsed.</i> | <i>20 Years lapsed.</i> | <i>21 Years lapsed.</i> |
|--------------------------------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|
| <i>Fines Payable.</i>                | <i>Fines Payable.</i>   | <i>Fines Payable.</i>   | <i>Fines Payable.</i>   | <i>Fines Payable.</i>   | <i>Fines Payable.</i>   |
| <i>l.</i>                            | <i>l. s. d. q.</i>      | <i>l. s. d. q.</i>      | <i>l. s. d. q.</i>      | <i>l. s. d. q.</i>      | <i>l. s. d. q.</i>      |
| 1                                    | 4 14 2 2                | 5 7 1 2                 | 6 1 6 1                 | 6 17 6 0                | 7 15 6 0                |
| 2                                    | 9 8 5 0                 | 10 14 2 3               | 12 3 0 1                | 13 14 11 3              | 15 11 0 0               |
| 3                                    | 14 2 7 3                | 16 1 4 1                | 18 4 6 2                | 20 12 5 3               | 23 6 6 0                |
| 4                                    | 18 16 10 1              | 21 8 5 2                | 24 6 0 3                | 27 9 11 3               | 31 2 0 0                |
| 5                                    | 23 11 0 3               | 26 15 7 0               | 30 7 6 3                | 34 7 5 3                | 38 17 6 0               |
| 6                                    | 28 5 3 2                | 32 2 8 2                | 36 9 1 0                | 41 4 11 2               | 46 13 0 0               |
| 7                                    | 32 19 6 0               | 37 9 9 3                | 42 10 7 1               | 48 2 5 2                | 54 8 6 0                |
| 8                                    | 37 13 8 2               | 42 16 11 1              | 48 12 1 1               | 54 19 11 2              | 62 4 0 0                |
| 9                                    | 42 7 11 0               | 48 4 0 2                | 54 13 7 2               | 61 17 5 2               | 69 19 6 0               |
| 10                                   | 47 2 1 3                | 53 11 2 0               | 60 15 1 3               | 68 14 11 1              | 77 15 0 0               |
| 20                                   | 94 4 3 1                | 107 2 4 0               | 121 10 3 1              | 137 9 10 3              | 155 10 0 1              |
| 30                                   | 141 6 5 0               | 160 13 6 0              | 182 5 5 0               | 206 4 10 0              | 233 5 0 2               |
| 40                                   | 188 8 6 3               | 214 4 8 1               | 243 0 6 3               | 274 19 9 2              | 311 0 0 3               |
| 50                                   | 235 10 8 1              | 267 15 10 1             | 303 15 8 1              | 343 14 8 3              | 388 15 0 3              |
| 60                                   | 282 12 10 0             | 321 7 0 1               | 364 10 10 0             | 412 9 8 0               | 466 10 1 0              |
| 70                                   | 329 14 11 2             | 374 18 2 1              | 425 5 11 2              | 481 4 7 2               | 544 5 1 1               |
| 80                                   | 376 17 1 1              | 428 9 4 1               | 486 1 1 1               | 549 19 6 3              | 622 0 1 1               |
| 90                                   | 423 19 3 0              | 482 0 6 1               | 546 16 3 0              | 618 14 6 1              | 699 15 1 2              |
| 100                                  | 471 1 4 2               | 535 11 8 2              | 607 11 4 2              | 687 9 5 2               | 777 10 1 3              |
| 200                                  | 942 2 9 1               | 1071 3 5 0              | 1215 2 9 1              | 1374 18 11 1            | 1555 0 3 2              |
| 300                                  | 1413 4 1 3              | 1606 15 1 1             | 1822 14 1 3             | 2062 8 4 3              | 2332 10 5 1             |
| 400                                  | 1884 5 6 2              | 2142 6 10 0             | 2430 5 6 2              | 2749 17 10 1            | 3110 0 7 0              |
| 500                                  | 2355 6 11 0             | 2677 18 6 1             | 3037 16 11 0            | 3437 7 3 3              | 3887 10 8 3             |

*The*

*The Use of this Table.*

*Example 1.]* What Fine is to be paid, to make up a Lease 21 Years, when 9 Years of 21 are lapsed (or let slip) supposing the Rent *l. 147 per Annum*?

|                                                            | <i>per Ann.</i> | <i>Fine.</i>   |
|------------------------------------------------------------|-----------------|----------------|
| In the Table under 9 Years lapsed against <i>l. 100</i> is | <i>l. 145</i>   | <i>13 : 9</i>  |
| against 40 —                                               | 58              | <i>5 : 6</i>   |
| 7 —                                                        | 10              | <i>3 : 11½</i> |

There are two Books that afford us a Table of this kind: The first was written by the Reverend *John Newton*, D. D. one of the King's Chaplains, Anno 1668. which contains only Fines, or Value of 1 *l. per Ann.* Rent, all in Decimals. The other is said to be approved of by a very great Author, (who perhaps never saw it) which is very tedious in its Use; as I shall shew by the Example above, which is performed by my Table: To find the Fine payable to make up 9 Years lapsed 21 Years; Rent 147 *l. per Ann.* his Answer is 1 : 1 : 2 : 5 Purchase, taken out of one Table; which he values by another thus:

| <i>per Ann.</i>            | <i>l.</i> | <i>s.</i> | <i>d.</i> |  | <i>Sums.</i> |
|----------------------------|-----------|-----------|-----------|--|--------------|
| <i>l. 147</i> for 1 Year   | =         | 147       | : 00 : 00 |  |              |
| 100 for 1 Quarter          | =         | 25        | : 00 : 00 |  |              |
| 2 Months                   | =         | 16        | : 13 : 4  |  |              |
| 5 Tenths of a Mon.         | =         | 4         | : 3 : 4   |  |              |
| <i>l. 40</i> for 1 Quarter | — —       | 10        | : 00 : 00 |  |              |
| 2 Months                   | — —       | 6         | : 13 : 4  |  |              |
| 5 Tenths of a Mon.         | =         | 1         | : 13 : 4  |  |              |
| <i>l. 7</i> for 1 Quarter  | — —       | 1         | : 15 : 00 |  |              |
| 2 Months                   | — —       | 1         | : 3 : 4   |  |              |
| 5 Tenths of a Mon.         | =         | 00        | : 5 : 10  |  |              |

}

*l. 147.*

*l. 214 : 7 : 6*

But 'tis plain the Use  
of my Table (as above)  
is much more brief,  
and will be allowed  
more accurate.

*Example 2.]* To make up a Lease of 14 Years any Number of Years less than 21: As suppose I would add 5 Years to 14 in a Lease of *l. 40 per Ann.* take 5 from 7 there rests 2; then take the Fine payable for 2 Years lapsed = *l. 8 : 10 : 1 : 1* from the Annual Rent, (here 40) and the Remainder is = *l. 31 : 9 : 10 : 3* = the Sum to be paid to add 5 Years.



*Example 3.]* To find the Value of a Church or College full Lease of Lands of 21 Years: as suppose *l. 80 per Ann.* The last Column in the Table against *l. 80* gives you *l. 622 : 00 : 1 : 1*, the Answer.

I need not tell the Reader, That in case of Improvement by the Seller, he may advance the Rent to the Buyers in proportion.

SECT. IX. *To Extract the Square and Cube Roots of Decimals and Mixt Numbers.*

1. *For the Square Root.* What is the Square Root of .000976541?

To what is said under the Extraction of the Square Root for Intire Numbers, it is only necessary to add for Decimals, that you begin to point over Seconds place of the Decimal given, and so proceed over every other towards the right hand: and if the Decimal Places given are not an even Number, you must make them so, by adding a Cypher towards the right hand.

.0009765410 (.03124 = Root.

6) 76

62) 1554

624) 31010

6034 refts.

2. That for every Decimal Place you would have in the Root, you must (by adding Cyphers towards the right hand, if need be) make twice so many Decimal Places in that given to have its Root extracted.

There are several Ways of extracting the Roots, but I chuse that natural one, according to the Algebraic Canon, as in whole Numbers foregoing.

*Secondly, For the Cube Root.*

*Example 1.] Of a Mixt Number.* What is the Cube Root of 32.934168093464? Answer 3.2054.

In these Cases you must point over the Thirds place of the Decimal, &c. over every Third toward the right hand adding Cyphers, if the Point fall not over the place next the right hand.

*Example 2.]* What is the Cube Root of 91? See the Work.

In Examples of this kind, where the whole Number is not a compleat Cube, you must add so many times 3 Cyphers towards the right hand, as you intend to have Decimal Places in the Root:

as

as here are 6 Cyphers for the 2 Decimal Places in the Root. And if you desire more, it is but putting 3 Cyphers towards the right hand of the Remainder 481151, and proceed to make that Root as exact as you please.

3. If you have the Square or Cube Roots of a Vulgar Fraction to extract, you may reduce it to a Decimal, and then proceed as in the Example above.

4. And if the Decimals have Cyphers before next the Point, keep to the Rule under the last Example of the Cube Root above for the Points over: as .014672, point thus .014672; .0014672, point thus, .001467200; and .00014672 point thus, .000146720, &c. and extract as in Whole Numbers: For in this Case of the Cube of Decimals, their Places must always be, or be made, 3, 6, or 9, &c. by putting Cyphers toward the right hand of the Decimal whose Root is required.

Having already shewn how to extract the Square, Cube, &c. Roots of Intire Numbers, and Fractions Vulgar and Decimal; I shall in the Use of Logarithms give a much more easy and short Way of Extraction, and also shew the Use of Logarithms in resolving Questions of Interest: on both which accounts, Logarithms (among many other of their Uses) are extremely to be valued.

*Here ends DECIMAL ARITHMETIC.*

CHAP.

91.000000 (4.49

27000

12

48

492

64

192

192

21184

5816000

132

5808

58212

729

10692

52272

5334849

481151 refts.

## CHAP. IV.

## DUODECIMAL ARITHMETIC.

**I. NUMERATION.]** I have shew'd under *Duodecimals*, in the Definition of Numbers, *Chap. I.* what these Fractions are; as in  $1:1:1:1:1$ . This is read 1 Foot, 1 Prime (or 12th of a Foot) 1 Second (or 12th of a Prime) 1 Third (or 12th of a Second) 1 Fourth (or 12 of a Third) &c. which is sufficient to know how to read or write Duodecimals. I shall proceed to

**II. Addition.]** This is no more of consequence to what is above, but to divide the Sum of each Series by 12, putting down the Remainder, and carrying the Quotes to the next. But that next the left hand is done as all other things that have but one Denomination. The Example makes it clear.

| Feet. | '    | "    | ''' | ''' | ''' |
|-------|------|------|-----|-----|-----|
| 321   | : 11 | : 10 | : 9 | : 8 | : 7 |
| 210   | : 10 | : 9  | : 8 | : 7 | : 6 |
| 109   | : 9  | : 8  | : 7 | : 6 | : 5 |
| 98    | : 8  | : 7  | : 6 | : 5 | : 4 |
| 87    | : 7  | : 6  | : 5 | : 4 | : 3 |
| 76    | : 6  | : 5  | : 4 | : 3 | : 2 |
| 65    | : 5  | : 4  | : 3 | : 2 | : 1 |

**III. Subtraction of Duodecimals.]** A Joiner having lined several Rooms with Cedar, as above; finds the Deductions for the Apertures, as Chimnies, Windows, Beauvets, Arches, Vistoes, &c. to be — — —

Sum 971 : 0 : 4 : 9 : 1 : 4

89 : 9 : 8 : 7 : 6 : 5

How many Feet must he be paid for?

Subtracting the lesser from the greater, adding 12 where the upper is too little, or 10 in that to the left the Remainder is the Answer — —

881 : 2 : 8 : 1 : 6 : 11

**IV. Multiplication.]** This is the principal Rule to be insisted on in this kind of Arithmetic, as being much the ofteneft in use, and consequently the most to be observed.

*Example*



*Example 1.]* Multiply 9 F. 10' by 8 : 8' Feet. <sup>1</sup>  
 8 : 8' Feet. See the Work in the Margin: Where I say 8 times 10 Primes is 80 or 6' to carry, and 8'' to be put down; then 8 times 9 is 72, and 6 is 78 Primes, which divided mentally by 12 is 6 F. 6' which I put down as you see. Then I multiply 9 : 10' by the 8 F, saying 8 times 10 is 80 Primes, which is 8' and 6 Foot to carry; then 8 times 9 is 72, and 6 carried is 78 : the Sum of which 2 Lines or Products is 85 : 2' : 8'' = the Answer.

$$\begin{array}{r}
 \text{Feet. } 1 \\
 9 : 10 \\
 8 : 8 \\
 \hline
 6 : 6 : 8 \\
 78 : 8 \\
 \hline
 \text{Foot } 85 : 2 : 8 = \text{Answ.}
 \end{array}$$

By which Example you may perceive the Necessity of having in your mind the Product of any 2 Numbers under 13, or not exceeding 12, by 12, as in the Multiplication Table.

*Example 2.]* Multiply 16 : 10 : 6 by 9 : 3. See the Operation. Here 'tis plain I multiply, 1st, the Seconds, then the Primes, and then the Integers by the 3, carrying the Quote of each Product divided by 12 to that towards the left hand; placing the Remainder in each Division under the Degrees of the Factors, as the Example indicateth.

$$\begin{array}{r}
 \text{F. } ' \text{ ''} \\
 16 : 10 : 6 \\
 9 : 3 : 0 \\
 \hline
 4 : 2 : 7 : 6 \\
 151 : 10 : 6
 \end{array}$$

Prod. 156 : 1 : 1 : 6

*Example 3.]* Multiply 372 : 11' : 4'' by 25 : 6' : 3''. I have inserted the whole Work. As, 1st, I multiply 4'' by 3 produceth 12, so I carry 1; 11' by 3 is 33, and 1 is 34, I put down 10 and carry 2; then I multiply 372 by the 3, adding the 2 makes 1118'', all which I put down as you see. Then I multiply 372 : 11 : 4 by 6', and it produceth 2237' : 8'' : 0''', as you see.

|                   |      | I    | II   | III | IIII |
|-------------------|------|------|------|-----|------|
|                   | 372  | 11   | 4    |     |      |
|                   | 25   | 6    | 3    |     |      |
| 1st Product       |      |      | 1118 | 10  |      |
| 2d Product        | 2237 |      | 8    |     |      |
| 3d Product        | 283  |      | 4    |     |      |
| Sum               | 2520 | 1130 | 10   |     |      |
| Quote             | 94   | 12   |      |     |      |
| Sum               | 2614 |      |      |     |      |
| Quote             | 217  | 10   |      |     |      |
| Prod. 372 by 25 = | 9300 |      |      |     |      |
| Add.              |      |      |      |     |      |
| Prod. or Answ. =  | 9517 | 10   | 2    | 10  |      |

And this I take to be the most easy and natural Way of working, when the Integers are large Numbers.

3dly.

3dly, I multiply  $372 : 11 : 4$  by  $25$  : as 4 times 25 is 100, or  $81 : 4''$  ; put 4 down, and carry 8 ; 11 times 5 is 55, and 8 is 63', put 3 down, and carry 6 ; 11 times 2 is 22', and 6 is 28, or  $283' : 4''$ .

4thly, I sum up those 3 Lines makes  $2520' : 1130'' : 10'''$ .

5thly, I divide the 1130 Seconds mentally by 12, produceth  $94'$ , which makes  $2614' : 2''$ .

6thly, I divide, as before,  $2614'$  by 12, produceth 217 Integers, and  $10'$  remains.

7thly, I multiply the 372 by the 25 at once, as taught in Decimals, and the Product is 9300: So the Sum or Answer is 9517 Integers  $10' : 2'' : 10''' : 0''''$ .

*Notes to be observed as appears by the abovesaid Examples.*

If you multiply 2ds by 2ds, the Product is 4ths ; Primes by 2ds place gives 3ds, Primes by Primes gives Seconds, and Integers by Primes gives Primes, and by 2ds gives 2ds, &c. And accordingly it is proper to place them Degrees towards the right hand, that so the Degrees of each part of the Operation may fall under the like in the given Duodecimals.

So also 2 Places given in order, as Integers and Primes, to be multiplied by 2 Places or Denominations, produce 3 Places ; 3 Places by 2 produce 4 Places ; and 3 Places by 3 produce 5 Denominations, or Degrees, in the Product ; *i.e.* 1 less than the Sum of the Exponents or Places, or Names in both the Factors given.

V. Division of Duodecimals.] Divide 156 F.

$1' : 1'' : 6''' : 9''''$  by 9.

See the Operation :

Where the rest is al-

ways reduced into the

next Denomination or

Degree, and the re-

spective Figures of each

Degree added.

F.

$9 (156 : 1 : 1 : 6 : 9 (17 : 4 : 1 : 6 : 1 = \text{Quote.}$

66

37

13

54

9

There is rarely occasion ('tis to be supposed) to divide by a Number of more than 1 Degree or Name : yet in case there should,

should, it may be done as in the Margin, where  $85 : 2' : 8''$  is divided by  $8 : 8'$ . Where I say, the Eights in 85 are 9 times, which put in the Quote, and say 9 times  $8'$  is 72, (which is 6 to carry, and put down 0) 9 times 8 Integers is 72, and 6 is 78,

which put down, and deducted, the Remainder is  $7 : 2'$  or  $86'$ ; to which bring down the  $8''$ , and say 8 in 86 is 10 times; 10 times  $8'$  is 80, or  $6' : 8$ , put the 8 down, and carry the 6, saying 10 times 8 is 80, and 6 is 86, which deducted, 0 remains: which proves the Truth of the first Example in Multiplication.

And by the same Rules the Example in the Margin, or any other, is performed. But it may be observed, that in this Division more than one Place of Figures may, and often must, be put in the Quote at once.

$$\begin{array}{r} F. \quad ' \quad ' \quad '' \quad ' \\ 8 : 8) 85 : 2 : 8 (9 : 10 = \text{the Quote.} \\ \underline{78 : 0} \end{array}$$

$$\begin{array}{r} 7 : 2 \\ 86 : 8 \\ \underline{86 : 8} \end{array}$$

0 refts.

$$\begin{array}{r} ' \quad ' \quad '' \quad '' \quad ' \quad '' \\ 9 : 3) 156 : 1 : 1 : 6 (16 : 10 : 6 \\ \underline{148 : 0} \quad = \text{Quote.} \end{array}$$

$$\begin{array}{r} 8 : 1 \\ 97 : 1 \\ \underline{92 : 6} \end{array}$$

$$\begin{array}{r} 4 : 7 \\ 55 : 6 \\ \underline{55 : 6} \end{array}$$

0 refts.



## CHAP. V.

## SEXAGESIMAL ARITHMETIC.

**T**H E S E Fractions, or 60th Parts, are by some called Astronomicals, because used in the Mensuration of Time and Motion in Astronomy: as  $1^h : 1' : 1'' : 1''' : 1''''$ , that is, 1 Hour, 1 Minute (or 60 part of an Hour) 1 Second (or 60th part of a Minute) 1 Third (or 60th part of a Second) 1 Fourth (or 60th of a Third) &c. of Time. And in Motion or Measure in the Heavens, Earth, or Sea,  $1^\circ : 1' : 1''$  is 1 Degree (or 60, but according to some 'tis 69 *English* Miles upon the Terrestrial Globe)  $1'$  is 1 Minute (or 60th part of a Degree)  $1''$  is 1 Second (or 60th of a Minute)  $1'''$  is the 60th of a Second, &c. See *Sexagesimal Numbers*, at the beginning of *Chap. I.*

*Addition* is only to sum up each Column, and divide them by 60, carrying the Quotes, &c. And *Subtraction* is so easy, that neither of them need an Example. But

*In Multiplication*

I think the best way is to work as *per* Margin; as being a methodical and easy Way, which does not charge the Memory: and I am sure 'tis more brief than any Way I have seen. The Rule is plain; for having multiplied the Denominations one in another, and placed the whole Products as you see, I add them; and find the Sum as under the second black Line.

Then I begin at the right hand to divide by 60, and find  $520''''$  to be  $8''' : 40''''$ , placed as you see; and so I proceed with the rest, not making a Digit more than what is down.

*Example*

$$\begin{array}{r} h. \quad ' \quad '' \\ 57 : 29 : 52 \\ 6 : 12 : 10 \end{array} \left. \vphantom{\begin{array}{r} h. \quad ' \quad '' \\ 57 : 29 : 52 \\ 6 : 12 : 10 \end{array}} \right\} \text{Multiply.}$$

---


$$\begin{array}{r} 570 : 290 : 520 \\ 684 : 348 : 624 \\ 342 : 174 : 312 \end{array}$$


---

$$\begin{array}{r} 342 : 858 : 1230 : 914 : 520 \\ 14 : 20 : 15 : 8 \end{array}$$


---

$$\text{Answer} = 356 : 38 : 45 : 22 : 40$$

Example 2.]  $75^b : 33' : 52'' : 27'''$  by  $32 : 35' : 24'' : 42'''$ . See the Operation just in the same Method with the last Example above.

|                                 |      |      |      |       |        |         |            |
|---------------------------------|------|------|------|-------|--------|---------|------------|
| Multiply. }                     | $b.$ | $'$  | $''$ | $'''$ | $''''$ | $'''''$ | $''''''$   |
|                                 | 75   | 33   | 52   | 27    |        |         |            |
| Multiplication of Sexagesimals. | 32   | 35   | 24   | 42    |        |         |            |
|                                 |      |      |      | 3150  | 1386   | 2184    | 1134       |
|                                 |      |      | 1800 | 792   | 1248   | 648     |            |
|                                 |      | 2625 | 1155 | 1820  | 945    |         |            |
|                                 | 2400 | 1056 | 1664 | 864   |        |         |            |
| Sums Totals                     | 2400 | 3681 | 4619 | 6626  | 3579   | 2832    | 1134       |
| Quotes                          | = 62 | 78   | 111  | 60    | 47     | 18      |            |
| Product =                       | 2462 | 39   | 50   | 26    | 26     | 30      | 54 Answer. |

Here it may be noted, that I multiply  $27'''$ ,  $52''$ , &c. by  $42'''$ , without making 2 Lines (as is taught in Multiplication of Decimals).

2dly, I begin at  $1134''''''$  to divide the Line of Totals by 60, (or all except Units place by 6) so the Quote is 18, and Remainder 54; then I divide  $\frac{2832}{18}$  by 60, and 47 is the Quotient, and 30 the Remainder, &c.

I shall give the Reader the Proof of this by Division, for a Conclusion, viz.

$32 : 35 : 24 : 42$   $2462 : 39 : 50 : 26 : 26 : 30 : 54$  ( $75 : 33 : 52 : 27 =$  Quote.  
 $2444 : 15 : 52 : 30 =$  the Divisor multiplied in the 75 in the Quote,  
 (and divide by 60.

Division of Sexagesimals.

$18 : 23 : 57 : 56$  the Difference between the 4 first places in the  
 (two last Lines.  
 $1103 : 57 : 56 : 26$  ditto with the Integ. reduc'd to Primes, &c.  
 $1075 : 28 : 35 : 6$  the Divisor multiplied in  $33'$ .

$28 : 29 : 21 : 20$  the Difference between the two last.  
 $1709 : 21 : 20 : 30$  ditto with the  $28'$  reduced to  $''$ , &c.  
 $1694 : 41 : 24 : 24$  the Divisor multip. in  $52''$  in the Quote.

$14 : 39 : 56 : 6$  the Difference between the two last.  
 $879 : 56 : 6 : 54$  ditto with the  $14''$  reduced to  $'''$ .  
 $879 : 56 : 6 : 54$  the Divisor multip. in  $27$  in Quote.

o Remains.

## C H A P. VI.

## POLITICAL ARITHMETIC.

**T**HIS Specie of Arithmetic has nothing new in it, as to the Nature of the Numbers themselves, nor as to the Manner of Operation; but only in the Application or Subject about which the Numbers are employ'd, which is Political (or relating to Polity or Government): As to discover the Riches and Strength of a Nation, by estimating its Income from Land, Trade, Commerce, Art and Labour, Exports, Imports, its Number of People, Males, Females, Fighting-Men, &c.

And there have been several Pieces written on this Subject, as Mr *Grant's* Observations on the Bills of Mortality of *London* in 1676, done at the request of the Royal Society; Sir *William Petty's* Political Arithmetic in 1687, and his Book of the Use of Duplicate Proportion; the Learned Dr *Halley's* Observations on the Bills of Mortality of *Breslaw* in 1692, whereby he computes the Value of Annuities for Lives at different Ages thereof, &c. as mentioned a little farther; and Dr *Davenant* and *Gregory King* Esq; on Revenues and Trade, &c. I shall give some Examples of this Way of Computing: As

*Prop. 1.* To find the Number of People in *England*, and how many are Males, Females, and Fighting-Men.

First, I find the Number of Houses within the Weekly Bills of *London*, by considering the Number of those Families who pay to the Poor in each Parish, and of the Poor themselves; or by the Poll-Books, King's-Tax-Books, &c. Which suppose I find 108000 (as they are thereabout) these at 5 in each Family, one with another, makes 540000 People in the said Compass. Now admit that by a Poll-Tax, &c. it has been found that the Heads in *London* are to those in all the rest of *England* as 1 to 9; there must at this rate be 4860000 Souls in *England*. And it having been found by many Years.



Years Observation upon the Weekly Bills, that there are 14 Males to 13 Females; so that of the said 4860000 Persons, there must be 2520000 Males, and 2340000 Females: for

27. 14 :: 4860000. 2520000. Or 27. 13 :: 4860000. 2340000

And hence to find the Fighting-Men, it has been computed, That 37.4 of 100 Souls are above 16, and under 60; therefore

100. 37.4 :: 2520000. 942480 = the Fighting-Men.

*Prop. 2.* To find the Number of People by the Coals imported: I find, suppose, by the Coal-Meeters Books 404200 Chaldron imported in one Year, *comm. Ann.* It may easily, by considering the Medium of the several Classes of Housekeepers, be found that each House, one with another, burneth about  $3\frac{1}{2}$  Chaldron *per Ann.* So that by Division I find 107786 Houses, which is but 214 short of the Number of Houses in the Weekly Bills, found as in the last Proportion.

*Prop. 3.* To find the Number of Houses within the Bills, by the Yards Square of the Ground. I find (suppose, as *per Mr Grant*) 54 Houses in every hundred Yards Square, and 2000 such Squares built upon within the Bills of Mortality; which multiplied together gives 108000 Houses, as *per* the first Method.

From these and such-like Reasonings by Political Arithmetic, the Vitality *per Cent.* of several Ages of Persons is found.

As that 64 of 100 born are alive at 6 Years old.

40 of 100 ——— at 16

25 of 100 ——— at 26

16 of 100 ——— at 36

10 of 100 ——— at 46

6 of 100 ——— at 56

3 of 100 ——— at 60

1 of 100 ——— at 76

And that the Income of *England per An.* is, from Trade l. 90000000

Land ——— 14000000

Art and Labour 20000000

So that the whole Annual Income is 43000000 l. *Sterl. per Ann.*

The Income of *France* l. 81000000

Of *Holland* l. 18000000

Also that the Exports of *England* in *English* Commodities are about l. 5000000 *per Ann.*

The Current Coin of *England* is about l. 70000000:

The

The Expence for the abovesaid Number of People at 8 *l.* per Head *per Ann.* is *l.* 38,880000 *per Ann.* So that *England* increaseth in Riches at that rate, *l.* 4120000 *per Ann.*

The Number of Acres of the Land of *England* has been by the very Judicious esteemed (of the several sorts of Land) as follows: *viz.*

|                      | N <sup>o</sup> of Acres. |                      | N <sup>o</sup> of Acres. |
|----------------------|--------------------------|----------------------|--------------------------|
| Of Arable Land —     | 9,000000                 | In the last Column — | 28,000000                |
| Pasture and Meadow — | 12,000000                | Heaths and Barren    | } 10,000000              |
| Wood and Coppice —   | 3,000000                 | Lands —              |                          |
| Forrests, Parks, and | } 3,000000               | Rivers, Lakes and    | } 500000                 |
| Commons —            |                          | Ponds —              |                          |
| Houses, Orchards,    | } 1,000000               | Roads and Waste-Land | 500000                   |
| and Church-yards     |                          |                      |                          |

So that by the above said Estimate there is  $8\frac{1}{486}$  Acres for each Head in *England*, and about *l.* 1 : 8 : 10 each in Cash.

Sum = the Acres in }  
*England* — } 39,000000  
 In *Scotland* 24,000000  
*Ireland* 29,000000

|                                            | N <sup>o</sup> of Acres. |                                                    | N <sup>o</sup> of Acres. |
|--------------------------------------------|--------------------------|----------------------------------------------------|--------------------------|
| There is computed to be in <i>France</i> } | 117,000000               | Acres in <i>Great Britain</i> and <i>Ireland</i> } | 92,000000                |
| The 17 Provinces —                         | 13,000000                | In last Column —                                   | 2652,000000              |
| <i>Spain</i> and <i>Portugal</i> —         | 78,000000                | <i>Mogul's Empire</i> —                            | 741,000000               |
| <i>Italy, Venice, and</i>                  | } 66,000000              | <i>China</i> and <i>Tonquin</i> —                  | 702,000000               |
| Isles thereabouts                          |                          | <i>Tartary</i> —                                   | 2730,000000              |
| <i>Germany</i> —                           | 120,000000               | <i>Georgia</i> —                                   | 117,000000               |
| <i>Sweden, Norway,</i>                     | } 156,000000             | <i>Madagascar</i> —                                | 78,000000                |
| and <i>Denmark</i> —                       |                          | <i>Sumatra, and Isles</i>                          | } 80,000000              |
| <i>Poland</i> about —                      | 116,000000               | thereabout —                                       |                          |
| <i>Muscovy</i> —                           | 468,000000               | <i>Japan</i> —                                     | 117,000000               |
| <i>Turkey</i> in <i>Europe</i> —           | 192,000000               | <i>All Africa</i> —                                | 4680,000000              |
| <i>Turkey</i> in <i>Asia</i> —             | 234,000000               | <i>America</i> —                                   | 3510,000000              |
| <i>Arabia</i> —                            | 585,000000               | Isles <i>Wight, Man,</i>                           | } 50,000000              |
| <i>Persia</i> —                            | 507,000000               | <i>Guernsey, Jersey, and</i>                       |                          |
|                                            |                          | in the <i>West-Indies</i>                          |                          |

So that the Acres on the whole Earth are computed — } 15549,000000  
 Area of the Earth and Sea, as the Superficies }  
 of a Sphere in Acres — } 127675,955000

So

So that the Area of the Sea in Acres is ——— 112126,000000

And the People of the whole World are com- }  
puted at ——— } 300,000000

But the People in the World, in proportion as the Acres of *England* is to the People of *England*, would be 1937,600000. For

39,000000. 4,860000 :: 15549,000000. 1937,600000

This Difference above 300,000000 (which is about  $\frac{1}{2}$  of the whole 1937, &c.) is caused by the vast Quantities of Barren Ground, as  $\frac{1}{3}$  of *Africa*,  $\frac{1}{4}$  *America*,  $\frac{1}{2}$  *Tartary*,  $\frac{1}{3}$  *Russia*,  $\frac{1}{3}$  *Arabia*, &c. which are probably desert and unpeopled.

The very Ingenious and Accurate Dr *Halley*, in his said judicious Remarks upon the *Breslaw* Bills of Mortality, (wherein both the Ages and Sexes of all that died were monthly delivered and compared with the Number of Births, for the Years 1687, 1688, 1689, 1690, and 1691) hath calculated a Table, (published in the *Philosophical Transactions* in 1692, and in *Miscellanea Curiosa*, Vol. I.) and hath shew'd these Uses thereof; as, 1<sup>st</sup>, In finding the Proportion of Men able to bear Arms in any Multitude, from 18 to 56 Years of Age. 2<sup>dly</sup>, The different Degrees of Vitality in all Ages; as at what Number of Years it is an even Lay, that a Person of any Age shall die: for instance, that a Man of 30 Years of Age liveth between 27 and 28 Years; That 'tis 80 to 1, that a Person of 25 Years does not die in a Year; That 'tis 5 $\frac{1}{2}$  to 1, that one of 40 lives 7 Years. 3<sup>dly</sup>, He computes the Value of Annuities for Lives, (as in the curious little Table after the Tables of Compound Interest foregoing) and the Price of Insurances; and that one half of those who are born, do not live above 17 Years, &c.

Mr *Grant* (besides what he computeth, as above, of the Vitality per Cent.) saith, That the People of *London* are about one 14th of the People of *England*. And

Sir *William Petty* saith, That there are more People living between the Ages of 16 and 26 than any other Ages; and thence infers, That the Square Root of every Person's Age under 16 (whose Square Root is 4) sheweth the Proportion of Probability of such Person's living to 70 Years: *i. e.* It is 4 times more likely that one of 16 Years of Age lives to be 70, than a Child of 1 Year; 'Tis thrice as probable that one of 9 Years lives to 70, than that a newborn Child does; That the odds is 5 to 4, that one of 25 dieth before 1 of 16 Years old; That 'tis 6 to 5, that one of 36 dieth before 1 of 25 Years: These Proportions being the Roots of 25,

16,



16, and 36 = the Ages, as is above said. He also says, That the Shipping of *Europe* is about 2000000 Tons, of which *England* hath 500000, the *Dutch* 900000, the *French* 100000, *Spain*, *Portugal*, and *Italy* 250000, &c.

And besides what is said above of the Number of Acres of each kind of Land in *England*, Messieurs *King* and *Davenant* say farther, That the Increase of the People of *England* is 9000 per Annum, Allowances being made for War, Plague, Shipping, and the Plantations. They reckon the Souls of *London* 530000, in the Cities and Market-Towns in *England* 870000, the Villages and Hamlets 4100000. The Annual Produce by Cattel in Butter, Cheese, and Milk, about £. 2500000; the Value of the Wool yearly shorn, about £. 2000000; of Horses bred yearly £. 250000; Value of Flesh yearly spent as Food £. 3350000; of the Tallow and Hides about £. 600000; Hay yearly consumed by Horses £. 1300000; by other Cattel £. 1000000, &c.



## CHAP. VII.

### LOGARITHMICAL ARITHMETIC.

**I**N this kind of Arithmetic the Work of Multiplication, Division, and Extraction of Roots, are performed not by the Numbers themselves, but by Artificial Numbers adapted to those given; so that their Addition performs Multiplication; Subtraction does the business of Division; and Dividing by 2, 3, 4, &c. of these Artificial Numbers, gives the Square, Cube, Biquadrate, &c. Roots of the respective Natural Numbers.

2. There are to every Natural or Common Number, an Artificial one proportioned; which being formed into a Table, where each Natural Number has its own Artificial standing right against it, 'tis called a Table of Artificial Numbers or Logarithms.

3. Of these Tables some have the Logarithms of all Natural Numbers from 1 to 1000, some to 10000, and some to 100000.

And

And again, these Logarithms consist of Places from 5 to 14; but one and the same Table has always but one Number of Places: and the more extensive any Table is, the more it is universally useful: and therefore I shall by and by give Rules for enlarging a Table.

4. There may be several kinds of Logarithms contrived; for any Series of Numbers in Arithmetical Progression are the Logarithms of those right against them in Geometrical Proportion: As

|                                             |                                                                             |
|---------------------------------------------|-----------------------------------------------------------------------------|
| 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024 | { Natural Numbers<br>in Geom. Progr.<br>Their Logarith.<br>in Arith. Progr. |
| 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.           |                                                                             |

And the aforesaid Uses of Logarithms will appear even in this small Example of a Table: As,

1. *In Multiplication.* As 32 by 16. Against these in the upper Series, you have 5 and 4 in the lower (or that of Logarithms); therefore add 5 to 4 maketh 9, which see in the Logarithms, and over it is 512 = the Product of 32 by 16, done by only adding their respective Logarithms 5 and 4. So 32 by 32 is 1024; 8 by 16 = 128, &c.

2. *In Division.* To divide 1024 by 64: Under 1024 stands the Logarithm 10; and under 64 stands the Log. = 6, therefore take 6 from 10, and the Remainder is the Log. = 4, over which stands the Quotient 16. And so 512 divided by 16, quotes 32, and proves the first Example in Multiplication, &c.

3. *In the Extraction of the Square Root.* Admit you would have the Square Root of 1024: Under that you have the Logar. 10. the half of which is 5, over which Log. is 32 = the Square Root of 1024, by only taking half its Logarithm; so the Root of 256 is 16, &c.

4. *In Extracting the Cube Root:* If you would have the Cube Root of 512, under that is the Log. 9. a 3d, of which is 3; over which Log. stands the Answer 8 = the Cube-Root sought, &c.

5. By this very small Specimen it may be observed how necessary (because exceeding useful) a Table of Artificial Numbers must needs be, when they are made such a universal Series, as to be fitted for all Numbers from 1 to 100000; and this was the happy Contrivance or Thought of the Famous Lord *Neper*, a *Scotch* Baron, *Anno* 1614: whose Memory will always be precious, especially to all Trigonometrical Calculators. But the first Table, as abovesaid, was made

K k

by

by that Learned Mathematician of our own Country, Mr *Henry Briggs*, sometime *Savilian* Professor of Geometry in the University of *Oxford*, for Natural Numbers to 100000, and the Log. to 14 places; which new Invention was extremely approved of and promoted by *Kepler*, and other Foreigners.

6. These Logarithms being a Series of Numbers in Arithmetical Proportion to answer 1, 2, 3, 4, &c. to 10 or 100 Thousand, must needs be Decimals, except the Index or first Figure towards the left hand, separated from the Logarithm by a Point: for these shew how many Places the Natural Number consists of, if it be not a Decimal; but

if a Decimal, the Index is marked with a Note of Negation, as —, put commonly under the Index or Characteristic: which Index, in case of Logarithms of Decimals, does shew how many Places from

| <i>The Index of the Log. of<br/>all Intire Numbers.<br/>from</i> | <i>In Decimals the<br/>Index of</i> |
|------------------------------------------------------------------|-------------------------------------|
| 1, to 10 excl. is 0                                              | .1, .2, .3, &c. is 1                |
| 10, to 100 — is 1                                                | .01, .02, .03, &c. 2                |
| 100, to 1000 . . is 2                                            | .001, .002, &c. 3                   |
| 1000, to 10000 is 3                                              | .0001, .0002, &c. 4                 |
| 10000, to 100000 = 4                                             | .00001, &c. — 5                     |
| &c.                                                              | &c.                                 |

the Point the first significant Figure stands.

And 'tis plain, that the Indexes of Logarithms of intire Numbers are always 1 less in Value, than the Number of Places in those of whole Numbers.

*Note*, That for taking the Logarithm out of my Table, &c. you may see *Sect.* 2, and 4. of this Chapter.

7. Indefatigable must the Trouble and Pains be supposed, which Mr *Briggs* had in his first Table; as must be granted, when the Method is understood of making but one commonly used, consisting of half the Number of Places. Which is this: They first found so many Mean Proportionals between 10 and 1, by the continual adding Cyphers, about 28, and half so many afterwards towards the right hand of the 10, and then extracting the Roots, till at last the Root was so small, as to have as many Cyphers next the Point in the Decimal, as they intended the Log. should consist of Places; which could not be done under 27 very long Extractions: these several



veral Means they put down in order, from the extreme 10 downward.

2<sup>dly</sup>, They found Logarithms to these Extreme and Means, by putting down against the Extreme 10.000, &c. the Log. 1.000, &c. (as is before said) and proceeding to take half that Log. continually. half of that half, &c. and placing them orderly against their respective Mean Proportionals found as before.

3<sup>dly</sup>, Then when they had a mind to make the Log. of any Number, they, by extracting the Square Root, found so many continual Means between the Number given and 1, that the last Mean was the least Root that had so many Cyphers next the Point, as the Logarithm was to have Places.

4<sup>thly</sup>, Then to find Logarithms to these Means; the Proportion is As the significant Figures of the Mean Proportional found between 10 and 1 (of the like place with the last continual Mean in these you are upon)

Is to its respective Logarithm (omitting the Cyphers)

So is the last Mean in the Series you are now a working,

To its Logarithm.

5<sup>thly</sup>, This Logarithm of the last continual Mean between the Number you are investigating the Log. of, and 1, being found, is to be doubled, and that, &c. Products doubled continually; putting the several Products against the respective Means last found, till at last the Product falling against the Number whose Log. is sought, gives you that Logarithm, by which several others are easily found: for the Log. of 4 multiplied by 2, gives the Log. of 16;  $\frac{1}{2}$  Log. of 4 is that of 2: so have you the Logarithms of all the Powers of 2; and the Log. of 10 (or 1.0) less the Log. of 2. leaveth the Log. of 5. So you have the Log. of all the Powers of 5, and Product by 2 and 5.

I did intend to have inserted here the Whole of finding the Log. of any Number between 1 and 10; but there being lately so many briefer Ways of finding the Logarithms, published by Mr *Mercator*, Mr *Gregory*, Mr *Speidal*, and especially by Dr *Halley*, in *Philosophical Transactions*, N<sup>o</sup> 216, that I thought it not worth while to give an Example of the old tedious Way above-mentioned.

It could hardly be expected, that in one Treatise handling so many different Parts of Arithmetic, I should insert a Table of Logarithms; which alone, with its Use, makes a considerable Volume: And especially considering there are so many Tables already publish'd; for besides Mr *Briggs's* Original, there are Transcripts by

Norwood, Gunter, Wingate, Moor, Newbouse, Leyburn, Sturmy, Taylor, Harris, Ward, &c. Englishmen; besides those of Kepler, Vlacq, Urfin, &c. Foreigners. But I have however (the better to make this Book answer to the Title) inserted a Table to 10000, and Rules to make it to 100000000.

SECT. II. *To Add, Subtract, Multiply, and Divide Logarithms; or to perform the Work of Multiplication, Division, to Square and Cube, and Extraction of Roots of Intire Numbers by the Logarithms.*

**B**UT it must first be observed, That

I. *To take any Logarithm in my Table belonging to any Natural Number.*

If the Natural Number consist of three Places only, you have them in the Column under *Natural Numbers*; and in the next Column toward the right hand are their respective Logarithms under (0).

But if the Number given consist of four Places, look for all but Units place under *Natural Numbers* as before, right against which, and under the Digit in Units place, is your Logarithm required. And in like manner for any Logarithm under 100, see for that in Tens place under *Natural Numbers*; and for that in Units at the head of the Column, and in the common Angle of meeting stands the Logarithm sought.

II. *To find the Natural Number to a Logarithm.*

This you will easily do by the help of your Index, and especially by considering the Nature of the Numbers which increase from the left hand towards the right; as the Digits at the head of the Columns, and those in the left hand Columns respectively increase where four Places is in the Natural Number. But downwards the Logarithms are to such Natural Numbers as increase by Tens. Thus the Logarithm 8123785. I find under 2, and against 649; so that it is the Logarithm of 6492: but if it were the Logarithm 8122447, I find that under (0) and against 649; so that 6490 is the Natural Number sought. And the same Logarithm is also for 649: As all are Logarithms for the Natural Number expressed in the Table, or for that Number multiplied by 1 or more Tens; but you are to be guided by your Index what places your Number is.

So

*So much for the Logarithm of Integers: And for those of Fractions and Mixt Numbers, see Sect. 3. following, and more of the above, Sect. 4.*

I. *Multiplication of Intire Numbers by the Logarithms.*

*Example 1.] Multiply 91 by 93.*

*Rule.* To the Logarithm of 91, which is = 1.9590414

Add the Logarithm of 93 ——— = 1.9684829

Product = 8463; the Nat. Numb. to the Log. 3.9275243 Sum.

*Example 2.] Multiply 4 by 177.*

*Rule.* To the Log. of 177, which is = 2.2479733 } Add.  
Add the Log. of 4, which is = 0.6020600 }

Product 708; its Logarithm = 2.8500333 Sum.

These two Examples are sufficient to shew how to multiply any intire Number by another; by the Logarithms, where I need not give many words to shew that against the Natural Numbers given stand their Logarithms; and against the Sum of those Logarithms stand the respective Natural Numbers 8463 and 708: *i. e.* In my Table you have the Log. 9275243 under 3, and against 846 (the Units place, where there are 4) being always over the Logarithm. See Sect. 4. and the Rules above for finding the Logarithms, and their Natural Numbers.

II. *Division by Logarithms.*

*Example 1.] Divide 8463 by 93.*

*Rule.* From the Logarithm of 8463, which is = 3.9275243

Deduct the Log. of 93, which is = 1.9684829

Remts the Log. of the Quote 91, which is = 1.9590414

*Example 2.] Divide 708 by 177.*

From the Logarithm of 708, which is = 2.8500333

Deduct the Log. of 177, which is = 2.2479733

Remts the Log. of the Quote 4; which is = 0.6020600

III. *To Square any Intire Number, and Extract the Square Root by the Logarithms.*

I. To square (or involve in itself) any whole Number.

*Example.]*



*Example.]* What is the Square of 99 ?

*Rule.* Multiply the Logarithm of 99, which is = 1.9956352  
by ——— 2

9801, the Square sought; being the Natural } 3.9912704  
Number to the Product \_\_\_\_\_ }

2. To evolve, or extract the Square Root of any *Intire Number*.

*Example.*] What is the Square Root of 9801?

*Rule.* Take half of the Logarithm of 9801. Log. is 3.9912704  
Half that Log. of 9801 is = 1.9956352

Which half is the Log. of 99 = the Root sought. And this proves the last Question.

IV. To Cube, or Extract the Cube Root of any Intire Number, by the Logarithms.

1. To Cube any *Whole Number*.

*Example.*] What is the Cube of 21?

*Rule.* Multiply the Log. of 21, which is  $\frac{\text{---}}{\text{---}} = 1.3222193$   
by  $\frac{\text{---}}{\text{---}} = 3$

And the Cube required is  $9261 =$  the Natural }  
 Number of the Log. ———— } 3.9666579

2. To extract the Cube Root of any *Intire Number*.

*Example.*] What is the Root of 9261?

The Log. of 9261 is as before = 3.9666579  
 $\frac{1}{3}$  of that Logarithm is = 1.3222193

Whose Natural Number is  $21 =$  the Cube Root required.

3. If you desire the Biquadrate of any Number, multiply its Log. by 4, gives the Log. of the Answer: And if you would have the Biquadrate or Surfolid Roots, &c. of a Number, divide its Logarithm by 4 or 5, &c. Examples are needless; but note, that if the Index is less than the Divisor, take so often as the Divisor can be had in 2 places.

SECT. III. *To perform the Work of Multiplication, Division, Square or Cube, &c. or Extract the Roots of Fractions and mixt Numbers, by the Logarithms.*

**B**EFORE I give Examples of Logarithms, it will be necessary to give a Rule for *Adding or Subtracting Negative Indices or Characteristics.*

*Note,*

*Note,* That when the Index is mark'd under thus (—) it is so much less than nothing as those beforegoing, or 2, 3, 1, &c.

1. The Rules for adding and subtracting the Indices of Logarithms are numerous, and not easy to retain in mind: I have therefore supposed (which taking the Reason of the thing along with it, easily solves all, without charging the Memory) when the Index is negative, that it is a Person owing so many *l.* 100 more than he is worth, as the Index-Digit expresseth.

| <i>Addition of Indices.</i> |                                        | <i>Subtraction.</i>                       |  |
|-----------------------------|----------------------------------------|-------------------------------------------|--|
| <i>Exam. 1.</i>             | Add the Index 3<br>to ——— 1<br>Sum = 2 | From ——— 2<br>Take ——— 1<br>Refts 3       |  |
| <i>Exam. 2.</i>             | Add ——— 3<br>to ——— 1<br>Sum = 2       | 2dly, From ——— 2<br>Take ——— 1<br>Refts 3 |  |
| <i>Exam. 3.</i>             | Add ——— 2<br>to ——— 3<br>Sum = 5       | 3dly, From ——— 5<br>Take ——— 2<br>Refts 3 |  |

*These prove the Truth of those in Addition.*

This being done, the adding to or taking from the upper Index according to the Nature of each, makes the Sum or Remainer plainly appear.

1. So in the first Example of Addition above, I consider that if I add *l.* 100 to a Person who is 3 worse than nothing, the Sum must be 2; that is, I then make him worth but 2 less than nothing.

2. In the second Example, if I add 1 less than (o) to; that is, if I take 1 from a Person who hath 3, the Sum is but 2; for that is all that is left. And if I add 3 less than nothing to him who before was 2 worse than (o) I leave him 5 less than (o), as in Example 3.

*So in Subtraction of Indices:*

3. If (as in the first Example) I take 1 from him who already is 2 worse, or less than (o), I leave him 3 less than nothing; and so 3 is the Remainer.

4. If

4. If (as in the 2d Example) I take 1 (or 1 less than nothing) from him that hath 2 in possession already, I then leave him worth 3, because I take 1 less than nothing from him: therefore I add 1 to him, that is, I make 2 into 3, which is therefore the Remainder. (See the words *Negative Arithmetic* in Algebra.) And all this consider'd right, is but one single, short, and easy Rule.

II. To find the Logarithm of a mixt Number, or of a Decimal, by the Table.

Prob. 1. To find the Logarithm of a mixt Number, as of 9.177.

Rule. Take the Log. of 9177, except the Index; and to that Logarithm prefix an Index according to the Number of Places in the integral Part of the Number given, as by the little Table under the 6th Head of the first Sect. of this Chapter, and you have the Answer.

Examp. Thus the Log. of 9177 (without the Index) is 9627007; and the proper Index for 1 Place being 0, therefore the Answer is 0.9627007.

So also the Log. of 91.77 is = 1.9627007, and of 917.7 'tis = 2.9627007.

Prob. 2. To find the Logarithm of a Decimal, as of .5

Rule. Take the Log. of 5, except the Index, and prefix to that Log. an Index agreeable to the Decimal, as per the said little Table under the 6th Head of the 1st Sect. of this Chapter.

Examples. The Log. of 5 is = 6989700, without its Index: to which Log. if you put its proper Index 1, (there being no Cyphers between the Point and the 5) the Answer is 1.6989700. And by the same Rule the Log. of .05 is 2.6989700; of .005 = 3.6989700; and of .000579 the Log. is 4.7626785; being the Log. of 759, with the Index of .000579 put before it.

III. To find Mixt Numbers or Decimals to Logarithms by the Table.

Prob. 1. To find a Mixt Number to a Logarithm.

Rule. Regard not the Index; but look for the Logarithm among those which would have the greatest Indexes, which having found, take the Natural Number standing against it, and point off for Integers according to the Index given.

Example. What is the Natural Number to the Log. 1.9627007?

My Table extending to 10000, the greatest Index is 3; among which I find my Logarithm given, and right against it I find

9177.



### SECT. III. *Multiplication by the Logarithms.*

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9177. Now the Index given being 1, requires that 2 of the 4 Places be Integers, so that the Answer is 91.77: And also the Natural Number to 2.9627007 must be 917.7, &c. as per the Rule for Indexes above.

Prop. 2. *To find the Decimal to a Logarithm.*

Rule. Look for your Logarithm in that part of the Table where 'tis to be found, as before in the last, and put down the Natural Number that stands right against it: then according to the Greatness of the Index given, make the Point of the Decimal.

Example. What is the Natural Number to 4.7626785?

Against this Logarithm I find 5790. Now because my Index given is 4, I place 5 in the 4th place from the Point, and say therefore that .000579 is the true Decimal sought.

### IV. *To Multiply Mixt Numbers and Decimals, by adding their Logarithms.*

I have so fully prepared the Reader for this, that I only need to give Examples.

Example 1. A Mixt Number by a Mixt, as 9.336 by 12.09  
To the Logarithm of 9.336, which is 0.9701608  
Add the Log. of 12.09, which is — 1.0824263

The Sum is the Log. of 112.9 the Product, viz. 2.0525871

Example 2. A Decimal by a Decimal, as .09 by .123  
To the Log. of .09, which is — 2.9542425  
Add the Log. of .123, which is — 1.0899051

The Sum is the Log. of the Product .01107, which is 2.0441476

Example 3. A Whole Number by a Decimal, as 765 by .00345  
To the Log. of 765, which is 2.8836614  
Add the Log. of .00345, which is 3.5378191

The Sum is the Log. of the Product 2.639, viz. 0.4214805

*Example 4.* A Mixt Number by a Decimal, as 1.23 by .00009.  
 To the Log. of 1.23, which is — 0.0899051  
 Add the Log. of .00009, which is 5.9542425

The Sum is the Log. of the Product .0001107 = 4.0441476

*Note,* That *Whensoever* 1 is carried to the Index, that 1 is affirmative, and must be added to the lower Index, and that Sum to the upper, as *per* the Rules and Examples of adding Indices at the beginning of this Section; and more particularly in the three last Examples.

V. To divide Mixt Numbers, Decimals, &c. by Subtracting their Logarithms.

In the following Scheme you have all the Cases that can happen under this Head: which will be easily performed, only observing the first and second Examples of subtracting Indices, and the Rule at the beginning of this Section.

*The*

| The 9 Cases of Division, done by Logarithms. |                                               |                                                      |
|----------------------------------------------|-----------------------------------------------|------------------------------------------------------|
| Cases.                                       | Numbers given, or Dividends and Divisors, &c. | Their Logarithms; Sub-<br>tract that of the Divisor. |
| 1.                                           | Divide 237.3<br>by 25.32<br>Quote = 9.376     | — 2.3732977<br>— 1.4034637<br>Refts 0.9718340        |
| 2.                                           | Divide 11.5<br>by 932<br>Quote .01234         | — 1.0606978<br>— 2.9694159<br>Refts 2.0912819        |
| 3.                                           | Divide .1234<br>by .321<br>Quote .3846        | — 1.0913151<br>— 1.5065050<br>Refts 1.5848101        |
| 4.                                           | Divide .463<br>by 3214<br>Quote = .000144     | — 1.6655810<br>— 3.5070459<br>Refts 4.1585351        |
| 5.                                           | Divide 2<br>by 3.214<br>Quote .6222           | — 0.3010300<br>— 0.5070459<br>Refts 1.7939841        |
| 6.                                           | Divide 5<br>by 365<br>Quote = .0137           | — 0.6989700<br>— 2.5622929<br>Refts 2.1366771        |
| 7.                                           | Divide 1.26<br>by .002<br>Quote = 630         | — 0.1003705<br>— 3.3010300<br>Refts 2.7993405        |
| 8.                                           | Divide .5<br>by 7.5<br>Quote = .06 r 1        | — 1.6989700<br>— 0.8750612<br>Refts 2.8239088        |
| 9.                                           | Divide 999<br>by .00013<br>Quo. 7684615.384   | — 2.9995655<br>— 4.1139433<br>Refts 6.8856222        |



VI. To Square any Mixt Number or Decimal, or to Extract the Square Root thereof.

1. To Square such Numbers.

*Rule.* Multiply the *Logarithm* of the Root given by 2; but if the Index be negative, out of the Product of that by the 2 deduct what Tens are carried from the place next toward the right hand, and put down the Remainder for the Index of the Product (or *Log.* of the Square).

*Examples.*

I. For Squaring Decimals, &c. by *Logarithms*.

| The Square Roots given. | The <i>Logarithms</i> of those Roots. | Double the last Col. or the <i>Log.</i> of the Squares. | The Squares sought. |
|-------------------------|---------------------------------------|---------------------------------------------------------|---------------------|
| 987.6.                  | 2.9945811                             | 5.9891622                                               | 975353.76           |
| 98.76.                  | 1.9945811                             | 3.9891622                                               | 9753.5376           |
| 9.876.                  | 0.9945811                             | 1.9891622                                               | 97.5353.            |
| .9876.                  | 1.9945811                             | 1.9891622                                               | .9753               |
| .09876.                 | 2.9945811                             | 3.9891622                                               | .009753             |
| .009876.                | 3.9945811                             | 5.9891622                                               | .00009753           |
| .0009876.               | 4.9945811                             | 7.9891622                                               | .0000009753         |
| .00009876.              | 5.9945811                             | 9.9891622                                               | .000000009753       |

Or, II. For the Square Roots; these Titles observe.

| Squ. Roots sought (or the Natural N <sup>o</sup> to the half <i>Logarithms</i> .) | $\frac{1}{2}$ the <i>Logarithms</i> of the Squares; or those of the Roots. | <i>Logarithms</i> of the Squares given. | Squares given, whose Roots are required. |
|-----------------------------------------------------------------------------------|----------------------------------------------------------------------------|-----------------------------------------|------------------------------------------|
|-----------------------------------------------------------------------------------|----------------------------------------------------------------------------|-----------------------------------------|------------------------------------------|

2. Observe in Extracting the Square Root, that is to say, in taking half of the *Log.* of the Square; if the Index be an odd Number, add 1 to it in your mind, and so proceed: As in taking half 3.989, &c. 1 say half of 4 is 2; of 19, = 9: of 18, = 9, &c.

VII. To

## VII. To Cube, or Extract the Cube Root of Decimals, &amp;c.

*Examples.*I. For the Cubing of Decimals by *Logarithms*.

| <i>The Cube<br/>Roots<br/>given.</i> | <i>Logarithms<br/>of those<br/>Roots.</i> | <i>Three times<br/>the Logari-<br/>thms.</i> | <i>The Cube Numbers<br/>sought.</i> |
|--------------------------------------|-------------------------------------------|----------------------------------------------|-------------------------------------|
| 3.1                                  | 0.4913617                                 | 1.4740851                                    | 29.791                              |
| .31                                  | 1.4913617                                 | 2.4740851                                    | .029791                             |
| .031                                 | 2.4913617                                 | 5.4740851                                    | .000029791                          |
| .0031                                | 3.4913617                                 | 8.4740851                                    | .000000029791                       |
| .00031                               | 4.4913617                                 | 11.4740851                                   | .000000000029791                    |
| .000031                              | 5.4913617                                 | 14.4740851                                   | .000000000000029791                 |

Or, II. For the Cube Root observe these Titles of the Columns.

| <i>Cube<br/>Roots<br/>sought.</i> | <i><math>\frac{1}{3}</math> of those Lo-<br/>garith. be-<br/>ing those of<br/>the Roots.</i> | <i>Logarithms<br/>of those Cube<br/>Numbers.</i> | <i>Cube Numbers given ;<br/>whose Roots are re-<br/>quired.</i> |
|-----------------------------------|----------------------------------------------------------------------------------------------|--------------------------------------------------|-----------------------------------------------------------------|
|-----------------------------------|----------------------------------------------------------------------------------------------|--------------------------------------------------|-----------------------------------------------------------------|

1. You see by the Examples, that to Cube any Number (as those next the left hand) is to multiply their *Logarithms* (as those in the 2d Column) by 3; and the Products are those in the 3d Column, whose Natural Numbers required are those in the 4th Column. And here note, That what Tens you carry, you must abate out of 3 times the Index; as taught for the Square Root.

2. For the Examples of Extracting the Cube Root: The Number in the Column next the right hand are Examples of Cube Numbers given, the next Column are their *Logarithms*, and the third Column are third parts of those *Logarithms*. Where note, That if your Index cannot be divided by 3 without a Remainder, you must add so many Units to it, as is necessary for that end: And so many Units as you borrow'd there, you must pay Tens to the

the next Figure, and then take a 3d of the whole Log. The Examples make all plain.

Or for finding the Biquadrate, Sur-solid, or Squared Cube Powers.

**Rule 1.** Multiply the Log. of the Root given by 4, 5, or 6; observing the Rules given before, abating what you carry, out of the Product of the Index respectively.

Or for finding the Biquadrate, Sur-solid, or Squared Cube Roots.

**Rule 2.** Divide the Log. of the Power by 4, 5, or 6; observing the Rule of adding so many to the Index, that you can divide it without a Remainder, and paying so many Tens as you added or borrow'd Units, to the next Figure.

Examples to Rule 1.

Whose Nat. N<sup>o</sup>  
Logarithm of are the Ans. or  
the Powers. Powers requir'd

|                                                                         |                         |
|-------------------------------------------------------------------------|-------------------------|
| Thus the Biquadrate of .3; its Log. being = 1.4771212; 4 times that —   | } = 3.9084848 = .0081   |
| The Sur-solid Power of .3; its Log. being = 1.4771212; 5 times that — — |                         |
| The Squared Cube of .3; its Log. being = 1.4771212; 6 times that — —    |                         |
|                                                                         | } = 3.3856060 = .00243  |
|                                                                         | } = 4.8627272 = .000729 |

And according to the last Rule.

|                                                                                          | Log. of the<br>Roots. | Roots<br>sought. |
|------------------------------------------------------------------------------------------|-----------------------|------------------|
| The Biquadrate Root of .0081, its Log. being 3.9084848; $\frac{1}{4}$ of that is — —     | } = 1.4771212         | = .3.            |
| The Sur-solid Root of .00243, its Log. being 3.3856060; $\frac{1}{5}$ of that is — —     |                       |                  |
| The Squared Cube Root of .000729, its Log. being 4.8627272; $\frac{1}{6}$ of that is — — |                       |                  |

Which later Examples prove the Truth of the former.

VIII. To find the Logarithms of Vulgar Fractions, and of the Operations thereby.

**Prop.** To find the Log. of it<sup>s</sup>.

**Rule.** From the Log. of the Numerator deduct the Log. of the Denominator, and the Remainder is the Log. for the Answer.

Example.



*Example.* From the Log. of 3456, which is = 3.5385737

Take the Log. of 4567, which is = 3.6596310

Remains the Log. of  $\frac{3456}{4567}$ , which is = 1.8789427

*Prop. 2.* To find the Log. of a compound Fraction.

*Rule.* Reduce the Compound to a Simple, and work as above.

*Example.* To find the Log. of  $\frac{4567}{3456}$  of  $\frac{123}{456}$ .

The Compound in a Simple being  $\frac{123 \times 4567}{456 \times 3456}$ , the Log. is as before;

Or Log. of  $\frac{4567}{3456}$  of  $\frac{123}{456}$  = 1.8789427.

*Prop. 3.* To find the Logarithm of a Mixt Number.

*Rule.* Reduce it to an Improper Fraction, and find the Logarithm as per the first Proposition. This needs no Example.

*Prop. 4.* To reduce a Vulgar Fraction to a Decimal.

*Rule.* Find the Log. of the Vulgar Fraction as per the first: then against that Log. in the Table you have the Decimal.

*Example.* To Reduce  $\frac{4567}{3456}$  to a Decimal. The Log. of this Vulgar Fraction, as before, is = 1.8789427; and the Natural Number to that, per the Table, is .7567 = the Answer.

And for reducing such Fractions as this given, whose Terms have so many places, I take this to be the most easy and speedy Way of reducing them to Decimals: and if your Table of Logarithms were to 100000, or where the Numerator is a place less than the Denominator, the Rule would in most Cases be exact enough.

*Prop. 5.* What is the Product of one Vulgar Fraction by another, giving the Answer in a Decimal Fraction?

*Rule.* Add the Log. of the one Fraction to the Log. of the other found as per the 1st Prop. and look for the total Log. in the Table, against which stands the Decimal required.

*Example.* What is the Product of  $\frac{4567}{3456}$  by  $\frac{234}{567}$ ?

$\frac{4567}{3456}$  its Log. as before = 1.8789427

$\frac{234}{567}$  its Log. — — = 2.8890803

The Product for Answer is .05862, whose Log. is 2.7680230

And thus 'tis plain, that knowing by the Rules delivered how to find the Logarithms of Vulgar and Decimal Fractions, they may be multiplied or divided simply or promiscuously. For,

6. Suppose it were required to divide .05862 by  $\frac{234}{567}$ :

If from the *Log.* of .05862, which is = 2.7680230

You take the *Log.*  $\frac{7}{87\frac{1}{2}}$ , which is = 2.8890803

The Remainder is the *Log.* of .7567, or } = 1.8789427  
of  $\frac{1}{13\frac{1}{2}}$ , viz. — — — — —

Which proves the Operation above.

7. By what is abovesaid, it will follow that the Powers or Roots of Vulgar Fractions are easily found by multiplying the *Log.* of the Vulgar Fraction assigned as a Root, by 2, 3, 4, 5, 6, &c. to obtain the Power, or by dividing the *Log.* of the Vulgar Fraction assigned as a Power, by 2, 3, &c. to obtain the respective Roots. And this I take also to be the easiest and briefest Way of finding the Powers or Roots of Vulgar Fractions; especially where the Terms consist of 3, 4, or 5 places, and your Table of *Logarithms* is large enough.

SECT. IV. *The Explanation of my Table of Logarithms, whose Method somewhat differs from Briggs, Norwood, and some others. 2dly, Of the Arithmetical Complements of Logarithms. 3dly, Of Parts Proportional and Differences: And, 4thly, How to enlarge the Tables of Logarithms several Ways.*

1. **T**HE easiest Tables for a Learner to use, are those that have the Indices put down, and *Logarithms* to the Natural Numbers which are intirely placed successively. But I would here explain to him another kind of Table placed without the Indices, and which have not the Units place of the Natural Number with the rest, but over the respective *Logarithm* (this is the best Method, and which I have therefore observed in my following Table of *Logarithms*) thus

| N <sup>o</sup> . | 0       | 1       | 2       | 3       | 4       | &c.<br>to 9 |
|------------------|---------|---------|---------|---------|---------|-------------|
| 522              | 7176705 | 7177537 | 7178369 | 7179200 | 7180032 |             |
| 523              | 7185017 | 7185847 | 7186677 | 7187507 | 7188337 |             |
| 524              | 7193313 | 7194142 | 7194970 | 7195799 | 7196627 |             |
| 525              | 7201593 | 7202420 | 7203247 | 7204074 | 7204901 |             |

In Case such a Table as this is to be used; you must, for example to find the Logarithm of 5244, put down, 1<sup>st</sup>, 3 for your Index, as *per* the Rules in the sixth Paragraph of the first Sect. of this Chapter.

2<sup>dly</sup>, Having found 524, (the three first Figures of your Natural Number given toward the left hand) cast your eye in the line toward the right hand, till you come under the Figure in Units place of the Number given, (here 4) where against 524, and under 4, you have the Log. required: for the Log. of 5244 is 3.7196627.

So the Log. of 5223 is 3.7179200; and that of 523 = 2.7185017: the Log. of the last Natural Number having all its places found together under Number; the Log. against 523, and under 0, is that sought, &c.

These Logarithms in Mr Brigg's first and large Table, also in Norwood and some others, stand thus; (tho' they take up much room.)

| N <sup>o</sup> | Log.      | N <sup>o</sup> | Log.      | N <sup>o</sup> | Log.           |
|----------------|-----------|----------------|-----------|----------------|----------------|
| 522            | 2.7176705 | 5221           | 3.7177537 | 5231           | 3.7185847      |
| 523            | 2.7185017 | 5222           | 3.7178369 | 5232           | 3.7186677      |
| 524            | 2.7193313 | 5223           | 3.7179200 | 5233           | 3.7187507      |
| 525            | 2.7201593 | 5224           | 3.7180032 | 5234           | 3.7188337, &c. |

II. *The Arithmetical Complement* of a Logarithm, is that which will make it 10.0000000; so that to find the Complement, the Logarithm given must be deducted from the 10.0, &c.

Now the best way to subtract this (not according to the common way, but) to take every Digit of the Subtrahend from 9, except that in Units place, which take from 10: And to do this, you may begin at either end to subtract.

$$\begin{array}{r} 10.0000000 \\ \text{Log. given} = 3.7204901 \\ \hline \end{array}$$

$$\text{Its Compl. Arith.} = 6.2795099$$

The Complement Arith. is often used in Analogies wrought by Artificial Numbers, to prevent Trouble and Confusion: For instead of subtracting a Logarithm from the Sum of two others, it is usual to add the Complement Arithmetical, and from the Sum of the three Logarithms to abate Radius or 10.0 from the Index of that Sum; which gives the same Answer as tho' you had deducted the Logarithm of the first Number.

To instance in a common Question in the Rule of Proportion: If I give £. 25 : 10 : — for 48 Yards of Cloth; what will 15 cost at that rate?

M m

Yards



Yards 48, its *Logarithm* = 1.6812412

l. 25.5, its *Logarithm* = 1.4065402

Yards 15, its *Logarithm* = 1.1760912

} Add.

Sum 2.5826314

From which deducting the first *Log.* the  
Remainder is that of l. 7.968, viz. — } = 0.9013902

This by the Complement Arithmetical is done thus:

48. Compl. Arith. = 8.3187588

25.5 : : its *Log.* — = 1.4065402

15, its *Log.* — = 1.1760912

Sum abating 10 in the Index, the *Log.* of  
7.968 the Answer as before — } = 0.9013902

These kinds of Operations are much used in Trigonometry, with the *Logarithms* of Natural Sines, Tangents, &c.

### III. Of Differences and Proportional Parts.

1. By the *Differences* is meant the Differences of the *Logarithms* of 2 Numbers immediately and numerically succeeding one another, or the Differences of the Products of such Numbers multiplied by 10.

2. By *Proportional Parts* you are to understand, one or more tenth Part of those Differences. And these are apply'd in Practice thus :

IV. When you would find the *Logarithm* of any Number that is greater than what is contained in the subsequent Table of *Logarithms*, and you would do this by the help of the Table of Differences and Parts Proportional.

Suppose you were to find the *Logarithm* of 51716 : see the Operation.

I find in the Table the *Log.* of 5171 or 51710, } = 7.135745  
which is —————

And of the next Number 5172 or 51720 = 7.136585

The Difference of these *Logarithms* is = 840 Diff.

Which found in the Table of Differences (following that of the *Logarithms*) right against that 840 and under 6 (the Digit in Units place

504 Add.

7.136249 Sum.

of

of the Number given) you find the Proportional Part 504; which added to the Logarithm of 5171, the Sum is that of 51716, which with its proper Index is 4.7136249. You may prove the Truth of this by dividing 51716 by 7, the Quote is = 7388; to the Log. of which if you add that of 7, the Sum is the Log. of 51716, as above.

2. To prove the Truth of any Logarithms in the Table, without the Table of Proportional Parts: as those of 7000 to 7010 exclusive.

The Difference between the Logarithms of 700 and 701, or 7000 and 7010, is 6200.

| <i>Tenths of the Difference</i> | <i>Are these Proportional Parts.</i> | <i>Log. of 7000 repeated.</i> | <i>Sums of the last Logarithms and Proport. Parts respectively.</i> | <i>The Natural Numbers to the last Logarithms.</i> |
|---------------------------------|--------------------------------------|-------------------------------|---------------------------------------------------------------------|----------------------------------------------------|
| 1                               | 620                                  | 8450981                       | 8451601                                                             | 7001                                               |
| 2                               | 1240                                 | 8450981                       | 8452221                                                             | 7002                                               |
| 3                               | 1860                                 | 8450981                       | 8452841                                                             | 7003                                               |
| 4                               | 2480                                 | 8450981                       | 8453461                                                             | 7004                                               |
| 5                               | 3100                                 | 8450981                       | 8454081                                                             | 7005                                               |
| 6                               | 3720                                 | 8450981                       | 8454701                                                             | 7006                                               |
| 7                               | 4340                                 | 8450981                       | 8455321                                                             | 7007                                               |
| 8                               | 4960                                 | 8450981                       | 8455941                                                             | 7008                                               |
| 9                               | 5580                                 | 8450981                       | 8456561                                                             | 7009                                               |

By this it appears, that the Difference between the Logarithms of 7000 and 7010 being divided into 10 parts, 1 Tenth is 620, the Part to add to the Logarithm of 7000, to give the Logarithm of 7001; 2 Tenths added to the Logarithm of 7000, gives the Logarithm of 7002; 3 Tenths of the Difference 6200, which is 1860, added to 8450981 (the Log. of 7000) giveth 8452841 = the Logarithm of 7003, &c.

And if you would have the Proportional Part immediately to make the Log. of 7000 that of 7008, it is only multiplying a Tenth of the Difference (which here is 620) by 8, and the Product is the Proportional Part 4960; which added to 8450981 = the Log. of 7000, the Sum is 8455941 = the Log. of 7008 required. And thus the 9 Logarithms between 7000 and 7010 exclusive, are examined and proved: And so may any other without a Table of Differences and Proportional Parts, or by even calculating or examining those at the same time.

3. To augment the Table of Logarithms, so as to make that which extends only to shew the Log. of any Natural Number under 10000, subservient to give that of any Number under 100000, &c. without the Table of Parts Proportional. For Example;

1. To find the Log. of 64736.

The Log. of 6473 or 64730 is = 8111056

Of 6474 or 64740 is = 8111727

The Difference = 671

A Tenth of that Difference (being the Proportional Part for 1, that is to make the Log. of 64731) — } 67.1

Which multiply by 6 (the Place of Units in the given Number) produceth — } 402.6

Or because the 5 is above one half, you may make the Proportional Part for 6 (in Units place of the Number given) — } 403

Which added to the Log. above of 64730, viz. — 8111056

The Sum is the Log. of 64736 or Answer = 4.8111459

For as 10. 671 :: 6. 403 *ferè*.

*i. e.* As the Number of Cyphers added to the Natural Number in your Table with a 1 before such (0) towards the left hand; Is to the Proportional Part, or Difference of the two Logarithms: So is the Figures towards the right hand exceeding the Places for which your Table is calculated, To the Proportional Part to be added to the Log. to answer those Figures.

2. To find (by a Table which has only the Logarithms to Numbers under 10000) what the Logarithms are for all Numbers under 1000000: For instance what is the Log. of 647365?

Log. of 647300 is = 8111056

of 647400 is = 8111727

Difference = 671

thereof, there being 100 difference between the Nat. N<sup>o</sup> of which these are Logarithms — } 67.1

This multiplied by 65 (the 2 places toward the right hand exceeding the Compass of the Table) — } 65

Produceth = 436.15

Which added to } 8111056  
Log. 647300 }

Sum = 8111492 = the Log. of 647365

For 100. 671 :: 65. 436. 15 the .15 may be omitted.

3. To



3. To find (by a Table which has only the *Logarithms* to Natural Numbers not exceeding 10000) what the *Logarithms* are for all Numbers not exceeding 10000000000. For example,

I would know the *Logarithm* of 2966820514.  
 As before, the *Log.* of the most places towards the left hand of the Number given, contained in my Table is of } *Nat. N<sup>o</sup>.* *Logarithms.*  
 2966, which is also of } 2966000000 = 4721711  
 The *Log.* of the next Number is of 2967 } 2967000000 = 4723175  
 or of — — — — — }

The Difference of these *Logarithms* is = 1464

Which Difference divided by 1000000 that each Unit may have its Proportional Part of that Difference, } = .001464  
 that for 1 is — — — — — }

But because the Numb. given requires 820514 of those Parts for its Part Proportional, therefore I multiply the Difference (as *per* the former Rules) by — } = 820514

And the Product is the Part Proport. for 820514 = 1201.232496

Which (omitting the Decimal as useless) added to the *Log.* of 2966000000 as above, which is — } 4721711

The Sum is the *Logarithm* sought, viz. 4722912

Which having a proper Index put before it, according to the Number given, as *per* the 5th Rule of the first Section of this Chapter:

The *Log.* of 2966820514 is found = 9.4722912

I have in this last Example made this matter so very plain that I need add nothing for that end; only to observe,

1<sup>st</sup>, That the *Log.* of any Number is the same, tho' that Number have never so many Cyphers added to its right hand. And therefore in

*Example 1.* Of this Head, I call the *Log.* of 6473, which is 8111056, the *Log.* of 64730.

In *Example 2.* I call the same *Logarithm* that of 647300. And

In *Example 3.* I call the *Log.* of 2966 (which is 4721711) the *Log.* of 2966000000, making up the rest of the Places toward the right hand (for which I have no *Logarithms* in my Table) with Cyphers.

2<sup>dly</sup>, (As in the last Example) whatever Number of Cyphers those are, I cut off so many Places of the Difference of the *Logarithms*, &c.

And

And thus I have endeavoured to make this Part (which has been omitted by most others) very intelligible, by shewing how to prove the Truth of *Logarithms*, or find those to Numbers that are large by the Difference and Parts Proportional, as they are found in Tables for that purpose: And have also most easily and plainly shew'd how *Logarithms* of large Numbers are found without such Tables of Differences and Proportional Parts. It remains next to shew,

1. *How to find Natural Numbers to large Logarithms by the Tables of Differences and Parts Proportional.*
2. *How to do the same without such Tables of Differences and Parts Proportional.*

4. *To find the Natural Numbers to Logarithms beyond the Compass of your Table, by the Difference and Parts Proportional.*

I would know the Natural Number to the *Logarithm* 4.8111459: See the Operation.

The *Logarithm* given  $\equiv$  8111459

The next *Log.* in the Table less than that given is the *Log.* of 6473, (which are the four first Places towards the left hand of the Number required) *viz.*  $\equiv$  8111056

The next *Log.* in the Table greater than the last (being that of 6474) is — —  $\equiv$  8111727

The Difference of the 2 first *Logarithms*  $\equiv$  403

Difference of the 2 last *Logarithms*  $\equiv$  671

Now to find the 5th Figure of the Number sought, or that in Units place, (the Index of the *Log.* given shewing that it hath but 5 places) look for the 2d Difference under [*Differences*] in the Table of Proportional Parts, and you'll find the next it (greater) to be 672. Then look in the same Line toward the right hand, and you'll find the 1st Difference 403; and at the head of that Column stands (6) which put in Units place, gives 64736 for the Number sought. This proves the Truth of the 1st Example, last above-said.

5. *To find the Natural Numbers (tho' very large) to any Log. given, without the Tables of Parts Proportional.*

What is the Natural Number to the *Logarithm* 9.4722912?

I find the *Logarithms*, and take the Differences as above thus:

The

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The *Log.* given (without the Index) is 4722912 = 1st Differ. 1201  
 The next less in the Table (which is the *Log.* of 2966) is ——— } 4721711  
 The next *Log.* (greater than the last) is ——— } 4723175 = 2d Differ. 1464

Now whereas the Index shews the Natural Number required to have 10 places, and there being only the first 4 found, there wants 6 places, which with a Unit towards the left hand is = 1000000; therefore I say, As the 2d Difference is to 1000000 ::

So is the 1st Difference to the rest of the Nat. N<sup>o</sup> required, i. e.

$$1464. 1000000 :: 1201. 820514.$$

Which 820514 placed towards the right hand of the 2966 found in the Table as above, is together 2966820514 = the Answer.

But the *Logarithms* of 7 places being too few to find so large a Natural Number by, makes the Differences of too few places; and consequently the 4th Proportional in the last Analogy will fall somewhat short, if done by a Table having only 7 places. However, the Example shews the true Method of proceeding in this Case, which is all here intended.

This last Example was designed to prove the Truth of the third; where .232496 being omitted, if that Decimal be annexed to the 1201, the third in the last Analogy, it will then be divided by the 1464, and produce the true Quotient 820514 exactly.

## SECT. V. The Use of Logarithms in solving Questions of Compound Interest.

Quest. 1. **W**HAT is the Amount of 1. 10 in 130 Years, at 5 per Cent?

*Rule.*] Multiply the *Log.* of 1.05 (which is 1 l. and its Interest for a Year, which in the subsequent Questions I call the Ratio) by 130, (the N<sup>o</sup>. of Years) and the Product is the *Log.* of the Amount of 1. 1 in 130 Years.

To this *Log.* add that of 1. 10 = the Principal; and the Sum is the *Log.* of the Answer, viz. of 1. 5684.409 = the Amount required.

*Example.*

1.05, its *Log.* is = 0.0211893

Multiply by Years 130

l. —————

Amount of 1 l. } = 568.4. }  
 in 130 Years. } its *Log.* } 2.7546090

To which add the *Log.* of the Princip. 10 l. } 1.0

Sum or *Log.* of 5684.409 — } 3.7546090

Quest.



Quest. 2. What is the present Worth of  $l. 5684 : 8 : 2 \frac{1}{4}$  (or of  $l. 5684.409$ ) due at the End of 130 Years?

Rule.] Find the Logarithm of the Amount of  $l. 1$  for 130 Years, as before.

Subtract the Log. of that Amount from the Log. of the Sum whose present Worth is required, and the Remainder is the Logarithm of the present Worth, as per Margin.

So I find  $l. 5684.409$  to be received 130 Years hence, is worth  $l. 10$ . in present Money; which proves the 1st.

Example.

Ratio = 1.05, its Log. = 0.0211893  
 Multiply by — — 130  
 Product, or Amount of  
 $l. 1$  in 130 Years = } 2.7546090  
 $568.4$ ; its Log. — }  
 which last, deduct from } 3.7546090  
 the Log. of  $l. 5684.409$  }  
 And the remaining Log. }  
 is that of  $l. 10$ , the } 1.0000000  
 Answer. — — }

Quest. 3. What is the Amount of the Lease or Annuity of  $l. 25$  per Ann. in 21 Years, according to its utmost Improvement at 5 per Cent?

Rule.] Find the Amount of  $l. 1$  for 21 Years, as above.

Example.

2. Then from the Log. of the Amount less 1, deduct the Log. of the Ratio less 1; and the Remainder is the Amount of  $l. 1$  Annuity in 21 Years.

3. Add to the last Log. that of 25 the Annuity, and the Sum is the Answer, or  $l. 893$ , the Amount of 25 per Ann. in 21 Years.

Ratio 1.05, its Log. = 0.0211893  
 Multiply by Years — — 21  
 Product, or Amount of  
 $l. 1$  in 21 Years 2.786; } 0.4449753  
 its Log. — — }  
 $1.786$  = the Amount } 0.2518815  
 less 1 its Log. — — }  
 $1.05$  = Ratio, less 1 } 2.6989700  
 = .05; its Log. — — }  
 Which last deducted, }  
 there rests the Log. of } 1.5529115  
 $l. 35.72$ , the Amount of }  
 $l. 1$  per An. in 21 Years. }  
 To which adding the } 1.3979400  
 Log. of 25 the Annu. }  
 The Sum is the Log. of }  
 $l. 893$  the Amount of } 2.9508515  
 the Annuity for Answ. }

Quest.

Quest. 4. What is the present Worth of a Lease, or Annuity of  $l. 25$  per Annum, to continue 21 Years at 5 per Cent. Interest?

Rule.] Find the Amount of  $l. 1$  in 21 Years, as in the last Question.

2dly, To the Log. of that Amount add the Log. of the Ratio, less 1.

3dly, Subtract that Sum from the Log. of the Amount, less 1; and the Remainder is the Log. of the present Worth of  $l. 1$  Annuity to continue 21 Years.

4thly, To the last Log. add that of the Annuity given, and the Sum is the Log. of the present Worth required.

Example.

|                                               |   |           |
|-----------------------------------------------|---|-----------|
| 2.786 = the Amount                            | } | 0.4449753 |
| of $l. 1$ in 21 Years (as above) its Log. — — |   |           |
| .05 = Ratio less 1, its                       | } | 2.6989700 |
| Log. — — — —                                  |   |           |
| Sum = — — —                                   |   | 1.1439453 |
| 1.786 = Amount, less                          | } | 0.2518815 |
| 1, its Log. — — —                             |   |           |
| Remts the Log. of 12.821                      | } | 1.1079362 |
| which is the present                          |   |           |
| Worth of $l. 1$ per Ann.                      |   |           |
| to continue 21 Years                          | } | 1.3979400 |
| To which add the Log.                         |   |           |
| of $l. 25$ the Annuity                        |   |           |
| And the Sum is the                            | } | 2.5058762 |
| Log. of the Answer                            |   |           |
| $l. 320.527$ , which is                       |   |           |

Quest. 5. What Annuity or Lease will the Sum of  $l. 320.527$ , or  $l. 320 : 10 : 6\frac{1}{2}$  purchase, to continue 21 Years, allowing the Purchaser 5 per Cent.

Rule.] Find the Amount of  $l. 1$  in 21 Years, as in the 1st, &c. Questions.

2dly, To the Log of that Amount add the Log. of the Ratio less 1, as in the last.

3dly, From that Sum deduct the Logarithm of the Amount less 1, and the Remainder is the Log. of the Annuity that  $l. 1$  will purchase for 21 Years

4thly, To the Log. of the Annuity of  $l. 1$  add the Log. of the Purchase-Money, and the Sum is the Log. of the Annuity sought here  $l. 25$  per Ann. as per the Work in the Margin.

Example.

|                                  |   |           |
|----------------------------------|---|-----------|
| 2.786 the Amount of $l. 1$       | } | 0.4449753 |
| in 21 Years, its Log. — —        |   |           |
| .05 = Ratio, less 1,             | } | 2.6989700 |
| its Log. — — — —                 |   |           |
| Sum — — — —                      |   | 1.1439453 |
| 1.786 = Amount, less             | } | 0.2518815 |
| 1, deduct its Log. — —           |   |           |
| Remts the Log. of the            | } | 2.8920638 |
| Annuity that $l. 1$ will         |   |           |
| purchase for 21 Years,           |   |           |
| viz. of .078, which              | } | 2.5058762 |
| Log. is — — — —                  |   |           |
| 320.527 the Purchase-Mo-         | } | 1.3979400 |
| ney, its Log. add =              |   |           |
| $l. 25$ per Ann. the Annuity re- |   |           |
| quired, whose Log. is the        | } |           |
| Sum of the 2 last Log.           |   |           |

N n

Quest.

Quest. 6. What is an Estate worth in Fee, which contains Arable Land *l.* 60 *per Ann.* Under-Wood which once in 15 Years is worth *l.* 200, and there is also Timber which 'tis computed after 30 Years Growth will be worth *l.* 1000?

Rule.] The Arable Land at 20 Years Purchase } *l.* 1200 : 00 : 00  
is worth \_\_\_\_\_

2dly, *l.* 200 (the first for the Under-Wood)  
due 15 Years hence at 6 *per Cent.* by the  
Logarithms (as per the 2d Question) is  
found thus : 1.06 its Log. = 0.0253058  
Multiply by Years \_\_\_\_\_ 15

2.397 = Amount of *l.* 1 in 15 } 0.3795870  
Years, its Log \_\_\_\_\_  
Which taken from the Log. of } 2.3010300  
*l.* 200 \_\_\_\_\_

The Remainder is that of the } 1.9214430 = Log of .83 : 9 : 00  
present Worth \_\_\_\_\_

3dly, By the same Rule the present Value of *l.* 200 } 34 : 16 : 00  
due 30 Years hence is \_\_\_\_\_

4thly, By ditto Rule the Value of the 3d 200 due } 14 : 10 : 00  
45 Years hence, is \_\_\_\_\_

5thly, The Value of the 4th *l.* 200 due 60 Years } 6 : 1 : 00  
hence, is *per ditto* Rule \_\_\_\_\_

6thly, The Value of *l.* 1000 for the Timber due } 174 : 2 : 00  
30 Years hence, is in present \_\_\_\_\_

The Sum of which six Articles is the Answer near } 1512 : 18 : 00  
enough (for 'tis needless in this Purchase to  
take notice of the 5th 200 *l.*) \_\_\_\_\_

Quest.



Quest. 7. What Annuity payable Quarterly, will  $l. 320.527$  purchase, to continue 21 Years at 5 per Cent.?

Rule.] First take a fourth of the Log. of the Ratio.

2<sup>dly</sup>, Multiply that 4<sup>th</sup> by the Quarters in the Years given, and that gives the Log. of the Amount of  $l. 1$  for those Quarters.

3<sup>dly</sup>, To the Log. of that Amount add that of the Rate per Quarter less 1.

4<sup>thly</sup>, From that Sum deduct the Log. of the Amount for the Quarters, less 1; and the Remainder is the Log. of the Annuity that  $l. 1$  will purchase for 21 Years payable quarterly.

5<sup>thly</sup>, To the last Log. add that of the Purchase-Money, and the Sum is the Log. of the Answer.

By this Example it is obvious, that the  $l. 320.527$  will purchase but 6 $l.$  per Quarter, at the Rate of 24 $l.$  per Ann. Whereas by the 5<sup>th</sup> Question it appears, that Sum will purchase  $l. 25$  per Ann. if payable yearly. So that there is  $l. 1$  per Ann. difference in that Purchase between receiving the Rent quarterly and yearly, the Purchaser having the Use of the Quarters Rents.

Example.

1.05 the Ratio, its Log. 0.0211893  
 A 4<sup>th</sup> of that Log. w<sup>ch</sup> }  
 is the Log. of 1.012 } 0.0052974  
 the Rate per Quarter. }  
 Which Log. multip. by }  
 the Quarters in 21 Y<sup>rs</sup> } — — 84  
 And that produceth the }  
 Log. of 2.786 = the }  
 Amount of  $l. 1$  for 84 } 0.4449816  
 Quarters, viz. ——— }  
 To which Log. add that }  
 of .012 = Rate per }  
 Quarter, less 1 ——— } 2.0791812  
 Sum — — — 2.5241628  
 1.786 = the Amount, }  
 less 1, its Log. deduct } 0.2518815  
 Rests the Log. of .018721 }  
 = the Annuity that  $l. 1$  }  
 will purchase, payable } 2.2722813  
 quarterly, viz. ——— }  
 To which Log. add that }  
 of  $l. 320.527$  the Pur- }  
 chase Money, which is } 2.5058762  
 And the Sum is the Log. }  
 of  $l. 6$ , the Quarterly }  
 payment required. } 0.7781575

Quest. 8. What is the present Worth of *l.* 24 per Ann. payable by equal Quarterly Payments, to continue 21 Years?

Rule.] First find the Amount of *l.* 1 for 84 Quarters, as above.

2dly, To that Log. add the Log. of the Rate per Quarter (found as above) less 1.

3dly, Deduct that Sum from the Log. of the Amount less 1, and the Remainder is the Log. of the present Worth of *l.* 1 per Quarter, to continue 84 Quarters.

4thly, To that Log. add that of *l.* 6 per Quarter, (as the 4th of 24 per Ann.) and the Sum is the Log. of the present Worth of the Annuity given, payable quarterly.

This proves the truth of the last Question.

Example.

|                                                                        |   |           |
|------------------------------------------------------------------------|---|-----------|
| Rate per Quarter found                                                 | } | 1.012     |
| as above, is — — —                                                     |   |           |
| 2.786 = the Amount of                                                  | } | 0.4449816 |
| <i>l.</i> 1 for 84 Qrs. (found as above) its Log. —                    |   |           |
| .012 = the Rate per                                                    | } | 2.0791812 |
| Quarter less 1, its Log. Add — — —                                     |   |           |
| Sum of those 2 Logs.                                                   |   | 2.5241628 |
| 1.786 = the Amount                                                     | } | 0.2518815 |
| above, less 1, deduct its Logarithm — — —                              |   |           |
| Rests the Log. of <i>l.</i> 53.42                                      | } | 1.7277187 |
| = present Worth of <i>l.</i> 1 per Quarter, to continue 21 Years — — — |   |           |
| 6 <i>l.</i> per Quarter (as per the Annuity given)                     | } | 0.7781575 |
| its Log. Add — — —                                                     |   |           |
| And the Sum is the Log. of 320.527 the Answ.                           |   | 2.5058762 |

Quest. 9. In an Account of Consequence 'tis found that *l.* 5684.409 is the Amount of 10 *l.* Compound Interest at 5 per Cent. what Time was required for that Increase?

Rule.] From the Log. of the Amount deduct that of the Principal put out.

2dly, Divide the remaining Log. by the Log. of the Ratio, and the Quotient is the Years required. See the Operation.

Example.

|                                           |                  |
|-------------------------------------------|------------------|
| <i>l.</i> 5684.409 its Log. — —           | 3.7546090        |
| 10 <i>l.</i> the Principal, its Log. } 1. |                  |
| Deduct — — —                              |                  |
| <i>l.</i> 05, its Log. .021893            | } 2.7546090 (130 |
| the Divisor — — —                         |                  |
| So the Answer is                          | 635670           |
| 130 Years.                                |                  |

Quest.

*Quest. 10.* In a certain Account *l.* 5684.409 is charged as the Amount of 10 *l.* in 130 Years Compound Interest: But upon a Trial the Simple Interest being only allowed according to the Rate agreed on, whence the said Amount is computed; now it not appearing by any Bond, or Entry, what Rate or Interest was reckoned, the Question is how that may be found out?

*Rule.]* From the Log. of the Amount deduct that of the Principal put forth.

2dly, Divide that remaining Log. by the Number of Years, and the Quote is the Log. of *l.* 1 and its Interest for a Year. Therefore say,

3dly, As *l.* 1 is to *l.* 1 and its Interest found:: So is 100 to 100 and its Interest. Whence taking 100, the rest must be the Rate of Interest required.

*Example.*

|                                   |   |           |
|-----------------------------------|---|-----------|
| 5684.409 the Amount,              | } | 3.7546090 |
| its Log. _____                    |   |           |
| 10 = the Principal,               | } | 1.        |
| deduct its Log. —                 |   |           |
| Divide by 130) rests —            |   | 2.7546090 |
| Quoteth the Log. of               | } | 0.0211893 |
| <i>l.</i> 1.05, which is —        |   |           |
| Therefore                         |   |           |
| As 1. 1.05 :: 100. 105            |   |           |
| And 105 less 100 = 5 the Interest |   |           |
| required <i>per Cent.</i>         |   |           |

*Quest. 11.* A Gentleman bought a Lease of *l.* 25 *per Ann.* to continue 21 Years for *l.* 320.527: At what Rate was Interest allowed him *per Cent*?

To do this by the Logarithms, take the Log. of

*l.* 320.527, which is \_\_\_\_\_ } 2.5058762

Whence deduct the Log. of the Annuity given 25,

which is \_\_\_\_\_ } 1.3979400

And the Remainder is the Log. of 12.821 (or the pre-

sent Worth of *l.* 1 *per Ann.* to continue 21 Years; } 1.1079362

which note. \_\_\_\_\_

Now to find what Interest this is reckon'd at, I make trial (as *per Quest. 4.*) and find at 6 *per Cent.* that the present Worth of *l.* 1 *per Ann.* to continue 21 Years is 11.764: which being less than the 12.821 (the present Worth of *l.* 1 *per Ann.* as above) I know that 6 *per Cent.* is too much, (for the more the present Worth is, the less is the Rate of Interest, and the contrary.)

Therefore I make trial at 4 *per Cent.* and (by the Rules under the 4th Question) I find the present Worth of *l.* 1

*per*



*per Ann.* at 4 *per Cent.* is (to continue 21 Years) 14.031 : which being more than the above 12.821, I know that 4 *per Cent.* is too little. So that it neither being at 6 nor 4, but finding the Rate to lie between them, I suppose it 5 *per Cent.* And trying by the Rules under the said 4th Question I find the Log. of the present Worth of *l.* 1 *per Ann.* to continue 21 Years is that of 12.821, the same as above, viz. \_\_\_\_\_ 1.1079362

And consequently I know that 5 *per Cent.* is the Rate sought.

Or thus to find the Rate :

Having found the present Worth at 4 and 6 *per Cent.* as above, proceed thus.

1. Take the Difference between the Years Purchase given (or found by the Logarithms) which Years Purchase as above are \_\_\_\_\_ } 12.821

And the Years Purchase at 4 *per Cent.* found \_\_\_\_\_ 14.031

Which Difference is \_\_\_\_\_ = 1.210

2. The Years Purchase at 4 *per Cent.* are found = 14.031

Ditto at 6 *per Cent.* as above are = 11.764

2d Difference = 2.267

3. Say 2.267. 1.210 :: 2. 1

i. e. As the 2d Difference is to the first: so is 2 the Difference between the Rates *per Cent.* (supposed) as before, to 1 = the Proportion to be added to the lesser Rate 4 to give the Rate sought for; in which case the 1st place in the 4th Proportional is sufficient, where (0) is next the Point in the Decimal, if the Division were carry'd on farther.

Quest. 12. What is the present Value of the Reversion of an Annuity or Lease of *l.* 50 *per Ann.* to continue 21 Years, to commence after the Expiration of 7 Years, at 5 *per Cent.* *per Ann.* allowed to the Purchaser.

Rule.] First add the Years (the 7 to the 21) make 28.

2dly, Find the present Worth of the Annuity to continue 28 Years by the 4th Question, which } *l.* 745 : 00 : 00  
you'll find \_\_\_\_\_

3dly, Find the present Worth of the *l.* 50 *per Ann.* }  
(by the Rules under the said 4th Quest.) to con- } *l.* 283 : 16 : 00  
tinue 7 Years, which is \_\_\_\_\_

4thly,

4thly, Deduct the later from the former, and the Remainder is the present Worth of the Reversion required  $\left. \begin{array}{l} \text{---} \\ \text{---} \\ \text{---} \end{array} \right\} \text{l. 461:14:0}$

*Example the 2d* shall be to find the Values of the Reversions of Church or College Leases of Houses; or the Fines payable for any Number of Years lapsed: As suppose 17 Years lapsed of 40, what Fine must be paid to make up the Lease again 40 Years: supposing the clear annual Rent 40 Pounds.

*Rule.*] Since this is the same thing as to find the Value of the Reversion of 17 Years after the Expiration of 23 in *esse*; and the Rate of Interest in these Cases is 1.116:

1st, Therefore find the present Worth of *l. 40 per Ann.* to continue 40 Years by the Rules above and as follows.

2dly, From that deduct the present Worth of *l. 40 per Ann.* to continue 23 Years, and the Remainder is the Fine or Value of the Reversion: See the whole Operation as follows.

$$\begin{array}{r} 1.116 \text{ the Rate, its Logarithm} = 0.0476642 \\ \text{multiply by Years} \quad \text{---} \quad \text{---} \quad \text{---} \quad 40 \end{array}$$

The Amount of *l. 1* in 40 Years or 80.64; its Log. 1.9065680

.116 the Rate less 1, add its Log. = 1.0644580

Sum of the 2 last 0.9710260

Which deduct from the Log. of the Amount } = 1.9011313  
less 1 = 79.64, its Log. ---

And the Remainder is the Log. of the present } 0.9301053  
Worth of *l. 1 per Ann.* to continue 40 Years }

To which add the Log. of the Annuity *l. 40* = 1.6020600.

The Sum is the Log. of 340.5 the present Worth } 2.5321653  
of *l. 40 per Ann.* to continue 40 Years ---

2dly for the present Worth of *l. 40* to continue 23 Years.

1.116 the Rate or Ratio, its Log. = 0.0476642

multiply by Years --- 23

The Amount of *l. 1* in 23 Years 12.48 its Log. = 1.0962766

.116 the Ratio less 1 add its Log. = 1.0644580

Sum of the 2 last --- 0.1607346

Which deduct from the Amount less 1, } = 1.0599419  
*viz.* 11.48, its Log. ---

The Remainder is the Log. of the present } = 0.8992073  
Worth of *l. 1* to continue 23 Years ---

To which add the Log. of the Annuity 40 = 1.6020600

And

And the Sum is the Log. of the present Worth }  
 of  $l. 40$  per Ann. to continue 23 Years, ——— } 2.5012673  
*Viz.* of 317.1, which deducted from the above ———  
 $l. 340.5$ , the Remainder is  $l. 23 : 8 : 0$  the Fine required.

13. *When it is required to purchase an Estate either in Fee or for Years mentioned, with Money due at any time to come:* as suppose I would purchase an Annuity to continue 14 Years, with  $l. 1000$  due to me at the end of 5 Years; the Annuity to commence presently at 5 per Cent.

*Rule.*] First, by the 2d Question find the present Worth of  $l. 1000$  due at the End of 5 Years, which is }  $l. 783 : 10 : 0$   
 2dly, By the 5th Question find what Annuity to continue 14 Years  $l. 783 : 10$  will purchase; which }  $l. 79 : 2$  per An.  
 for Answer you'll find at 5 per Cent. ——— } 14 Years.

I need not tell the Reader, that in case the Purchase had been required in Fee, to divide the present Worth of  $l. 1000 = l. 783.5$  by the Number of Years Purchase agreed on; as at 20 it will be ———  $l. 39 : 3 : 6$   
 per Ann. for ever.

14. *When you would Fine off Rent:* As suppose a Lease is  $l. 90$  per Ann. and the Tenant is willing to Fine off  $l. 40$  per Ann. for his Lease of 21 Years at 5 per Cent. in this Case there is nothing to be done but by the 4th Quest. find the present Worth of  $l. 40$  per Ann. to continue 21 Years; which present Worth  $l. 512 : 16 : 11$  must be given the Landlord as a Fine, and  $l. 50$  per Ann. must be paid Annual Rent. On the contrary:

15. *When you would Rent off a Fine:* As suppose a Lease for 21 Years is to be let at  $l. 50$  per Ann. with a Fine of  $l. 512.846$ ; but the Lessee not having so much ready Money to spare, he is willing to pay an Equivalent Rent, and no Fine. To answer this, you need only to consider by the Rules under the 5th Question, what Annuity to continue 21 Years at 5 per Cent. the said Fine =  $l. 512.846$  will purchase; which you'll find  $l. 40$  per Ann. which added to the  $l. 50$ , shews that he must pay  $l. 90$  per Ann. if he pays no Fine.

These 15 Questions and Proportions contain the Fundamental Rules for solving all Questions of Compound Interest whatever; and whosoever doth thoroughly understand them, may be able to give a true Answer to any Question relating to Selling or Purchasing Estates, &c. without the help of Tables for that purpose; provided he has well learnt the Logarithms.

*The End of Logarithmical Arithmetic, and its Use.*

A T A B L E



A  
T A B L E  
OF THE  
LOGARITHMS  
TO ALL  
NUMBERS,

Not exceeding 10000, or 4 Places, whether  
they be Intire, Broken, or Mixt Numbers.

And the Differences of Logarithms, and the  
Parts proportional, whereby the Logarithm of any  
Number is produced to 100000.

Particularly useful in Extracting the Square,  
Cube, &c. Roots, and solving Questions in Com-  
pound Interest, &c.

O o

*Artificial Numbers: Or,*

| Natural<br>Numbers. | 0       | 1       | 2       | 3       | 4       |
|---------------------|---------|---------|---------|---------|---------|
| 1                   | 0000000 | 0413927 | 0791812 | 1139433 | 1461280 |
| 2                   | 3010300 | 3222193 | 3424227 | 3617278 | 3802112 |
| 3                   | 4771212 | 4913617 | 5051500 | 5185139 | 5314789 |
| 4                   | 6020600 | 6127838 | 6232493 | 6334684 | 6434527 |
| 5                   | 6989700 | 7075702 | 7160033 | 7242759 | 7323937 |
| 6                   | 7781512 | 7853298 | 7923917 | 7993405 | 8061800 |
| 7                   | 8450980 | 8512583 | 8573325 | 8633229 | 8692317 |
| 8                   | 9030900 | 9084850 | 9138138 | 9190781 | 9242793 |
| 9                   | 9542425 | 9590414 | 9637878 | 9684829 | 9731278 |
| 100                 | 0000000 | 0004341 | 0008677 | 0013009 | 0017337 |
| 101                 | 0043214 | 0047511 | 0051805 | 0056094 | 0060379 |
| 102                 | 0086002 | 0090257 | 0094509 | 0098756 | 0102999 |
| 103                 | 0128372 | 0132587 | 0136794 | 0141003 | 0145205 |
| 104                 | 0170333 | 0174507 | 0178677 | 0182843 | 0187005 |
| 105                 | 0211893 | 0216027 | 0220157 | 0224284 | 0228406 |
| 106                 | 0253059 | 0257154 | 0261245 | 0265333 | 0269416 |
| 107                 | 0293838 | 0297895 | 0301948 | 0305997 | 0310043 |
| 108                 | 0334237 | 0338257 | 0342273 | 0346284 | 0350293 |
| 109                 | 0374265 | 0378247 | 0382226 | 0386202 | 0390173 |
| 110                 | 0413927 | 0417873 | 0421816 | 0425755 | 0429691 |
| 111                 | 0453230 | 0457140 | 0461048 | 0464952 | 0468852 |
| 112                 | 0492180 | 0496056 | 0499928 | 0503797 | 0507663 |
| 113                 | 0530784 | 0534626 | 0538464 | 0542299 | 0546130 |
| 114                 | 0569048 | 0572856 | 0576661 | 0580462 | 0584260 |
| 115                 | 0606978 | 0610753 | 0614525 | 0618293 | 0622058 |
| 116                 | 0644580 | 0648322 | 0652061 | 0655797 | 0659530 |
| 117                 | 0681859 | 0685569 | 0689276 | 0692980 | 0696681 |
| 118                 | 0718820 | 0722499 | 0726175 | 0729847 | 0733517 |
| 119                 | 0755470 | 0759118 | 0762762 | 0766404 | 0770043 |
| 120                 | 0791812 | 0795430 | 0799045 | 0802656 | 0806265 |
| 121                 | 0827854 | 0831441 | 0835026 | 0838608 | 0842187 |
| 122                 | 0863598 | 0867157 | 0870712 | 0874264 | 0877814 |
| 123                 | 0899051 | 0902580 | 0906107 | 0909631 | 0913151 |
| 124                 | 0934217 | 0937718 | 0941216 | 0944711 | 0948204 |
| 125                 | 0969100 | 0972573 | 0976043 | 0979511 | 0982975 |
| 126                 | 1003705 | 1007151 | 1010593 | 1014033 | 1017471 |
| 127                 | 1038037 | 1041455 | 1044871 | 1048284 | 1051694 |
| 128                 | 1072100 | 1075491 | 1078880 | 1082266 | 1085650 |
| 129                 | 1105897 | 1109262 | 1112625 | 1115985 | 1119343 |
| 130                 | 1139433 | 1142773 | 1146110 | 1149444 | 1152776 |
| 131                 | 1172713 | 1176027 | 1179338 | 1182647 | 1185954 |
| 132                 | 1205739 | 1209028 | 1212314 | 1215598 | 1218880 |
| 133                 | 1238516 | 1241780 | 1245042 | 1248301 | 1251558 |
| 134                 | 1271048 | 1274288 | 1277525 | 1280760 | 1283993 |

*Logarithms (to 1349.)*

| Natural<br>Numbers. | 5       | 6       | 7       | 8       | 9       |
|---------------------|---------|---------|---------|---------|---------|
| 1                   | 1760912 | 2041200 | 2304489 | 2552725 | 2787536 |
| 2                   | 3979400 | 4149733 | 4313637 | 4471580 | 4623980 |
| 3                   | 5440680 | 5563025 | 5682017 | 5797836 | 5910646 |
| 4                   | 6532125 | 6627578 | 6720978 | 6812412 | 6901961 |
| 5                   | 7403627 | 7481880 | 7558748 | 7634280 | 7708520 |
| 6                   | 8129133 | 8195439 | 8260748 | 8325089 | 8388491 |
| 7                   | 8750613 | 8808136 | 8864907 | 8920946 | 8976271 |
| 8                   | 9294189 | 9344984 | 9395192 | 9444827 | 9493900 |
| 9                   | 9777236 | 9822712 | 9867717 | 9912261 | 9956352 |
| 100                 | 0021661 | 0025980 | 0030295 | 0034605 | 0038912 |
| 101                 | 0064660 | 0068937 | 0073209 | 0077478 | 0081742 |
| 102                 | 0107239 | 0111473 | 0115704 | 0119931 | 0124154 |
| 103                 | 0149403 | 0153597 | 0157787 | 0161973 | 0166155 |
| 104                 | 0191163 | 0195317 | 0199467 | 0203613 | 0207755 |
| 105                 | 0232524 | 0236639 | 0240750 | 0244857 | 0248960 |
| 106                 | 0273496 | 0277572 | 0281644 | 0285712 | 0289777 |
| 107                 | 0314085 | 0318123 | 0322157 | 0326188 | 0330214 |
| 108                 | 0354297 | 0358298 | 0362295 | 0366289 | 0370279 |
| 109                 | 0394141 | 0398105 | 0402066 | 0406023 | 0409977 |
| 110                 | 0433623 | 0437551 | 0441476 | 0445398 | 0449315 |
| 111                 | 0472749 | 0476642 | 0480532 | 0484418 | 0488301 |
| 112                 | 0511525 | 0515384 | 0519239 | 0523091 | 0526939 |
| 113                 | 0549958 | 0553783 | 0557605 | 0561423 | 0565237 |
| 114                 | 0588055 | 0591846 | 0595634 | 0599419 | 0603200 |
| 115                 | 0625820 | 0629578 | 0633334 | 0637085 | 0640834 |
| 116                 | 0663259 | 0666985 | 0670708 | 0674428 | 0678145 |
| 117                 | 0700379 | 0704073 | 0707765 | 0711453 | 0715138 |
| 118                 | 0737183 | 0740847 | 0744507 | 0748164 | 0751818 |
| 119                 | 0773679 | 0777312 | 0780941 | 0784568 | 0788192 |
| 120                 | 0809870 | 0813473 | 0817073 | 0820669 | 0824263 |
| 121                 | 0845763 | 0849336 | 0852906 | 0856473 | 0860037 |
| 122                 | 0881361 | 0884905 | 0888446 | 0891984 | 0895519 |
| 123                 | 0916669 | 0920185 | 0923696 | 0927206 | 0930712 |
| 124                 | 0951693 | 0955180 | 0958664 | 0962146 | 0965624 |
| 125                 | 0986437 | 0989896 | 0993353 | 0996806 | 1000257 |
| 126                 | 1020905 | 1024337 | 1027766 | 1031192 | 1034616 |
| 127                 | 1055102 | 1058506 | 1061909 | 1065308 | 1068705 |
| 128                 | 1089031 | 1092410 | 1095785 | 1099159 | 1102529 |
| 129                 | 1122698 | 1126050 | 1129400 | 1132746 | 1136091 |
| 130                 | 1156105 | 1159432 | 1162756 | 1166077 | 1169396 |
| 131                 | 1189257 | 1192559 | 1195858 | 1199154 | 1202448 |
| 132                 | 1222159 | 1225435 | 1228709 | 1231981 | 1235250 |
| 133                 | 1254813 | 1258064 | 1261314 | 1264561 | 1267806 |
| 134                 | 1287223 | 1290450 | 1293676 | 1296890 | 1300110 |



*Artificial Numbers: Or,*

| Natural<br>Numbers. | 0       | 1       | 2       | 3       | 4       |
|---------------------|---------|---------|---------|---------|---------|
| 135                 | 1303338 | 1306553 | 1309767 | 1312978 | 1316187 |
| 136                 | 1335389 | 1338581 | 1341771 | 1344958 | 1348144 |
| 137                 | 1367206 | 1370374 | 1373541 | 1376705 | 1379867 |
| 138                 | 1398791 | 1401937 | 1405080 | 1408222 | 1411361 |
| 139                 | 1430148 | 1433271 | 1436392 | 1439511 | 1442628 |
| 140                 | 1461280 | 1464381 | 1467480 | 1470577 | 1473671 |
| 141                 | 1492191 | 1495270 | 1498347 | 1501422 | 1504494 |
| 142                 | 1522883 | 1525941 | 1528996 | 1532049 | 1535100 |
| 143                 | 1553360 | 1556396 | 1559430 | 1562462 | 1565491 |
| 144                 | 1583625 | 1586640 | 1589653 | 1592663 | 1595672 |
| 145                 | 1613680 | 1616674 | 1619666 | 1622656 | 1625644 |
| 146                 | 1643528 | 1646502 | 1649474 | 1652443 | 1655411 |
| 147                 | 1673173 | 1676127 | 1679078 | 1682027 | 1684975 |
| 148                 | 1702617 | 1705550 | 1708482 | 1711411 | 1714339 |
| 149                 | 1731863 | 1734776 | 1737688 | 1740598 | 1743506 |
| 150                 | 1760913 | 1763807 | 1766699 | 1769590 | 1772478 |
| 151                 | 1789769 | 1792645 | 1795518 | 1798389 | 1801259 |
| 152                 | 1818436 | 1821292 | 1824146 | 1826999 | 1829850 |
| 153                 | 1846941 | 1849752 | 1852588 | 1855421 | 1858253 |
| 154                 | 1875207 | 1878026 | 1880844 | 1883659 | 1886473 |
| 155                 | 1903317 | 1906118 | 1908917 | 1911714 | 1914510 |
| 156                 | 1931246 | 1934029 | 1936810 | 1939590 | 1942367 |
| 157                 | 1958996 | 1961762 | 1964525 | 1967287 | 1970047 |
| 158                 | 1986571 | 1989319 | 1992065 | 1994809 | 1997552 |
| 159                 | 2013971 | 2016702 | 2019431 | 2022158 | 2024883 |
| 160                 | 2041200 | 2043913 | 2046625 | 2049335 | 2052044 |
| 161                 | 2068259 | 2070955 | 2073650 | 2076344 | 2079035 |
| 162                 | 2095150 | 2097830 | 2100508 | 2103185 | 2105860 |
| 163                 | 2121876 | 2124540 | 2127201 | 2129862 | 2132521 |
| 164                 | 2148438 | 2151086 | 2153732 | 2156376 | 2159018 |
| 165                 | 2174839 | 2177471 | 2180100 | 2182727 | 2185355 |
| 166                 | 2201081 | 2203696 | 2206310 | 2208922 | 2211533 |
| 167                 | 2227165 | 2229764 | 2232363 | 2234959 | 2237554 |
| 168                 | 2253093 | 2255677 | 2258260 | 2260841 | 2263421 |
| 169                 | 2278867 | 2281436 | 2284003 | 2286570 | 2289134 |
| 170                 | 2304489 | 2307043 | 2309596 | 2312146 | 2314696 |
| 171                 | 2329961 | 2332500 | 2335038 | 2337574 | 2340108 |
| 172                 | 2355284 | 2357809 | 2360331 | 2362853 | 2365373 |
| 173                 | 2380461 | 2382971 | 2385479 | 2387986 | 2390491 |
| 174                 | 2405492 | 2407988 | 2410481 | 2412974 | 2415465 |
| 175                 | 2430380 | 2432861 | 2435341 | 2437819 | 2440296 |
| 176                 | 2455127 | 2457593 | 2460059 | 2462523 | 2464986 |
| 177                 | 2479733 | 2482186 | 2484637 | 2487085 | 2489536 |
| 178                 | 2504200 | 2506639 | 2509077 | 2511513 | 2513948 |

*Logarithms (to 1789.)*

| Natural<br>Numbers. | 5       | 6       | 7       | 8       | 9       |
|---------------------|---------|---------|---------|---------|---------|
| 135                 | 1319393 | 1322597 | 1325798 | 1328998 | 1332194 |
| 136                 | 1351326 | 1354507 | 1357685 | 1360861 | 1364034 |
| 137                 | 1383027 | 1386184 | 1389339 | 1392492 | 1395643 |
| 138                 | 1414498 | 1417632 | 1420765 | 1423895 | 1427022 |
| 139                 | 1445742 | 1448854 | 1451964 | 1455072 | 1458177 |
| 140                 | 1476763 | 1479853 | 1482941 | 1486026 | 1489110 |
| 141                 | 1507564 | 1510632 | 1513698 | 1516762 | 1519824 |
| 142                 | 1538149 | 1541195 | 1544240 | 1547282 | 1550322 |
| 143                 | 1568519 | 1571544 | 1574568 | 1577589 | 1580608 |
| 144                 | 1598678 | 1601683 | 1604685 | 1607686 | 1610684 |
| 145                 | 1628630 | 1631614 | 1634595 | 1637575 | 1640553 |
| 146                 | 1658376 | 1661340 | 1664301 | 1667260 | 1670218 |
| 147                 | 1687920 | 1690863 | 1693805 | 1696744 | 1699682 |
| 148                 | 1717264 | 1720188 | 1723110 | 1726029 | 1728947 |
| 149                 | 1746412 | 1749316 | 1752218 | 1755118 | 1758016 |
| 150                 | 1775365 | 1778250 | 1781132 | 1784013 | 1786892 |
| 151                 | 1804126 | 1806992 | 1809856 | 1812718 | 1815578 |
| 152                 | 1832698 | 1835545 | 1838390 | 1841233 | 1844075 |
| 153                 | 1861084 | 1863912 | 1866739 | 1869563 | 1872386 |
| 154                 | 1889285 | 1892095 | 1894903 | 1897709 | 1900514 |
| 155                 | 1917304 | 1920096 | 1922886 | 1925674 | 1928461 |
| 156                 | 1945143 | 1947917 | 1950690 | 1953460 | 1956229 |
| 157                 | 1972806 | 1975562 | 1978317 | 1981070 | 1983821 |
| 158                 | 2000293 | 2003032 | 2005769 | 2008505 | 2011239 |
| 159                 | 2027607 | 2030329 | 2033049 | 2035768 | 2038485 |
| 160                 | 2054750 | 2057455 | 2060159 | 2062869 | 2065560 |
| 161                 | 2081725 | 2084413 | 2087100 | 2089785 | 2092468 |
| 162                 | 2108534 | 2111203 | 2113876 | 2116544 | 2119211 |
| 163                 | 2135178 | 2137833 | 2140487 | 2143139 | 2145789 |
| 164                 | 2161659 | 2164298 | 2166936 | 2169572 | 2172206 |
| 165                 | 2187980 | 2190603 | 2193225 | 2195845 | 2198464 |
| 166                 | 2214142 | 2216750 | 2219356 | 2221960 | 2224563 |
| 167                 | 2240148 | 2242740 | 2245331 | 2247920 | 2250507 |
| 168                 | 2265999 | 2268576 | 2271151 | 2273724 | 2276296 |
| 169                 | 2291697 | 2294258 | 2296818 | 2299377 | 2301934 |
| 170                 | 2317244 | 2319790 | 2322335 | 2324879 | 2327421 |
| 171                 | 2342641 | 2345173 | 2347703 | 2350232 | 2352759 |
| 172                 | 2367891 | 2370408 | 2372923 | 2375437 | 2377950 |
| 173                 | 2392995 | 2395497 | 2397998 | 2400498 | 2402996 |
| 174                 | 2417954 | 2420442 | 2422929 | 2425414 | 2427898 |
| 175                 | 2442771 | 2445245 | 2447718 | 2450189 | 2452658 |
| 176                 | 2467447 | 2469907 | 2472365 | 2474823 | 2477278 |
| 177                 | 2491984 | 2494430 | 2496874 | 2499317 | 2501759 |
| 178                 | 2516382 | 2518814 | 2521246 | 2523675 | 2526103 |

*Artificial Numbers: Or,*

| Natural<br>Numbers. | 0       | 1       | 2       | 3       | 4       |
|---------------------|---------|---------|---------|---------|---------|
| 179                 | 2528530 | 2530956 | 2533380 | 2535803 | 2538224 |
| 180                 | 2552725 | 2555137 | 2557548 | 2559957 | 2562365 |
| 181                 | 2576786 | 2579184 | 2581582 | 2583978 | 2586373 |
| 182                 | 2600714 | 2603099 | 2605484 | 2607867 | 2610248 |
| 183                 | 2624511 | 2626883 | 2629255 | 2631625 | 2633993 |
| 184                 | 2648178 | 2650538 | 2652896 | 2655253 | 2657609 |
| 185                 | 2671717 | 2674064 | 2676410 | 2678752 | 2681097 |
| 186                 | 2695129 | 2697464 | 2699797 | 2702128 | 2704459 |
| 187                 | 2718416 | 2720738 | 2723058 | 2725378 | 2727696 |
| 188                 | 2741578 | 2743888 | 2746196 | 2748503 | 2750809 |
| 189                 | 2764618 | 2766915 | 2769211 | 2771506 | 2773800 |
| 190                 | 2787536 | 2789821 | 2792105 | 2794388 | 2796669 |
| 191                 | 2810334 | 2812607 | 2814879 | 2817150 | 2819419 |
| 192                 | 2833012 | 2835274 | 2837534 | 2839793 | 2842051 |
| 193                 | 2855573 | 2857823 | 2860071 | 2862318 | 2864565 |
| 194                 | 2878017 | 2880255 | 2882492 | 2884728 | 2886963 |
| 195                 | 2900346 | 2902573 | 2904798 | 2907022 | 2909246 |
| 196                 | 2922561 | 2924776 | 2926990 | 2929203 | 2931415 |
| 197                 | 2944662 | 2946866 | 2949069 | 2951271 | 2953471 |
| 198                 | 2966652 | 2968845 | 2971036 | 2973227 | 2975417 |
| 199                 | 2988531 | 2990713 | 2992893 | 2995073 | 2997251 |
| 200                 | 3010300 | 3012471 | 3014641 | 3016809 | 3018977 |
| 201                 | 3031961 | 3034121 | 3036280 | 3038438 | 3040595 |
| 202                 | 3053514 | 3055663 | 3057811 | 3059959 | 3062105 |
| 203                 | 3074960 | 3077099 | 3079237 | 3081374 | 3083509 |
| 204                 | 3096302 | 3098430 | 3100557 | 3102684 | 3104809 |
| 205                 | 3117539 | 3119657 | 3121774 | 3123889 | 3126004 |
| 206                 | 3138672 | 3140780 | 3142887 | 3144992 | 3147097 |
| 207                 | 3159703 | 3161801 | 3163897 | 3165993 | 3168087 |
| 208                 | 3180633 | 3182721 | 3184807 | 3186893 | 3188977 |
| 209                 | 3201463 | 3203540 | 3205617 | 3207692 | 3209767 |
| 210                 | 3222193 | 3224260 | 3226327 | 3228393 | 3230457 |
| 211                 | 3242825 | 3244882 | 3246939 | 3248995 | 3251050 |
| 212                 | 3263359 | 3265407 | 3267454 | 3269500 | 3271545 |
| 213                 | 3283796 | 3285834 | 3287872 | 3289909 | 3291944 |
| 214                 | 3304138 | 3306167 | 3308195 | 3310222 | 3312248 |
| 215                 | 3324385 | 3326404 | 3328423 | 3330440 | 3332457 |
| 216                 | 3344537 | 3346548 | 3348557 | 3350565 | 3352572 |
| 217                 | 3364597 | 3366598 | 3368598 | 3370597 | 3372595 |
| 218                 | 3384565 | 3386557 | 3388547 | 3390537 | 3392526 |
| 219                 | 3404441 | 3406424 | 3408405 | 3410386 | 3412366 |
| 220                 | 3424227 | 3426200 | 3428173 | 3430145 | 3432116 |
| 221                 | 3443923 | 3445887 | 3447851 | 3449814 | 3451776 |
| 222                 | 3463530 | 3465486 | 3467441 | 3469395 | 3471348 |



*Logarithms (to 2229.)*

| Natural<br>Numbers. | 5       | 6       | 7       | 8       | 9       |
|---------------------|---------|---------|---------|---------|---------|
| 179                 | 2540645 | 2543063 | 2545481 | 2547897 | 2550312 |
| 180                 | 2564772 | 2567177 | 2569581 | 2571984 | 2574386 |
| 181                 | 2588766 | 2591158 | 2593549 | 2595939 | 2598327 |
| 182                 | 2612629 | 2615008 | 2617385 | 2619762 | 2622137 |
| 183                 | 2636361 | 2638727 | 2641092 | 2643455 | 2645817 |
| 184                 | 2659964 | 2662317 | 2664669 | 2667020 | 2669369 |
| 185                 | 2683439 | 2685780 | 2688119 | 2690457 | 2692794 |
| 186                 | 2706788 | 2709116 | 2711443 | 2713769 | 2716093 |
| 187                 | 2730013 | 2732328 | 2734643 | 2736956 | 2739268 |
| 188                 | 2753113 | 2755417 | 2757719 | 2760020 | 2762320 |
| 189                 | 2776092 | 2778383 | 2780673 | 2782952 | 2785250 |
| 190                 | 2798950 | 2801229 | 2803507 | 2805784 | 2808059 |
| 191                 | 2821688 | 2823955 | 2826221 | 2828486 | 2830750 |
| 192                 | 2844307 | 2846563 | 2848817 | 2851070 | 2853322 |
| 193                 | 2866810 | 2869054 | 2871296 | 2873538 | 2875778 |
| 194                 | 2889196 | 2891428 | 2893659 | 2895889 | 2898118 |
| 195                 | 2911468 | 2913688 | 2915908 | 2918127 | 2920344 |
| 196                 | 2933626 | 2935835 | 2938044 | 2940251 | 2942457 |
| 197                 | 2955671 | 2957869 | 2960067 | 2962263 | 2964458 |
| 198                 | 2977605 | 2979792 | 2981979 | 2984164 | 2986348 |
| 199                 | 2999420 | 3001605 | 3003781 | 3005955 | 3008128 |
| 200                 | 3021141 | 3023309 | 3025474 | 3027637 | 3029799 |
| 201                 | 3042751 | 3044905 | 3047059 | 3049212 | 3051363 |
| 202                 | 3064250 | 3066394 | 3068537 | 3070679 | 3072820 |
| 203                 | 3085644 | 3087778 | 3089910 | 3092042 | 3094171 |
| 204                 | 3106933 | 3109056 | 3111178 | 3113299 | 3115420 |
| 205                 | 3128118 | 3130231 | 3132343 | 3134454 | 3136563 |
| 206                 | 3149200 | 3151303 | 3153405 | 3155505 | 3157605 |
| 207                 | 3170181 | 3172273 | 3174365 | 3176455 | 3178545 |
| 208                 | 3191061 | 3193143 | 3195224 | 3197305 | 3199384 |
| 209                 | 3211840 | 3213913 | 3215984 | 3218055 | 3220124 |
| 210                 | 3232521 | 3234584 | 3236645 | 3238706 | 3240766 |
| 211                 | 3253104 | 3255157 | 3257209 | 3259260 | 3261310 |
| 212                 | 3273589 | 3275633 | 3277675 | 3279716 | 3281757 |
| 213                 | 3293979 | 3296012 | 3298045 | 3300077 | 3302108 |
| 214                 | 3314273 | 3316297 | 3318320 | 3320343 | 3322364 |
| 215                 | 3334473 | 3336488 | 3338501 | 3340514 | 3342526 |
| 216                 | 3354579 | 3356585 | 3358589 | 3360593 | 3362596 |
| 217                 | 3374593 | 3376589 | 3378584 | 3380579 | 3382572 |
| 218                 | 3394514 | 3396501 | 3398488 | 3400473 | 3402458 |
| 219                 | 3414345 | 3416323 | 3418301 | 3420277 | 3422252 |
| 220                 | 3434086 | 3436055 | 3438023 | 3439991 | 3441957 |
| 221                 | 3453737 | 3455698 | 3457657 | 3459615 | 3461573 |
| 222                 | 3473300 | 3475252 | 3477202 | 3479152 | 3481101 |

*Artificial Numbers: Or,*

| Natural<br>Numbers. | 0       | 1       | 2       | 3       | 4       |
|---------------------|---------|---------|---------|---------|---------|
| 223                 | 3483049 | 3484996 | 3486942 | 3488887 | 3490832 |
| 224                 | 3502480 | 3504419 | 3506356 | 3508293 | 3510228 |
| 225                 | 3521815 | 3523755 | 3525684 | 3527612 | 3529539 |
| 226                 | 3541084 | 3543006 | 3544926 | 3546845 | 3548764 |
| 227                 | 3560259 | 3562171 | 3564083 | 3565994 | 3567905 |
| 228                 | 3579348 | 3581253 | 3583156 | 3585059 | 3586961 |
| 229                 | 3598355 | 3600251 | 3602146 | 3604040 | 3605934 |
| 230                 | 3617278 | 3619166 | 3621053 | 3622939 | 3624825 |
| 231                 | 3636120 | 3638000 | 3639878 | 3641756 | 3643633 |
| 232                 | 3654880 | 3656751 | 3658622 | 3660492 | 3662361 |
| 233                 | 3673559 | 3675423 | 3677285 | 3679147 | 3681008 |
| 234                 | 3692159 | 3694014 | 3695869 | 3697723 | 3699576 |
| 235                 | 3710679 | 3712526 | 3714373 | 3716219 | 3718065 |
| 236                 | 3729120 | 3730960 | 3732799 | 3734637 | 3736475 |
| 237                 | 3747483 | 3749316 | 3751147 | 3752977 | 3754807 |
| 238                 | 3765769 | 3767594 | 3769418 | 3771240 | 3773062 |
| 239                 | 3783979 | 3785796 | 3787612 | 3789427 | 3791241 |
| 240                 | 3802112 | 3803922 | 3805730 | 3807538 | 3809345 |
| 241                 | 3820170 | 3821972 | 3823773 | 3825573 | 3827373 |
| 242                 | 3838154 | 3839948 | 3841741 | 3843534 | 3845326 |
| 243                 | 3856063 | 3857850 | 3859636 | 3861421 | 3863206 |
| 244                 | 3873898 | 3875678 | 3877457 | 3879235 | 3881012 |
| 245                 | 3891661 | 3893433 | 3895205 | 3896975 | 3898746 |
| 246                 | 3909351 | 3911116 | 3912880 | 3914644 | 3916407 |
| 247                 | 3926969 | 3928727 | 3930485 | 3932241 | 3933997 |
| 248                 | 3944517 | 3946268 | 3948018 | 3949767 | 3951516 |
| 249                 | 3961993 | 3963734 | 3965480 | 3967223 | 3968964 |
| 250                 | 3979400 | 3981137 | 3982873 | 3984608 | 3986343 |
| 251                 | 3996737 | 3998467 | 4000196 | 4001925 | 4003653 |
| 252                 | 4014005 | 4015728 | 4017451 | 4019172 | 4020893 |
| 253                 | 4031205 | 4032921 | 4034637 | 4036352 | 4038066 |
| 254                 | 4048337 | 4050047 | 4051755 | 4053464 | 4055171 |
| 255                 | 4065402 | 4067105 | 4068807 | 4070508 | 4072209 |
| 256                 | 4082400 | 4084096 | 4085791 | 4087486 | 4089180 |
| 257                 | 4099331 | 4101021 | 4102710 | 4104398 | 4106085 |
| 258                 | 4116197 | 4117880 | 4119562 | 4121244 | 4122925 |
| 259                 | 4132998 | 4134674 | 4136350 | 4138025 | 4139700 |
| 260                 | 4149733 | 4151404 | 4153073 | 4154742 | 4156410 |
| 261                 | 4166405 | 4168069 | 4169732 | 4171394 | 4173056 |
| 262                 | 4183013 | 4184670 | 4186327 | 4187983 | 4189638 |
| 263                 | 4199557 | 4201208 | 4202859 | 4204509 | 4206158 |
| 264                 | 4216039 | 4217684 | 4219328 | 4220972 | 4222614 |
| 265                 | 4232459 | 4234097 | 4235735 | 4237372 | 4239009 |
| 266                 | 4248816 | 4250449 | 4252080 | 4253712 | 4255342 |

*Logarithms (to 2669.)*

| Natural<br>Numbers. | 5       | 6       | 7       | 8       | 9       |
|---------------------|---------|---------|---------|---------|---------|
| 223                 | 3492775 | 3494718 | 3496660 | 3498601 | 3500541 |
| 224                 | 3512163 | 3514098 | 3516031 | 3517963 | 3519895 |
| 225                 | 3531465 | 3533391 | 3535316 | 3537239 | 3539162 |
| 226                 | 3550682 | 3552599 | 3554515 | 3556430 | 3558345 |
| 227                 | 3569813 | 3571723 | 3573630 | 3575537 | 3577443 |
| 228                 | 3588862 | 3590762 | 3592662 | 3594560 | 3596458 |
| 229                 | 3607827 | 3609719 | 3611610 | 3613500 | 3615390 |
| 230                 | 3626709 | 3628593 | 3630476 | 3632358 | 3634239 |
| 231                 | 3645510 | 3647386 | 3649260 | 3651134 | 3653007 |
| 232                 | 3664230 | 3666097 | 3667964 | 3669830 | 3671695 |
| 233                 | 3682869 | 3684728 | 3686587 | 3688445 | 3690302 |
| 234                 | 3701428 | 3703280 | 3705131 | 3706981 | 3708830 |
| 235                 | 3719909 | 3721753 | 3723596 | 3725438 | 3727279 |
| 236                 | 3738311 | 3740147 | 3741983 | 3743817 | 3745651 |
| 237                 | 3756636 | 3758464 | 3760292 | 3762118 | 3763944 |
| 238                 | 3774884 | 3776704 | 3778524 | 3780343 | 3782161 |
| 239                 | 3793055 | 3794868 | 3796680 | 3798492 | 3800302 |
| 240                 | 3811151 | 3812956 | 3814761 | 3816565 | 3818368 |
| 241                 | 3829171 | 3830969 | 3832766 | 3834563 | 3836359 |
| 242                 | 3847117 | 3848908 | 3850698 | 3852487 | 3854275 |
| 243                 | 3864990 | 3866773 | 3868555 | 3870337 | 3872118 |
| 244                 | 3882789 | 3884565 | 3886340 | 3888114 | 3889888 |
| 245                 | 3900515 | 3902284 | 3904052 | 3905819 | 3907585 |
| 246                 | 3918169 | 3919931 | 3921691 | 3923452 | 3925211 |
| 247                 | 3935752 | 3937506 | 3939260 | 3941013 | 3942765 |
| 248                 | 3953264 | 3955011 | 3956758 | 3958504 | 3960249 |
| 249                 | 3970705 | 3972446 | 3974185 | 3975924 | 3977662 |
| 250                 | 3988077 | 3989811 | 3991543 | 3993275 | 3995007 |
| 251                 | 4005380 | 4007106 | 4008832 | 4010557 | 4012282 |
| 252                 | 4022614 | 4024333 | 4026052 | 4027771 | 4029488 |
| 253                 | 4039780 | 4041492 | 4043205 | 4044916 | 4046627 |
| 254                 | 4056878 | 4058584 | 4060289 | 4061994 | 4063698 |
| 255                 | 4073909 | 4075608 | 4077307 | 4079005 | 4080703 |
| 256                 | 4090874 | 4092567 | 4094259 | 4095950 | 4097641 |
| 257                 | 4107772 | 4109459 | 4111144 | 4112829 | 4114513 |
| 258                 | 4124605 | 4126285 | 4127964 | 4129643 | 4131320 |
| 259                 | 4141374 | 4143047 | 4144719 | 4146391 | 4148063 |
| 260                 | 4158077 | 4159744 | 4161410 | 4163076 | 4164741 |
| 261                 | 4174717 | 4176377 | 4178037 | 4179696 | 4181355 |
| 262                 | 4191293 | 4192947 | 4194601 | 4196254 | 4197906 |
| 263                 | 4207806 | 4209454 | 4211101 | 4212748 | 4214394 |
| 264                 | 4224257 | 4225898 | 4227539 | 4229180 | 4230820 |
| 265                 | 4240645 | 4242281 | 4243915 | 4245550 | 4247183 |
| 266                 | 4256972 | 4258601 | 4260230 | 4261858 | 4263486 |



*Artificial Numbers: Or,*

| Natural<br>Numbers. | 0       | 1       | 2       | 3       | 4       |
|---------------------|---------|---------|---------|---------|---------|
| 267                 | 4265113 | 4266739 | 4268365 | 4269990 | 4271614 |
| 268                 | 4281348 | 4282968 | 4284588 | 4286207 | 4287825 |
| 269                 | 4297523 | 4299137 | 4300751 | 4302364 | 4303976 |
| 270                 | 4313638 | 4315246 | 4316853 | 4318460 | 4320067 |
| 271                 | 4329693 | 4331295 | 4332897 | 4334498 | 4336098 |
| 272                 | 4345689 | 4347285 | 4348881 | 4350476 | 4352071 |
| 273                 | 4361626 | 4363217 | 4364807 | 4366396 | 4367985 |
| 274                 | 4377506 | 4379090 | 4380674 | 4382258 | 4383841 |
| 275                 | 4393327 | 4394906 | 4396484 | 4398062 | 4399639 |
| 276                 | 4409091 | 4410664 | 4412237 | 4413809 | 4415380 |
| 277                 | 4424798 | 4426365 | 4427932 | 4429499 | 4431065 |
| 278                 | 4440448 | 4442010 | 4443571 | 4445132 | 4446692 |
| 279                 | 4456042 | 4457598 | 4459154 | 4460709 | 4462264 |
| 280                 | 4471580 | 4473131 | 4474681 | 4476231 | 4477780 |
| 281                 | 4487063 | 4488608 | 4490153 | 4491697 | 4493241 |
| 282                 | 4502491 | 4504031 | 4505570 | 4507109 | 4508647 |
| 283                 | 4517864 | 4519399 | 4520932 | 4522466 | 4523998 |
| 284                 | 4533183 | 4534712 | 4536241 | 4537769 | 4539296 |
| 285                 | 4548449 | 4549972 | 4551495 | 4553018 | 4554540 |
| 286                 | 4563660 | 4565179 | 4566696 | 4568213 | 4569731 |
| 287                 | 4578819 | 4580332 | 4581844 | 4583356 | 4584868 |
| 288                 | 4593925 | 4595433 | 4596940 | 4598446 | 4599953 |
| 289                 | 4608978 | 4610481 | 4611983 | 4613484 | 4614985 |
| 290                 | 4623980 | 4625477 | 4626974 | 4628470 | 4629966 |
| 291                 | 4638930 | 4640422 | 4641914 | 4643405 | 4644895 |
| 292                 | 4653828 | 4655316 | 4656802 | 4658288 | 4659774 |
| 293                 | 4668676 | 4670158 | 4671640 | 4673120 | 4674601 |
| 294                 | 4683473 | 4684950 | 4686427 | 4687903 | 4689378 |
| 295                 | 4698220 | 4699692 | 4701163 | 4702634 | 4704105 |
| 296                 | 4712917 | 4714384 | 4715850 | 4717317 | 4718782 |
| 297                 | 4727564 | 4729027 | 4730488 | 4731949 | 4733410 |
| 298                 | 4742163 | 4743620 | 4745076 | 4746533 | 4747988 |
| 299                 | 4756712 | 4758164 | 4759616 | 4761067 | 4762518 |
| 300                 | 4771212 | 4772660 | 4774107 | 4775553 | 4776999 |
| 301                 | 4785665 | 4787108 | 4788550 | 4789991 | 4791432 |
| 302                 | 4800069 | 4801507 | 4802945 | 4804381 | 4805818 |
| 303                 | 4814426 | 4815859 | 4817292 | 4818724 | 4820156 |
| 304                 | 4828736 | 4830164 | 4831592 | 4833019 | 4834446 |
| 305                 | 4842998 | 4844422 | 4845845 | 4847268 | 4848690 |
| 306                 | 4857214 | 4858633 | 4860052 | 4861470 | 4862888 |
| 307                 | 4871384 | 4872798 | 4874212 | 4875626 | 4877039 |
| 308                 | 4885507 | 4886917 | 4888326 | 4889735 | 4891144 |
| 309                 | 4899585 | 4900990 | 4902395 | 4903799 | 4905203 |
| 310                 | 4913617 | 4915018 | 4916418 | 4917818 | 4919217 |

*Logarithms (to 3109.)*

| Natural<br>Numbers. | 5       | 6       | 7       | 8       | 9       |
|---------------------|---------|---------|---------|---------|---------|
| 267                 | 4273238 | 4274861 | 4276484 | 4278106 | 4279727 |
| 268                 | 4289443 | 4291060 | 4292677 | 4294293 | 4295908 |
| 269                 | 4305588 | 4307199 | 4308809 | 4310419 | 4312029 |
| 270                 | 4321673 | 4323278 | 4324883 | 4326487 | 4328090 |
| 271                 | 4337698 | 4339298 | 4340896 | 4342494 | 4344092 |
| 272                 | 4353665 | 4355258 | 4356851 | 4358444 | 4360035 |
| 273                 | 4369573 | 4371161 | 4372748 | 4374334 | 4375920 |
| 274                 | 4385423 | 4387005 | 4388587 | 4390167 | 4391747 |
| 275                 | 4401216 | 4402792 | 4404368 | 4405943 | 4407517 |
| 276                 | 4416951 | 4418522 | 4420092 | 4421661 | 4423229 |
| 277                 | 4432630 | 4434195 | 4435759 | 4437322 | 4438885 |
| 278                 | 4448252 | 4449811 | 4451370 | 4452928 | 4454485 |
| 279                 | 4463818 | 4465372 | 4466925 | 4468477 | 4470029 |
| 280                 | 4479329 | 4480877 | 4482424 | 4483971 | 4485517 |
| 281                 | 4494784 | 4496326 | 4497868 | 4499410 | 4500951 |
| 282                 | 4510184 | 4511721 | 4513258 | 4514794 | 4516329 |
| 283                 | 4525531 | 4527062 | 4528593 | 4530124 | 4531654 |
| 284                 | 4540823 | 4542349 | 4543875 | 4545400 | 4546924 |
| 285                 | 4556061 | 4557582 | 4559102 | 4560622 | 4562142 |
| 286                 | 4571246 | 4572762 | 4574277 | 4575791 | 4577305 |
| 287                 | 4586378 | 4587889 | 4589399 | 4590908 | 4592417 |
| 288                 | 4601458 | 4602963 | 4604468 | 4605972 | 4607475 |
| 289                 | 4616486 | 4617986 | 4619485 | 4620984 | 4622482 |
| 290                 | 4631461 | 4632956 | 4634450 | 4635944 | 4637437 |
| 291                 | 4646386 | 4647875 | 4649364 | 4650853 | 4652341 |
| 292                 | 4661259 | 4662743 | 4664227 | 4665711 | 4667194 |
| 293                 | 4676081 | 4677560 | 4679039 | 4680518 | 4681996 |
| 294                 | 4690853 | 4692327 | 4693801 | 4695275 | 4696748 |
| 295                 | 4705575 | 4707044 | 4708513 | 4709982 | 4711450 |
| 296                 | 4720247 | 4721711 | 4723175 | 4724639 | 4726102 |
| 297                 | 4734870 | 4736329 | 4737788 | 4739247 | 4740705 |
| 298                 | 4749443 | 4750898 | 4752352 | 4753806 | 4755259 |
| 299                 | 4763968 | 4765418 | 4766867 | 4768316 | 4769765 |
| 300                 | 4778445 | 4779890 | 4781334 | 4782778 | 4784222 |
| 301                 | 4792873 | 4794313 | 4795754 | 4797192 | 4798631 |
| 302                 | 4807254 | 4808689 | 4810124 | 4811559 | 4812993 |
| 303                 | 4821587 | 4823018 | 4824448 | 4825878 | 4827307 |
| 304                 | 4835873 | 4837299 | 4838725 | 4840150 | 4841574 |
| 305                 | 4850112 | 4851533 | 4852954 | 4854375 | 4855795 |
| 306                 | 4864305 | 4865721 | 4867138 | 4868554 | 4869969 |
| 307                 | 4878451 | 4879863 | 4881275 | 4882686 | 4884097 |
| 308                 | 4892552 | 4893959 | 4895366 | 4896773 | 4898179 |
| 309                 | 4906607 | 4908009 | 4909412 | 4910814 | 4912216 |
| 310                 | 4920616 | 4922014 | 4923413 | 4924810 | 4926207 |

*Artificial Numbers: Or,*

| Natural<br>Numbers. | 0       | 1       | 2       | 3       | 4       |
|---------------------|---------|---------|---------|---------|---------|
| 311                 | 4927604 | 4929000 | 4930396 | 4931791 | 4933186 |
| 312                 | 4941546 | 4942938 | 4944329 | 4945720 | 4947110 |
| 313                 | 4955443 | 4956831 | 4958218 | 4959604 | 4960990 |
| 314                 | 4969296 | 4970679 | 4972062 | 4973444 | 4974825 |
| 315                 | 4983106 | 4984484 | 4985862 | 4987240 | 4988617 |
| 316                 | 4996871 | 4998245 | 4999619 | 5000992 | 5002365 |
| 317                 | 5010593 | 5011962 | 5013332 | 5014701 | 5016069 |
| 318                 | 5024271 | 5025637 | 5027002 | 5028366 | 5029731 |
| 319                 | 5037907 | 5039268 | 5040629 | 5041989 | 5043349 |
| 320                 | 5051500 | 5052857 | 5054213 | 5055569 | 5056925 |
| 321                 | 5065050 | 5066403 | 5067755 | 5069107 | 5070459 |
| 322                 | 5078559 | 5079907 | 5081255 | 5082603 | 5083950 |
| 323                 | 5092025 | 5093370 | 5094713 | 5096057 | 5097400 |
| 324                 | 5105450 | 5106790 | 5108130 | 5109469 | 5110808 |
| 325                 | 5118834 | 5120170 | 5121505 | 5122841 | 5124175 |
| 326                 | 5132176 | 5133508 | 5134840 | 5136171 | 5137501 |
| 327                 | 5145478 | 5146805 | 5148133 | 5149460 | 5150787 |
| 328                 | 5158738 | 5160062 | 5161386 | 5162709 | 5164031 |
| 329                 | 5171959 | 5173279 | 5174598 | 5175917 | 5177236 |
| 330                 | 5185139 | 5186455 | 5187771 | 5189086 | 5190400 |
| 331                 | 5198280 | 5199592 | 5200903 | 5202214 | 5203525 |
| 332                 | 5211381 | 5212689 | 5213996 | 5215303 | 5216610 |
| 333                 | 5224442 | 5225746 | 5227050 | 5228353 | 5229656 |
| 334                 | 5237465 | 5238765 | 5240064 | 5241364 | 5242663 |
| 335                 | 5250448 | 5251744 | 5253040 | 5254335 | 5255631 |
| 336                 | 5263393 | 5264685 | 5265977 | 5267269 | 5268560 |
| 337                 | 5276299 | 5277588 | 5278876 | 5280163 | 5281451 |
| 338                 | 5289167 | 5290452 | 5291736 | 5293020 | 5294303 |
| 339                 | 5301997 | 5303278 | 5304558 | 5305839 | 5307118 |
| 340                 | 5314789 | 5316066 | 5317343 | 5318619 | 5319895 |
| 341                 | 5327544 | 5328817 | 5330090 | 5331363 | 5332635 |
| 342                 | 5340261 | 5341531 | 5342800 | 5344069 | 5345338 |
| 343                 | 5352941 | 5354207 | 5355473 | 5356738 | 5358003 |
| 344                 | 5365584 | 5366847 | 5368109 | 5369370 | 5370631 |
| 345                 | 5378191 | 5379450 | 5380708 | 5381966 | 5383223 |
| 346                 | 5390761 | 5392016 | 5393271 | 5394525 | 5395779 |
| 347                 | 5403295 | 5404546 | 5405797 | 5407048 | 5408298 |
| 348                 | 5415792 | 5417040 | 5418288 | 5419535 | 5420781 |
| 349                 | 5428254 | 5429498 | 5430742 | 5431986 | 5433229 |
| 350                 | 5440680 | 5441921 | 5443161 | 5444401 | 5445641 |
| 351                 | 5453071 | 5454308 | 5455545 | 5456781 | 5458017 |
| 352                 | 5465427 | 5466660 | 5467894 | 5469126 | 5470359 |
| 353                 | 5477747 | 5478977 | 5480207 | 5481436 | 5482665 |
| 354                 | 5490033 | 5491259 | 5492486 | 5493712 | 5494937 |



*Logarithms (to 3549.)*

| Natural<br>Numbers. | 5       | 6       | 7       | 8       | 9       |
|---------------------|---------|---------|---------|---------|---------|
| 311                 | 4934580 | 4935974 | 4937368 | 4938761 | 4940154 |
| 312                 | 4948500 | 4949890 | 4951279 | 4952667 | 4954056 |
| 313                 | 4962375 | 4963761 | 4965145 | 4966529 | 4967913 |
| 314                 | 4976206 | 4977587 | 4978967 | 4980347 | 4981727 |
| 315                 | 4989994 | 4991370 | 4992746 | 4994121 | 4995496 |
| 316                 | 5003737 | 5005109 | 5006481 | 5007852 | 5009222 |
| 317                 | 5017437 | 5018805 | 5020172 | 5021539 | 5022905 |
| 318                 | 5031094 | 5032458 | 5033821 | 5035183 | 5036545 |
| 319                 | 5044709 | 5046068 | 5047426 | 5048785 | 5050142 |
| 320                 | 5058280 | 5059635 | 5060990 | 5062344 | 5063697 |
| 321                 | 5071810 | 5073160 | 5074511 | 5075860 | 5077210 |
| 322                 | 5085297 | 5086644 | 5087990 | 5089335 | 5090680 |
| 323                 | 5098743 | 5100085 | 5101427 | 5102768 | 5104109 |
| 324                 | 5112147 | 5113485 | 5114823 | 5116160 | 5117497 |
| 325                 | 5125510 | 5126844 | 5128178 | 5129511 | 5130844 |
| 326                 | 5138832 | 5140162 | 5141491 | 5142820 | 5144149 |
| 327                 | 5152113 | 5153439 | 5154764 | 5156089 | 5157414 |
| 328                 | 5165354 | 5166676 | 5167997 | 5169318 | 5170639 |
| 329                 | 5178554 | 5179872 | 5181189 | 5182506 | 5183823 |
| 330                 | 5191715 | 5193028 | 5194342 | 5195655 | 5196968 |
| 331                 | 5204835 | 5206145 | 5207455 | 5208764 | 5210073 |
| 332                 | 5217916 | 5219222 | 5220528 | 5221833 | 5223138 |
| 333                 | 5230958 | 5232260 | 5233562 | 5234863 | 5236164 |
| 334                 | 5243961 | 5245259 | 5246557 | 5247854 | 5249151 |
| 335                 | 5256925 | 5258219 | 5259513 | 5260807 | 5262100 |
| 336                 | 5269851 | 5271141 | 5272431 | 5273721 | 5275010 |
| 337                 | 5282738 | 5284024 | 5285311 | 5286596 | 5287882 |
| 338                 | 5295587 | 5296869 | 5298152 | 5299434 | 5300716 |
| 339                 | 5308398 | 5309677 | 5310955 | 5312234 | 5313512 |
| 340                 | 5321171 | 5322446 | 5323721 | 5324996 | 5326270 |
| 341                 | 5333907 | 5335179 | 5336450 | 5337721 | 5338991 |
| 342                 | 5346606 | 5347874 | 5349141 | 5350408 | 5351675 |
| 343                 | 5359267 | 5360532 | 5361795 | 5363059 | 5364322 |
| 344                 | 5371892 | 5373153 | 5374413 | 5375672 | 5376932 |
| 345                 | 5384481 | 5385737 | 5386994 | 5388250 | 5389506 |
| 346                 | 5397032 | 5398286 | 5399538 | 5400791 | 5402043 |
| 347                 | 5409548 | 5410798 | 5412047 | 5413296 | 5414544 |
| 348                 | 5422028 | 5423274 | 5424519 | 5425765 | 5427010 |
| 349                 | 5434472 | 5435714 | 5436956 | 5438198 | 5439439 |
| 350                 | 5446880 | 5448119 | 5449358 | 5450596 | 5451834 |
| 351                 | 5459253 | 5460489 | 5461724 | 5462958 | 5464193 |
| 352                 | 5471591 | 5472823 | 5474055 | 5475286 | 5476517 |
| 353                 | 5483894 | 5485123 | 5486351 | 5487578 | 5488806 |
| 354                 | 5406162 | 5497387 | 5498612 | 5499836 | 5501060 |

*Artificial Numbers: Or,*

| Natural<br>Numbers. | 0       | 1       | 2       | 3       | 4       |
|---------------------|---------|---------|---------|---------|---------|
| 355                 | 5502283 | 5503507 | 5504730 | 5505952 | 5507174 |
| 356                 | 5514500 | 5515720 | 5516939 | 5518158 | 5519377 |
| 357                 | 5526682 | 5527898 | 5529114 | 5530330 | 5531545 |
| 358                 | 5538830 | 5540043 | 5541256 | 5542468 | 5543680 |
| 359                 | 5550944 | 5552154 | 5553362 | 5554572 | 5555781 |
| 360                 | 5563025 | 5564231 | 5565437 | 5566643 | 5567848 |
| 361                 | 5575072 | 5576275 | 5577477 | 5578680 | 5579881 |
| 362                 | 5587086 | 5588285 | 5589484 | 5590683 | 5591882 |
| 363                 | 5599066 | 5600262 | 5601458 | 5602654 | 5603849 |
| 364                 | 5611014 | 5612207 | 5613399 | 5614592 | 5615784 |
| 365                 | 5622929 | 5624118 | 5625308 | 5626497 | 5627685 |
| 366                 | 5634811 | 5635997 | 5637183 | 5638369 | 5639555 |
| 367                 | 5646661 | 5647844 | 5649027 | 5650209 | 5651392 |
| 368                 | 5658478 | 5659658 | 5660838 | 5662017 | 5663196 |
| 369                 | 5670264 | 5671440 | 5672617 | 5673793 | 5674969 |
| 370                 | 5682017 | 5683191 | 5684364 | 5685537 | 5686710 |
| 371                 | 5693739 | 5694910 | 5696080 | 5697249 | 5698419 |
| 372                 | 5705429 | 5706597 | 5707764 | 5708930 | 5710097 |
| 373                 | 5717088 | 5718252 | 5719416 | 5720580 | 5721743 |
| 374                 | 5728716 | 5729877 | 5731038 | 5732198 | 5733358 |
| 375                 | 5740313 | 5741471 | 5742628 | 5743786 | 5744943 |
| 376                 | 5751878 | 5753033 | 5754188 | 5755342 | 5756496 |
| 377                 | 5763413 | 5764565 | 5765717 | 5766868 | 5768019 |
| 378                 | 5774917 | 5776067 | 5777215 | 5778363 | 5779511 |
| 379                 | 5786392 | 5787538 | 5788683 | 5789828 | 5790973 |
| 380                 | 5797836 | 5798979 | 5800121 | 5801263 | 5802405 |
| 381                 | 5809250 | 5810389 | 5811529 | 5812668 | 5813807 |
| 382                 | 5820634 | 5821770 | 5822907 | 5824043 | 5825179 |
| 383                 | 5831988 | 5833122 | 5834255 | 5835388 | 5836521 |
| 384                 | 5843312 | 5844443 | 5845574 | 5846704 | 5847834 |
| 385                 | 5854617 | 5855735 | 5856863 | 5857990 | 5859117 |
| 386                 | 5865873 | 5866998 | 5868123 | 5869247 | 5870371 |
| 387                 | 5877110 | 5878232 | 5879353 | 5880475 | 5881596 |
| 388                 | 5888317 | 5889436 | 5890555 | 5891674 | 5892792 |
| 389                 | 5899496 | 5900612 | 5901728 | 5902844 | 5903959 |
| 390                 | 5910646 | 5911759 | 5912873 | 5913985 | 5915098 |
| 391                 | 5921768 | 5922878 | 5923988 | 5925098 | 5926208 |
| 392                 | 5932861 | 5933968 | 5935076 | 5936183 | 5937290 |
| 393                 | 5943925 | 5945030 | 5946135 | 5947239 | 5948344 |
| 394                 | 5954962 | 5956064 | 5957166 | 5958268 | 5959369 |
| 395                 | 5965971 | 5967070 | 5968169 | 5969268 | 5970367 |
| 396                 | 5976952 | 5978048 | 5979145 | 5980241 | 5981336 |
| 397                 | 5987905 | 5988999 | 5990092 | 5991186 | 5992279 |
| 398                 | 5998831 | 5999922 | 6001013 | 6002103 | 6003193 |

*Logarithms (to 3989.)*

| Natural<br>Numbers. | 5       | 6       | 7       | 8       | 9       |
|---------------------|---------|---------|---------|---------|---------|
| 355                 | 5508396 | 5509618 | 5510839 | 5512059 | 5513280 |
| 356                 | 5520595 | 5521813 | 5523031 | 5524248 | 5525465 |
| 357                 | 5532760 | 5533975 | 5535189 | 5536403 | 5537617 |
| 358                 | 5544892 | 5546103 | 5547314 | 5548524 | 5549735 |
| 359                 | 5556989 | 5558197 | 5559404 | 5560612 | 5561818 |
| 360                 | 5569053 | 5570257 | 5571461 | 5572665 | 5573869 |
| 361                 | 5581083 | 5582284 | 5583485 | 5584686 | 5585886 |
| 362                 | 5593080 | 5594278 | 5595476 | 5596673 | 5597870 |
| 363                 | 5605044 | 5606239 | 5607433 | 5608627 | 5609820 |
| 364                 | 5616975 | 5618167 | 5619358 | 5620548 | 5621739 |
| 365                 | 5628875 | 5630062 | 5631250 | 5632437 | 5633624 |
| 366                 | 5640740 | 5641925 | 5643109 | 5644293 | 5645477 |
| 367                 | 5652573 | 5653755 | 5654936 | 5656117 | 5657298 |
| 368                 | 5664375 | 5665553 | 5666731 | 5667909 | 5669087 |
| 369                 | 5676144 | 5677320 | 5678494 | 5679669 | 5680843 |
| 370                 | 5687882 | 5689054 | 5690226 | 5691397 | 5692568 |
| 371                 | 5699588 | 5700757 | 5701926 | 5703094 | 5704262 |
| 372                 | 5711263 | 5712428 | 5713594 | 5714759 | 5715924 |
| 373                 | 5722906 | 5724069 | 5725231 | 5726393 | 5727555 |
| 374                 | 5734518 | 5735678 | 5736837 | 5737996 | 5739154 |
| 375                 | 5746099 | 5747256 | 5748412 | 5749568 | 5750723 |
| 376                 | 5757650 | 5758803 | 5759956 | 5761109 | 5762261 |
| 377                 | 5769169 | 5770320 | 5771470 | 5772620 | 5773769 |
| 378                 | 5780659 | 5781806 | 5782953 | 5784100 | 5785246 |
| 379                 | 5792118 | 5793262 | 5794406 | 5795550 | 5796693 |
| 380                 | 5803547 | 5804688 | 5805829 | 5806969 | 5808110 |
| 381                 | 5814945 | 5816084 | 5817222 | 5818359 | 5819497 |
| 382                 | 5826314 | 5827450 | 5828585 | 5829719 | 5830854 |
| 383                 | 5837654 | 5838786 | 5839918 | 5841050 | 5842181 |
| 384                 | 5848963 | 5850093 | 5851222 | 5852351 | 5853479 |
| 385                 | 5860244 | 5861370 | 5862496 | 5863622 | 5864748 |
| 386                 | 5871495 | 5872618 | 5873742 | 5874865 | 5875987 |
| 387                 | 5882717 | 5883838 | 5884958 | 5886078 | 5887198 |
| 388                 | 5893910 | 5895028 | 5896145 | 5897262 | 5898379 |
| 389                 | 5905075 | 5906189 | 5907304 | 5908418 | 5909532 |
| 390                 | 5916210 | 5917322 | 5918434 | 5919546 | 5920657 |
| 391                 | 5927318 | 5928427 | 5929536 | 5930644 | 5931753 |
| 392                 | 5938397 | 5939503 | 5940609 | 5941715 | 5942820 |
| 393                 | 5949447 | 5950551 | 5951654 | 5952757 | 5953860 |
| 394                 | 5960470 | 5961571 | 5962671 | 5963771 | 5964871 |
| 395                 | 5971465 | 5972563 | 5973660 | 5974758 | 5975855 |
| 396                 | 5982432 | 5983527 | 5984622 | 5985717 | 5986811 |
| 397                 | 5993371 | 5994464 | 5995556 | 5996648 | 5997739 |
| 398                 | 6004283 | 6005373 | 6006462 | 6007551 | 6008640 |



*Artificial Numbers: Or,*

| Natural<br>Numbers. | 0       | 1       | 2       | 3       | 4       |
|---------------------|---------|---------|---------|---------|---------|
| 399                 | 6009729 | 6010817 | 6011905 | 6012993 | 6014081 |
| 400                 | 6020600 | 6021685 | 6022771 | 6023856 | 6024941 |
| 401                 | 6031444 | 6032527 | 6033609 | 6034692 | 6035774 |
| 402                 | 6042261 | 6043341 | 6044421 | 6045500 | 6046580 |
| 403                 | 6053050 | 6054128 | 6055205 | 6056282 | 6057359 |
| 404                 | 6063814 | 6064888 | 6065963 | 6067037 | 6068111 |
| 405                 | 6074550 | 6075622 | 6076694 | 6077766 | 6078837 |
| 406                 | 6085260 | 6086330 | 6087399 | 6088468 | 6089537 |
| 407                 | 6095944 | 6097011 | 6098078 | 6099144 | 6100210 |
| 408                 | 6106602 | 6107666 | 6108730 | 6109794 | 6110857 |
| 409                 | 6117233 | 6118295 | 6119356 | 6120417 | 6121478 |
| 410                 | 6127839 | 6128898 | 6129957 | 6131015 | 6132073 |
| 411                 | 6138418 | 6139475 | 6140531 | 6141587 | 6142643 |
| 412                 | 6148972 | 6150026 | 6151080 | 6152133 | 6153187 |
| 413                 | 6159501 | 6160552 | 6161603 | 6162654 | 6163705 |
| 414                 | 6170003 | 6171052 | 6172101 | 6173149 | 6174197 |
| 415                 | 6180481 | 6181527 | 6182573 | 6183619 | 6184665 |
| 416                 | 6190933 | 6191977 | 6193021 | 6194064 | 6195107 |
| 417                 | 6201360 | 6202402 | 6203443 | 6204484 | 6205524 |
| 418                 | 6211763 | 6212802 | 6213840 | 6214879 | 6215917 |
| 419                 | 6222140 | 6223177 | 6224213 | 6225149 | 6226284 |
| 420                 | 6232493 | 6233527 | 6234560 | 6235594 | 6236627 |
| 421                 | 6242821 | 6243852 | 6244884 | 6245915 | 6246945 |
| 422                 | 6253124 | 6254153 | 6255182 | 6256211 | 6257239 |
| 423                 | 6263404 | 6264430 | 6265457 | 6266483 | 6267509 |
| 424                 | 6273659 | 6274683 | 6275707 | 6276730 | 6277754 |
| 425                 | 6283889 | 6284911 | 6285933 | 6286954 | 6287975 |
| 426                 | 6294096 | 6295115 | 6296134 | 6297153 | 6298172 |
| 427                 | 6304279 | 6305296 | 6306312 | 6307329 | 6308345 |
| 428                 | 6314438 | 6315452 | 6316467 | 6317481 | 6318495 |
| 429                 | 6324573 | 6325585 | 6326597 | 6327609 | 6328620 |
| 430                 | 6334685 | 6335694 | 6336704 | 6337713 | 6338723 |
| 431                 | 6344773 | 6345780 | 6346788 | 6347795 | 6348801 |
| 432                 | 6354837 | 6355843 | 6356848 | 6357852 | 6358857 |
| 433                 | 6364879 | 6365882 | 6366884 | 6367887 | 6368889 |
| 434                 | 6374897 | 6375898 | 6376898 | 6377898 | 6378898 |
| 435                 | 6384893 | 6385891 | 6386889 | 6387887 | 6388884 |
| 436                 | 6394865 | 6395861 | 6396857 | 6397852 | 6398847 |
| 437                 | 6474814 | 6405808 | 6406802 | 6407795 | 6408788 |
| 438                 | 6414741 | 6415733 | 6416724 | 6417715 | 6418705 |
| 439                 | 6424645 | 6425634 | 6426623 | 6427612 | 6428601 |
| 440                 | 6434527 | 6435514 | 6436500 | 6437487 | 6438473 |
| 441                 | 6444386 | 6445371 | 6446355 | 6447339 | 6448323 |
| 442                 | 6454223 | 6455205 | 6456187 | 6457169 | 6458151 |

*Logarithms (to 4429.)*

| Natural<br>Numbers. | 5       | 6       | 7       | 8       | 9       |
|---------------------|---------|---------|---------|---------|---------|
| 399                 | 6015168 | 6016255 | 6017341 | 6018428 | 6019514 |
| 400                 | 6026025 | 6027109 | 6028193 | 6029277 | 6030361 |
| 401                 | 6036855 | 6037937 | 6039018 | 6040099 | 6041180 |
| 402                 | 6047659 | 6048738 | 6049816 | 6050895 | 6051973 |
| 403                 | 6058435 | 6059512 | 6060587 | 6061663 | 6062738 |
| 404                 | 6069185 | 6070259 | 6071332 | 6072405 | 6073478 |
| 405                 | 6079909 | 6080979 | 6082050 | 6083120 | 6084190 |
| 406                 | 6090605 | 6091674 | 6092742 | 6093809 | 6094877 |
| 407                 | 6101276 | 6102342 | 6103407 | 6104472 | 6105537 |
| 408                 | 6111921 | 6112948 | 6114046 | 6115109 | 6116171 |
| 409                 | 6122539 | 6123599 | 6124660 | 6125720 | 6126779 |
| 410                 | 6133132 | 6134189 | 6135247 | 6136304 | 6137361 |
| 411                 | 6143698 | 6144754 | 6145809 | 6146863 | 6147918 |
| 412                 | 6154240 | 6155292 | 6156345 | 6157397 | 6158449 |
| 413                 | 6164755 | 6165805 | 6166855 | 6167905 | 6168954 |
| 414                 | 6175245 | 6176293 | 6177340 | 6178387 | 6179434 |
| 415                 | 6185710 | 6186755 | 6187800 | 6188845 | 6189889 |
| 416                 | 6196150 | 6197193 | 6198235 | 6199277 | 6200319 |
| 417                 | 6206565 | 6207605 | 6208645 | 6209684 | 6210724 |
| 418                 | 6216955 | 6217992 | 6219030 | 6220067 | 6221104 |
| 419                 | 6227320 | 6228355 | 6229300 | 6230424 | 6231459 |
| 420                 | 6237660 | 6238693 | 6239725 | 6240757 | 6241789 |
| 421                 | 6247976 | 6249006 | 6250036 | 6251066 | 6252095 |
| 422                 | 6258267 | 6259295 | 6260322 | 6261350 | 6262377 |
| 423                 | 6268534 | 6269559 | 6270585 | 6271610 | 6272634 |
| 424                 | 6278777 | 6279800 | 6280823 | 6281845 | 6282867 |
| 425                 | 6288996 | 6290016 | 6291036 | 6292057 | 6293076 |
| 426                 | 6299190 | 6300208 | 6301226 | 6302244 | 6303262 |
| 427                 | 6309361 | 6310377 | 6311392 | 6312408 | 6313423 |
| 428                 | 6319508 | 6320522 | 6321535 | 6322548 | 6323560 |
| 429                 | 6329632 | 6330643 | 6331653 | 6332664 | 6333674 |
| 430                 | 6339732 | 6340740 | 6341749 | 6342757 | 6343765 |
| 431                 | 6349808 | 6350814 | 6351820 | 6352826 | 6353832 |
| 432                 | 6359861 | 6360865 | 6361869 | 6362872 | 6363876 |
| 433                 | 6369891 | 6370893 | 6371894 | 6372895 | 6373896 |
| 434                 | 6379898 | 6380897 | 6381896 | 6382895 | 6383894 |
| 435                 | 6389882 | 6390879 | 6391876 | 6392872 | 6393869 |
| 436                 | 6399842 | 6400837 | 6401832 | 6402826 | 6403820 |
| 437                 | 6409781 | 6410773 | 6411765 | 6412758 | 6413749 |
| 438                 | 6419696 | 6420686 | 6421676 | 6422666 | 6423656 |
| 439                 | 6429589 | 6430577 | 6431565 | 6432552 | 6433540 |
| 440                 | 6439459 | 6440445 | 6441430 | 6442416 | 6443401 |
| 441                 | 6449307 | 6450291 | 6451274 | 6452257 | 6453240 |
| 442                 | 6459133 | 6460114 | 6461095 | 6462076 | 6463057 |

*Artificial Numbers: Or,*

| Natural<br>Numbers. | 0       | 1       | 2       | 3       | 4       |
|---------------------|---------|---------|---------|---------|---------|
| 443                 | 6464037 | 6465017 | 6465997 | 6466977 | 6467957 |
| 444                 | 6473830 | 6474808 | 6475785 | 6476763 | 6477740 |
| 445                 | 6483600 | 6484576 | 6485552 | 6486527 | 6487502 |
| 446                 | 6493349 | 6494322 | 6495296 | 6496269 | 6497242 |
| 447                 | 6503075 | 6504047 | 6505018 | 6505989 | 6506960 |
| 448                 | 6512780 | 6513749 | 6514719 | 6515687 | 6516656 |
| 449                 | 6522463 | 6523430 | 6524397 | 6525364 | 6526331 |
| 450                 | 6532125 | 6533090 | 6534055 | 6535019 | 6535984 |
| 451                 | 6541765 | 6542728 | 6543691 | 6544653 | 6545616 |
| 452                 | 6551384 | 6552345 | 6553306 | 6554266 | 6555226 |
| 453                 | 6560982 | 6561941 | 6562899 | 6563857 | 6564815 |
| 454                 | 6570559 | 6571515 | 6572471 | 6573427 | 6574383 |
| 455                 | 6580114 | 6581068 | 6582023 | 6582976 | 6583930 |
| 456                 | 6589648 | 6590601 | 6591553 | 6592505 | 6593456 |
| 457                 | 6599162 | 6600112 | 6601062 | 6602012 | 6602962 |
| 458                 | 6608655 | 6609603 | 6610551 | 6611499 | 6612446 |
| 459                 | 6618127 | 6619073 | 6620019 | 6620964 | 6621910 |
| 460                 | 6627578 | 6628522 | 6629466 | 6630410 | 6631353 |
| 461                 | 6637009 | 6637951 | 6638893 | 6639835 | 6640776 |
| 462                 | 6646420 | 6647360 | 6648299 | 6649239 | 6650178 |
| 463                 | 6655810 | 6656748 | 6657685 | 6658623 | 6659560 |
| 464                 | 6665180 | 6666116 | 6667051 | 6667987 | 6668922 |
| 465                 | 6674530 | 6675463 | 6676397 | 6677331 | 6678264 |
| 466                 | 6683859 | 6684791 | 6685723 | 6686654 | 6687585 |
| 467                 | 6693169 | 6694099 | 6695028 | 6695958 | 6696887 |
| 468                 | 6702459 | 6703386 | 6704314 | 6705242 | 6706169 |
| 469                 | 6711728 | 6712654 | 6713580 | 6714506 | 6715431 |
| 470                 | 6720979 | 6721903 | 6722826 | 6723750 | 6724673 |
| 471                 | 6730209 | 6731131 | 6732053 | 6732974 | 6733896 |
| 472                 | 6739420 | 6740340 | 6741260 | 6742179 | 6743099 |
| 473                 | 6748611 | 6749529 | 6750447 | 6751365 | 6752283 |
| 474                 | 6757783 | 6758700 | 6759615 | 6760531 | 6761447 |
| 475                 | 6766936 | 6767850 | 6768764 | 6769678 | 6770592 |
| 476                 | 6776069 | 6776982 | 6777894 | 6778806 | 6779718 |
| 477                 | 6785184 | 6786094 | 6787004 | 6787914 | 6788824 |
| 478                 | 6794279 | 6795187 | 6796096 | 6797004 | 6797912 |
| 479                 | 6803355 | 6804262 | 6805168 | 6806074 | 6806980 |
| 480                 | 6812412 | 6813317 | 6814222 | 6815126 | 6816030 |
| 481                 | 6821451 | 6822354 | 6823256 | 6824159 | 6825061 |
| 482                 | 6830470 | 6831371 | 6832272 | 6833173 | 6834073 |
| 483                 | 6839471 | 6840370 | 6841269 | 6842168 | 6843066 |
| 484                 | 6848454 | 6849351 | 6850248 | 6851145 | 6852041 |
| 485                 | 6857417 | 6858313 | 6859208 | 6860103 | 6860998 |
| 486                 | 6866363 | 6867256 | 6868149 | 6869043 | 6869936 |



*Logarithms (to 4869.)*

| Natural<br>Numbers. | 5       | 6       | 7       | 8       | 9       |
|---------------------|---------|---------|---------|---------|---------|
| 443                 | 6468936 | 6469915 | 6470894 | 6471873 | 6472851 |
| 444                 | 6478718 | 6479695 | 6480671 | 6481648 | 6482624 |
| 445                 | 6488477 | 6489452 | 6490426 | 6491401 | 6492375 |
| 446                 | 6498215 | 6499187 | 6500160 | 6501132 | 6502104 |
| 447                 | 6507930 | 6508901 | 6509871 | 6510841 | 6511811 |
| 448                 | 6517624 | 6518593 | 6519561 | 6520528 | 6521496 |
| 449                 | 6527297 | 6528263 | 6529229 | 6530195 | 6531160 |
| 450                 | 6536948 | 6537912 | 6538876 | 6539839 | 6540802 |
| 451                 | 6546578 | 6547539 | 6548501 | 6549462 | 6550423 |
| 452                 | 6556186 | 6557145 | 6558105 | 6559064 | 6560023 |
| 453                 | 6565773 | 6566730 | 6567688 | 6568645 | 6569602 |
| 454                 | 6575339 | 6576294 | 6577250 | 6578205 | 6579159 |
| 455                 | 6584884 | 6585837 | 6586790 | 6587743 | 6588696 |
| 456                 | 6594408 | 6595359 | 6596310 | 6597261 | 6598212 |
| 457                 | 6603911 | 6604860 | 6605809 | 6606758 | 6607706 |
| 458                 | 6613393 | 6614340 | 6615287 | 6616234 | 6617181 |
| 459                 | 6622855 | 6623800 | 6624745 | 6625690 | 6626634 |
| 460                 | 6632296 | 6633239 | 6634182 | 6635125 | 6636067 |
| 461                 | 6641717 | 6642658 | 6643599 | 6644539 | 6645480 |
| 462                 | 6651117 | 6652056 | 6652995 | 6653933 | 6654872 |
| 463                 | 6660497 | 6661434 | 6662371 | 6663307 | 6664244 |
| 464                 | 6669857 | 6670792 | 6671727 | 6672661 | 6673595 |
| 465                 | 6679197 | 6680130 | 6681062 | 6681995 | 6682927 |
| 466                 | 6688516 | 6689447 | 6690378 | 6691308 | 6692239 |
| 467                 | 6697816 | 6698745 | 6699674 | 6700602 | 6701530 |
| 468                 | 6707096 | 6708023 | 6708950 | 6709876 | 6710802 |
| 469                 | 6716356 | 6717281 | 6718206 | 6719130 | 6720054 |
| 470                 | 6725596 | 6726519 | 6727442 | 6728365 | 6729287 |
| 471                 | 6734817 | 6735738 | 6736659 | 6737574 | 6738490 |
| 472                 | 6744018 | 6744937 | 6745856 | 6746775 | 6747693 |
| 473                 | 6753200 | 6754117 | 6755034 | 6755951 | 6756867 |
| 474                 | 6762362 | 6763277 | 6764192 | 6765107 | 6766022 |
| 475                 | 6771505 | 6772418 | 6773332 | 6774244 | 6775157 |
| 476                 | 6780629 | 6781540 | 6782452 | 6783362 | 6784273 |
| 477                 | 6789734 | 6790643 | 6791552 | 6792461 | 6793370 |
| 478                 | 6798819 | 6799727 | 6800634 | 6801541 | 6802448 |
| 479                 | 6807886 | 6808792 | 6809697 | 6810602 | 6811507 |
| 480                 | 6816934 | 6817838 | 6818741 | 6819645 | 6820548 |
| 481                 | 6825963 | 6826865 | 6827766 | 6828668 | 6829569 |
| 482                 | 6834973 | 6835873 | 6836773 | 6837673 | 6838572 |
| 483                 | 6843965 | 6844863 | 6845761 | 6846659 | 6847556 |
| 484                 | 6852938 | 6853834 | 6854730 | 6855626 | 6856522 |
| 485                 | 6861892 | 6862787 | 6863681 | 6864575 | 6865469 |
| 486                 | 6870828 | 6871721 | 6872613 | 6873506 | 6874398 |

*Artificial Numbers: Or,*

| Natural<br>Numbers. | 0       | 1       | 2       | 3       | 4       |
|---------------------|---------|---------|---------|---------|---------|
| 487                 | 6875290 | 6876181 | 6877073 | 6877964 | 6878855 |
| 488                 | 6884198 | 6885088 | 6885978 | 6886867 | 6887757 |
| 489                 | 6893089 | 6893977 | 6894864 | 6895752 | 6896640 |
| 490                 | 6901961 | 6902847 | 6903733 | 6904616 | 6905505 |
| 491                 | 6910815 | 6911699 | 6912584 | 6913468 | 6914352 |
| 492                 | 6919651 | 6920534 | 6921416 | 6922298 | 6923180 |
| 493                 | 6928469 | 6929350 | 6930231 | 6931111 | 6931991 |
| 494                 | 6937269 | 6938148 | 6939027 | 6939906 | 6940785 |
| 495                 | 6946052 | 6946929 | 6947806 | 6948683 | 6949560 |
| 496                 | 6954817 | 6955692 | 6956568 | 6957443 | 6958318 |
| 497                 | 6963564 | 6964438 | 6965311 | 6966185 | 6967058 |
| 498                 | 6972293 | 6973165 | 6974037 | 6974909 | 6975780 |
| 499                 | 6981005 | 6981876 | 6982746 | 6983616 | 6984485 |
| 500                 | 6989700 | 6990569 | 6991437 | 6992305 | 6993173 |
| 501                 | 6998377 | 6999244 | 7000111 | 7000977 | 7001843 |
| 502                 | 7007037 | 7007902 | 7008767 | 7009632 | 7010496 |
| 503                 | 7015680 | 7016543 | 7017406 | 7018269 | 7019132 |
| 504                 | 7024305 | 7025167 | 7026028 | 7026890 | 7027751 |
| 505                 | 7032914 | 7033774 | 7034633 | 7035493 | 7036352 |
| 506                 | 7041505 | 7042363 | 7043221 | 7044079 | 7044937 |
| 507                 | 7050080 | 7050936 | 7051792 | 7052649 | 7053505 |
| 508                 | 7058637 | 7059492 | 7060347 | 7061201 | 7062055 |
| 509                 | 7067178 | 7068031 | 7068884 | 7069737 | 7070589 |
| 510                 | 7075702 | 7076553 | 7077405 | 7078256 | 7079107 |
| 511                 | 7084206 | 7085059 | 7085908 | 7086758 | 7087607 |
| 512                 | 7092700 | 7093548 | 7094396 | 7095244 | 7096091 |
| 513                 | 7101174 | 7102020 | 7102866 | 7103713 | 7104559 |
| 514                 | 7109631 | 7110476 | 7111321 | 7112165 | 7113010 |
| 515                 | 7118072 | 7118915 | 7119759 | 7120601 | 7121444 |
| 516                 | 7126497 | 7127339 | 7128180 | 7129021 | 7129862 |
| 517                 | 7134905 | 7135745 | 7136585 | 7137425 | 7138264 |
| 518                 | 7143298 | 7144136 | 7144974 | 7145812 | 7146650 |
| 519                 | 7151674 | 7152510 | 7153347 | 7154183 | 7155019 |
| 520                 | 7160033 | 7160869 | 7161703 | 7162538 | 7163373 |
| 521                 | 7168377 | 7169211 | 7170044 | 7170877 | 7171710 |
| 522                 | 7176705 | 7177537 | 7178369 | 7179200 | 7180032 |
| 523                 | 7185017 | 7185847 | 7186677 | 7187507 | 7188337 |
| 524                 | 7193313 | 7194142 | 7194970 | 7195799 | 7196627 |
| 525                 | 7201593 | 7202420 | 7203247 | 7204074 | 7204901 |
| 526                 | 7209857 | 7210683 | 7211508 | 7212334 | 7213159 |
| 527                 | 7218106 | 7218930 | 7219754 | 7220578 | 7221401 |
| 528                 | 7226339 | 7227162 | 7227984 | 7228806 | 7229628 |
| 529                 | 7234557 | 7235378 | 7236198 | 7237019 | 7237839 |
| 530                 | 7242759 | 7243578 | 7244397 | 7245216 | 7246035 |

*Logarithms (to 5309.)*

| Natural<br>Numbers. | 5       | 6       | 7       | 8       | 9       |
|---------------------|---------|---------|---------|---------|---------|
| 487                 | 6879746 | 6880637 | 6881528 | 6882418 | 6883308 |
| 488                 | 6888646 | 6889535 | 6890423 | 6891312 | 6892200 |
| 489                 | 6897527 | 6898414 | 6899301 | 6900188 | 6901074 |
| 490                 | 6906390 | 6907275 | 6908161 | 6909046 | 6909930 |
| 491                 | 6915235 | 6916119 | 6917002 | 6917885 | 6918768 |
| 492                 | 6924062 | 6924944 | 6925826 | 6926707 | 6927588 |
| 493                 | 6932872 | 6933752 | 6934631 | 6935511 | 6936390 |
| 494                 | 6941663 | 6942541 | 6943419 | 6944297 | 6945174 |
| 495                 | 6950437 | 6951313 | 6952189 | 6953065 | 6953941 |
| 496                 | 6959193 | 6960067 | 6960942 | 6961816 | 6962690 |
| 497                 | 6967931 | 6968804 | 6969676 | 6970549 | 6971421 |
| 498                 | 6976652 | 6977523 | 6978394 | 6979264 | 6980135 |
| 499                 | 6985355 | 6986224 | 6987093 | 6987963 | 6988831 |
| 500                 | 6994041 | 6994908 | 6995776 | 6996643 | 6997510 |
| 501                 | 7002709 | 7003575 | 7004441 | 7005307 | 7006172 |
| 502                 | 7011361 | 7012225 | 7013089 | 7013953 | 7014816 |
| 503                 | 7019995 | 7020857 | 7021719 | 7022582 | 7023444 |
| 504                 | 7028612 | 7029472 | 7030333 | 7031193 | 7032054 |
| 505                 | 7037212 | 7038071 | 7038929 | 7039788 | 7040647 |
| 506                 | 7045793 | 7046652 | 7047509 | 7048366 | 7049223 |
| 507                 | 7054360 | 7055216 | 7056072 | 7056927 | 7057782 |
| 508                 | 7062910 | 7063764 | 7064617 | 7065471 | 7066324 |
| 509                 | 7071442 | 7072294 | 7073146 | 7073998 | 7074850 |
| 510                 | 7079957 | 7080808 | 7081659 | 7082509 | 7083359 |
| 511                 | 7088456 | 7089305 | 7090154 | 7091003 | 7091851 |
| 512                 | 7096939 | 7097786 | 7098633 | 7099480 | 7100327 |
| 513                 | 7105404 | 7106250 | 7107096 | 7107941 | 7108786 |
| 514                 | 7113854 | 7114698 | 7115542 | 7116385 | 7117229 |
| 515                 | 7122287 | 7123129 | 7123971 | 7124813 | 7125655 |
| 516                 | 7130703 | 7131544 | 7132385 | 7133225 | 7134065 |
| 517                 | 7139104 | 7139943 | 7140782 | 7141620 | 7142459 |
| 518                 | 7147488 | 7148325 | 7149162 | 7150000 | 7150837 |
| 519                 | 7155856 | 7156691 | 7157527 | 7158363 | 7159198 |
| 520                 | 7164207 | 7165042 | 7165876 | 7166710 | 7167544 |
| 521                 | 7172543 | 7173376 | 7174208 | 7175041 | 7175873 |
| 522                 | 7180863 | 7181694 | 7182525 | 7183356 | 7184186 |
| 523                 | 7189167 | 7189996 | 7190826 | 7191655 | 7192484 |
| 524                 | 7197455 | 7198283 | 7199111 | 7199938 | 7200766 |
| 525                 | 7205727 | 7206554 | 7207380 | 7208206 | 7209032 |
| 526                 | 7213984 | 7214809 | 7215633 | 7216458 | 7217282 |
| 527                 | 7222225 | 7223048 | 7223871 | 7224694 | 7225517 |
| 528                 | 7230450 | 7231272 | 7232093 | 7232914 | 7233736 |
| 529                 | 7238660 | 7239480 | 7240300 | 7241120 | 7241939 |
| 530                 | 7246854 | 7247672 | 7248491 | 7249309 | 7250127 |



*Artificial Numbers: Or,*

| Natural<br>Numbers. | 0       | 1       | 2       | 3       | 4       |
|---------------------|---------|---------|---------|---------|---------|
| 531                 | 7250945 | 7251763 | 7252581 | 7253398 | 7254215 |
| 532                 | 7259116 | 7259933 | 7260749 | 7261565 | 7262380 |
| 533                 | 7267272 | 7268087 | 7268901 | 7269716 | 7270531 |
| 534                 | 7275413 | 7276226 | 7277039 | 7277852 | 7278664 |
| 535                 | 7283538 | 7284349 | 7285161 | 7285972 | 7286784 |
| 536                 | 7291648 | 7292458 | 7293268 | 7294078 | 7294888 |
| 537                 | 7299743 | 7300551 | 7301360 | 7302168 | 7302977 |
| 538                 | 7307823 | 7308630 | 7309437 | 7310244 | 7311051 |
| 539                 | 7315888 | 7316693 | 7317499 | 7318304 | 7319109 |
| 540                 | 7323938 | 7324749 | 7325546 | 7326350 | 7327153 |
| 541                 | 7331973 | 7332775 | 7333578 | 7334380 | 7335182 |
| 542                 | 7339993 | 7340794 | 7341595 | 7342396 | 7343197 |
| 543                 | 7347998 | 7348798 | 7349598 | 7350397 | 7351196 |
| 544                 | 7355989 | 7356787 | 7357585 | 7358383 | 7359181 |
| 545                 | 7363965 | 7364762 | 7365558 | 7366355 | 7367151 |
| 546                 | 7371926 | 7372722 | 7373517 | 7374312 | 7375107 |
| 547                 | 7379873 | 7380667 | 7381461 | 7382254 | 7383048 |
| 548                 | 7387806 | 7388598 | 7389390 | 7390182 | 7390974 |
| 549                 | 7395723 | 7396514 | 7397305 | 7398096 | 7398886 |
| 550                 | 7403627 | 7404416 | 7405206 | 7405995 | 7406784 |
| 551                 | 7411516 | 7412304 | 7413092 | 7413880 | 7414668 |
| 552                 | 7419391 | 7420177 | 7420964 | 7421750 | 7422537 |
| 553                 | 7427251 | 7428037 | 7428822 | 7429607 | 7430392 |
| 554                 | 7435098 | 7435881 | 7436665 | 7437449 | 7438232 |
| 555                 | 7442930 | 7443712 | 7444495 | 7445277 | 7446059 |
| 556                 | 7450748 | 7451529 | 7452310 | 7453091 | 7453871 |
| 557                 | 7458552 | 7459332 | 7460111 | 7460890 | 7461670 |
| 558                 | 7466342 | 7467120 | 7467898 | 7468676 | 7469454 |
| 559                 | 7474118 | 7474895 | 7475672 | 7476448 | 7477225 |
| 560                 | 7481880 | 7482656 | 7483431 | 7484206 | 7484981 |
| 561                 | 7489629 | 7490403 | 7491177 | 7491950 | 7492724 |
| 562                 | 7497363 | 7498136 | 7498908 | 7499681 | 7500453 |
| 563                 | 7505087 | 7505855 | 7506626 | 7507398 | 7508168 |
| 564                 | 7512791 | 7513561 | 7514331 | 7515100 | 7515870 |
| 565                 | 7520484 | 7521253 | 7522022 | 7522790 | 7523558 |
| 566                 | 7528164 | 7528932 | 7529699 | 7530466 | 7531232 |
| 567                 | 7535831 | 7536596 | 7537362 | 7538128 | 7538893 |
| 568                 | 7543483 | 7544248 | 7545012 | 7545777 | 7546541 |
| 569                 | 7551123 | 7551886 | 7552649 | 7553412 | 7554178 |
| 570                 | 7558749 | 7559510 | 7560279 | 7561034 | 7561795 |
| 571                 | 7566361 | 7567122 | 7567882 | 7568642 | 7569402 |
| 572                 | 7573960 | 7574719 | 7575479 | 7576237 | 7576996 |
| 573                 | 7581546 | 7582304 | 7583062 | 7583819 | 7584577 |
| 574                 | 7589119 | 7589875 | 7590632 | 7591388 | 7592144 |

*Logarithms (to 5749)*

| Natural<br>Numbers. | 5       | 6       | 7       | 8       | 9       |
|---------------------|---------|---------|---------|---------|---------|
| 531                 | 7255033 | 7255850 | 7256667 | 7257483 | 7258300 |
| 532                 | 7263196 | 7264012 | 7264827 | 7265642 | 7266457 |
| 533                 | 7271344 | 7272158 | 7272972 | 7273786 | 7274599 |
| 534                 | 7279477 | 7280290 | 7281101 | 7281914 | 7282726 |
| 535                 | 7287595 | 7288406 | 7289216 | 7290027 | 7290838 |
| 536                 | 7295697 | 7296506 | 7297316 | 7298125 | 7298934 |
| 537                 | 7303785 | 7304593 | 7305400 | 7306208 | 7307015 |
| 538                 | 7311857 | 7312663 | 7313470 | 7314276 | 7315082 |
| 539                 | 7319914 | 7320719 | 7321524 | 7322329 | 7323133 |
| 540                 | 7327957 | 7328760 | 7329564 | 7330367 | 7331170 |
| 541                 | 7335985 | 7336787 | 7337588 | 7338390 | 7339191 |
| 542                 | 7343997 | 7344798 | 7345598 | 7346398 | 7347198 |
| 543                 | 7351995 | 7352794 | 7353593 | 7354392 | 7355191 |
| 544                 | 7359979 | 7360776 | 7361574 | 7362371 | 7363168 |
| 545                 | 7367948 | 7368744 | 7369540 | 7370335 | 7371131 |
| 546                 | 7375902 | 7376696 | 7377491 | 7378285 | 7379079 |
| 547                 | 7383841 | 7384634 | 7385427 | 7386220 | 7387013 |
| 548                 | 7391766 | 7392558 | 7393350 | 7394141 | 7394932 |
| 549                 | 7399677 | 7400467 | 7401257 | 7402047 | 7402837 |
| 550                 | 7407573 | 7408362 | 7409151 | 7409939 | 7410728 |
| 551                 | 7415455 | 7416243 | 7417030 | 7417817 | 7418604 |
| 552                 | 7423323 | 7424109 | 7424895 | 7425680 | 7426466 |
| 553                 | 7431176 | 7431961 | 7432745 | 7433530 | 7434314 |
| 554                 | 7439015 | 7439799 | 7440582 | 7441365 | 7442147 |
| 555                 | 7446841 | 7447622 | 7448404 | 7449187 | 7449967 |
| 556                 | 7454652 | 7455432 | 7456212 | 7456992 | 7457772 |
| 557                 | 7462449 | 7463228 | 7464006 | 7464785 | 7465564 |
| 558                 | 7470232 | 7471009 | 7471787 | 7472564 | 7473341 |
| 559                 | 7478001 | 7478777 | 7479553 | 7480329 | 7481105 |
| 560                 | 7485756 | 7486531 | 7487306 | 7488080 | 7488854 |
| 561                 | 7493498 | 7494271 | 7495044 | 7495817 | 7496590 |
| 562                 | 7501225 | 7501997 | 7502769 | 7503541 | 7504312 |
| 563                 | 7508939 | 7509710 | 7510480 | 7511251 | 7512021 |
| 564                 | 7516639 | 7517409 | 7518178 | 7518947 | 7519716 |
| 565                 | 7524326 | 7525094 | 7525862 | 7526629 | 7527397 |
| 566                 | 7531999 | 7532766 | 7533532 | 7534298 | 7535065 |
| 567                 | 7539659 | 7540424 | 7541189 | 7541954 | 7542719 |
| 568                 | 7547305 | 7548069 | 7548832 | 7549596 | 7550359 |
| 569                 | 7554937 | 7555700 | 7556462 | 7557224 | 7557987 |
| 570                 | 7562556 | 7563318 | 7564079 | 7564840 | 7565600 |
| 571                 | 7570162 | 7570922 | 7571682 | 7572441 | 7573201 |
| 572                 | 7577755 | 7578513 | 7579272 | 7580030 | 7580788 |
| 573                 | 7585334 | 7586091 | 7586848 | 7587605 | 7588362 |
| 574                 | 7592900 | 7593656 | 7594412 | 7595168 | 7595923 |

*Artificial Numbers: Or,*

| Natural<br>Numbers. | 0       | 1       | 2       | 3       | 4       |
|---------------------|---------|---------|---------|---------|---------|
| 575                 | 7596678 | 7597434 | 7598189 | 7598944 | 7599699 |
| 576                 | 7604225 | 7604979 | 7605733 | 7609486 | 7607240 |
| 577                 | 7611758 | 7612511 | 7613263 | 7614016 | 7614768 |
| 578                 | 7619278 | 7620030 | 7620781 | 7621532 | 7622283 |
| 579                 | 7626786 | 7627536 | 7628286 | 7629035 | 7629785 |
| 580                 | 7634280 | 7635029 | 7635777 | 7636526 | 7637274 |
| 581                 | 7641761 | 7642509 | 7643256 | 7644003 | 7644750 |
| 582                 | 7649230 | 7649976 | 7650722 | 7651468 | 7652214 |
| 583                 | 7656686 | 7657430 | 7658175 | 7658920 | 7659664 |
| 584                 | 7664128 | 7664872 | 7665616 | 7666359 | 7667102 |
| 585                 | 7671559 | 7672301 | 7673043 | 7673785 | 7674527 |
| 586                 | 7678976 | 7679717 | 7680458 | 7681199 | 7681940 |
| 587                 | 7686381 | 7687121 | 7687860 | 7688600 | 7689339 |
| 588                 | 7693773 | 7694512 | 7695250 | 7695988 | 7696727 |
| 589                 | 7701153 | 7701890 | 7702627 | 7703364 | 7704101 |
| 590                 | 7708520 | 7709256 | 7709992 | 7710728 | 7711463 |
| 591                 | 7715875 | 7716610 | 7717344 | 7718079 | 7718813 |
| 592                 | 7723217 | 7723951 | 7724684 | 7725417 | 7726150 |
| 593                 | 7730547 | 7731279 | 7732011 | 7732743 | 7733475 |
| 594                 | 7737864 | 7738596 | 7739326 | 7740057 | 7740788 |
| 595                 | 7745170 | 7745899 | 7746629 | 7747359 | 7748088 |
| 596                 | 7752463 | 7753191 | 7753920 | 7754648 | 7755376 |
| 597                 | 7759743 | 7760471 | 7761198 | 7761925 | 7762652 |
| 598                 | 7767012 | 7767738 | 7768464 | 7769190 | 7769916 |
| 599                 | 7774268 | 7774993 | 7775718 | 7776443 | 7777167 |
| 600                 | 7781513 | 7782236 | 7782960 | 7783683 | 7784407 |
| 601                 | 7788745 | 7789467 | 7790190 | 7790912 | 7791634 |
| 602                 | 7795965 | 7796686 | 7797408 | 7798129 | 7798850 |
| 603                 | 7803173 | 7803893 | 7804613 | 7805333 | 7806053 |
| 604                 | 7810369 | 7811088 | 7811807 | 7812526 | 7813245 |
| 605                 | 7817554 | 7818272 | 7818989 | 7819707 | 7820424 |
| 606                 | 7824726 | 7825443 | 7826159 | 7826876 | 7827592 |
| 607                 | 7831887 | 7832602 | 7833318 | 7834033 | 7834748 |
| 608                 | 7839036 | 7839750 | 7840464 | 7841178 | 7841892 |
| 609                 | 7846173 | 7846886 | 7847599 | 7848312 | 7849024 |
| 610                 | 7853298 | 7854010 | 7854722 | 7855434 | 7856145 |
| 611                 | 7860412 | 7861123 | 7861833 | 7862544 | 7863254 |
| 612                 | 7867514 | 7868224 | 7868933 | 7869643 | 7870352 |
| 613                 | 7874605 | 7875313 | 7876021 | 7876730 | 7877438 |
| 614                 | 7881684 | 7882391 | 7883098 | 7883805 | 7884512 |
| 615                 | 7888751 | 7889457 | 7890163 | 7890869 | 7891575 |
| 616                 | 7895807 | 7896512 | 7897217 | 7897922 | 7898626 |
| 617                 | 7902852 | 7903555 | 7904259 | 7904963 | 7905666 |
| 618                 | 7909885 | 7910587 | 0911290 | 7911992 | 7912695 |



*Logarithms (to 6189.)*

| Natural<br>Numbers. | 5       | 6       | 7       | 8       | 9       |
|---------------------|---------|---------|---------|---------|---------|
| 575                 | 7600453 | 7601208 | 7601962 | 7602717 | 7603471 |
| 576                 | 7607993 | 7608746 | 7609500 | 7610253 | 7611005 |
| 577                 | 7615520 | 7616272 | 7617024 | 7617775 | 7618527 |
| 578                 | 7623034 | 7623784 | 7624535 | 7625285 | 7626035 |
| 579                 | 7630534 | 7631284 | 7632033 | 7632782 | 7633531 |
| 580                 | 7638022 | 7638770 | 7639518 | 7640266 | 7641014 |
| 581                 | 7645497 | 7646244 | 7646991 | 7647737 | 7648484 |
| 582                 | 7652959 | 7653705 | 7654450 | 7655195 | 7655941 |
| 583                 | 7660409 | 7661153 | 7661897 | 7662641 | 7663385 |
| 584                 | 7667845 | 7668588 | 7669331 | 7670074 | 7670816 |
| 585                 | 7675269 | 7676011 | 7676752 | 7677494 | 7678235 |
| 586                 | 7682680 | 7683421 | 7684161 | 7684901 | 7685641 |
| 587                 | 7690079 | 7690818 | 7691557 | 7692296 | 7693035 |
| 588                 | 7697465 | 7698203 | 7698940 | 7699678 | 7700416 |
| 589                 | 7704838 | 7705575 | 7706311 | 7707048 | 7707784 |
| 590                 | 7712199 | 7712934 | 7713670 | 7714405 | 7715140 |
| 591                 | 7719547 | 7720282 | 7721016 | 7721750 | 7722483 |
| 592                 | 7726884 | 7727616 | 7728349 | 7729082 | 7729814 |
| 593                 | 7734207 | 7734939 | 7735670 | 7736402 | 7737133 |
| 594                 | 7741519 | 7742249 | 7742979 | 7743710 | 7744440 |
| 595                 | 7748818 | 7749547 | 7750276 | 7751005 | 7751734 |
| 596                 | 7756104 | 7756832 | 7757560 | 7758288 | 7759016 |
| 597                 | 7763379 | 7764106 | 7764833 | 7765559 | 7766286 |
| 598                 | 7770642 | 7771367 | 7772093 | 7772818 | 7773543 |
| 599                 | 7777892 | 7778616 | 7779340 | 7780065 | 7780789 |
| 600                 | 7785130 | 7785853 | 7786576 | 7787299 | 7788022 |
| 601                 | 7792356 | 7793078 | 7793800 | 7794522 | 7795243 |
| 602                 | 7799571 | 7800291 | 7801012 | 7801732 | 7802453 |
| 603                 | 7806773 | 7807492 | 7808212 | 7808931 | 7809650 |
| 604                 | 7813963 | 7814681 | 7815400 | 7816118 | 7816836 |
| 605                 | 7821141 | 7821859 | 7822576 | 7823293 | 7824010 |
| 606                 | 7828308 | 7829024 | 7829740 | 7830456 | 7831171 |
| 607                 | 7835463 | 7836178 | 7836892 | 7837607 | 7838321 |
| 608                 | 7842606 | 7843319 | 7844033 | 7844746 | 7845460 |
| 609                 | 7849737 | 7850450 | 7851162 | 7851874 | 7852586 |
| 610                 | 7856857 | 7857568 | 7858279 | 7858990 | 7859701 |
| 611                 | 7863965 | 7864675 | 7865385 | 7866095 | 7866805 |
| 612                 | 7871061 | 7871770 | 7872479 | 7873188 | 7873896 |
| 613                 | 7878146 | 7878853 | 7879561 | 7880269 | 7880976 |
| 614                 | 7885219 | 7885926 | 7886632 | 7887339 | 7888045 |
| 615                 | 7892281 | 7892986 | 7893691 | 7894397 | 7895102 |
| 616                 | 7899331 | 7900035 | 7900739 | 7901444 | 7902148 |
| 617                 | 7906370 | 7907073 | 7907776 | 7908479 | 7909182 |
| 618                 | 7913397 | 7914099 | 7914801 | 7915503 | 7916205 |

*Artificial Numbers: Or,*

| Natural<br>Numbers. | 0       | 1       | 2       | 3       | 4       |
|---------------------|---------|---------|---------|---------|---------|
| 619                 | 7916906 | 7917608 | 7918309 | 7919011 | 7919712 |
| 620                 | 7923917 | 7924617 | 7925318 | 7926018 | 7926718 |
| 621                 | 7930916 | 7931615 | 7932314 | 7933014 | 7933712 |
| 622                 | 7937904 | 7938602 | 7939300 | 7939998 | 7940696 |
| 623                 | 7944880 | 7945578 | 7946274 | 7946971 | 7947668 |
| 624                 | 7951846 | 7952542 | 7953238 | 7953933 | 7954629 |
| 625                 | 7958800 | 7959495 | 7960190 | 7960884 | 7961578 |
| 626                 | 7965743 | 7966437 | 7967131 | 7967824 | 7968517 |
| 627                 | 7972675 | 7973368 | 7974060 | 7974753 | 7975445 |
| 628                 | 7979596 | 7980288 | 7980979 | 7981671 | 7982362 |
| 629                 | 7986506 | 7987197 | 7987887 | 7988577 | 7989267 |
| 630                 | 7993405 | 7994097 | 7994784 | 7995473 | 7996162 |
| 631                 | 8000294 | 8000982 | 8001670 | 8002358 | 8003046 |
| 632                 | 8007171 | 8007858 | 8008545 | 8009232 | 8009919 |
| 633                 | 8014037 | 8014723 | 8015409 | 8016095 | 8016781 |
| 634                 | 8020893 | 8021578 | 8022262 | 8022947 | 8023632 |
| 635                 | 8027737 | 8028421 | 8029105 | 8029789 | 8030472 |
| 636                 | 8034571 | 8035254 | 8035937 | 8036619 | 8037302 |
| 637                 | 8041394 | 8042076 | 8042758 | 8043439 | 8044121 |
| 638                 | 8048207 | 8048887 | 8049568 | 8050248 | 8050929 |
| 639                 | 8055009 | 8055688 | 8056368 | 8057047 | 8057726 |
| 640                 | 8061800 | 8062478 | 8063157 | 8063835 | 8064513 |
| 641                 | 8068580 | 8069258 | 8069935 | 8070612 | 8071290 |
| 642                 | 8075350 | 8076027 | 8076703 | 8077379 | 8078055 |
| 643                 | 8082110 | 8082785 | 8083460 | 8084136 | 8084811 |
| 644                 | 8088859 | 8089533 | 8090207 | 8090881 | 8091555 |
| 645                 | 8095597 | 8096270 | 8096944 | 8097617 | 8098290 |
| 646                 | 8102325 | 8102997 | 8103670 | 8104342 | 8105013 |
| 647                 | 8109043 | 8109714 | 8110385 | 8111056 | 8111727 |
| 648                 | 8115750 | 8116420 | 8117090 | 8117760 | 8118430 |
| 649                 | 8122447 | 8123116 | 8123785 | 8124454 | 8125123 |
| 650                 | 8129134 | 8129802 | 8130470 | 8131138 | 8131805 |
| 651                 | 8135810 | 8136477 | 8137144 | 8137811 | 8138478 |
| 652                 | 8142476 | 8143142 | 8143808 | 8144474 | 8145140 |
| 653                 | 8149132 | 8149797 | 8150462 | 8151127 | 8151791 |
| 654                 | 8155777 | 8156441 | 8157105 | 8157769 | 8158433 |
| 655                 | 8162413 | 8163076 | 8163739 | 8164402 | 8165064 |
| 656                 | 8169038 | 8169700 | 8170362 | 8171024 | 8171686 |
| 657                 | 8175654 | 8176315 | 8176976 | 8177636 | 8178297 |
| 658                 | 8182259 | 8182919 | 8183579 | 8184239 | 8184898 |
| 659                 | 8188854 | 8189513 | 8190172 | 8190831 | 8191489 |
| 660                 | 8195439 | 8196097 | 8196755 | 8197413 | 8198071 |
| 661                 | 8202015 | 8202672 | 8203328 | 8203987 | 8204642 |
| 662                 | 8208580 | 8209236 | 8209892 | 8210548 | 8211203 |

Logarithms (to 6629.)

| Natural<br>Numbers. | 5       | 6       | 7       | 8       | 9       |
|---------------------|---------|---------|---------|---------|---------|
| 619                 | 7920413 | 7921114 | 7921815 | 7922516 | 7923216 |
| 620                 | 7927418 | 7928118 | 7928817 | 7929517 | 7930217 |
| 621                 | 7934411 | 7935110 | 7935809 | 7936507 | 7937205 |
| 622                 | 7941394 | 7942091 | 7942789 | 7943486 | 7944183 |
| 623                 | 7948365 | 7949061 | 7949757 | 7950454 | 7951150 |
| 624                 | 7955324 | 7956020 | 7956715 | 7957410 | 7958105 |
| 625                 | 7962273 | 7962967 | 7963662 | 7964356 | 7965050 |
| 626                 | 7969211 | 7969904 | 7970597 | 7971290 | 7971983 |
| 627                 | 7976137 | 7976829 | 7977521 | 7978213 | 7978905 |
| 628                 | 7983053 | 7983744 | 7984435 | 7985125 | 7985816 |
| 629                 | 7989957 | 7990647 | 7991337 | 7992027 | 7992716 |
| 630                 | 7996851 | 7997540 | 7998228 | 7998917 | 7999605 |
| 631                 | 8003734 | 8004421 | 8005109 | 8005796 | 8006484 |
| 632                 | 8010605 | 8011292 | 8011978 | 8012665 | 8013351 |
| 633                 | 8017466 | 8018152 | 8018837 | 8019522 | 8020208 |
| 634                 | 8024316 | 8025001 | 8025685 | 8026369 | 8027053 |
| 635                 | 8031156 | 8031839 | 8032522 | 8033205 | 8033888 |
| 636                 | 8037984 | 8038666 | 8039348 | 8040031 | 8040712 |
| 637                 | 8044802 | 8045483 | 8046164 | 8046845 | 8047526 |
| 638                 | 8051609 | 8052289 | 8052969 | 8053649 | 8054329 |
| 639                 | 8058405 | 8059085 | 8059763 | 8060442 | 8061121 |
| 640                 | 8065191 | 8065869 | 8066547 | 8067225 | 8067903 |
| 641                 | 8071967 | 8072643 | 8073320 | 8073997 | 8074674 |
| 642                 | 8078731 | 8079407 | 8080083 | 8080759 | 8081434 |
| 643                 | 8085485 | 8086160 | 8086835 | 8087510 | 8088184 |
| 644                 | 8092229 | 8092903 | 8093577 | 8094250 | 8094924 |
| 645                 | 8098962 | 8099635 | 8100308 | 8100980 | 8101653 |
| 646                 | 8105685 | 8106357 | 8107029 | 8107700 | 8108371 |
| 647                 | 8112398 | 8113068 | 8113739 | 8114409 | 8115080 |
| 648                 | 8119100 | 8119769 | 8120439 | 8121108 | 8121778 |
| 649                 | 8125792 | 8126460 | 8127129 | 8127797 | 8128465 |
| 650                 | 8132473 | 8133141 | 8133808 | 8134475 | 8135143 |
| 651                 | 8139144 | 8139811 | 8140477 | 8141144 | 8141810 |
| 652                 | 8145805 | 8146471 | 8147136 | 8147801 | 8148467 |
| 653                 | 8152456 | 8153120 | 8153785 | 8154449 | 8155113 |
| 654                 | 8159096 | 8159760 | 8160423 | 8161087 | 8161750 |
| 655                 | 8165727 | 8166389 | 8167052 | 8167714 | 8168376 |
| 656                 | 8172347 | 8173009 | 8173670 | 8174331 | 8174993 |
| 657                 | 8178958 | 8179618 | 8180278 | 8180939 | 8181599 |
| 658                 | 8185558 | 8186217 | 8186877 | 8187536 | 8188195 |
| 659                 | 8192146 | 8192806 | 8193465 | 8194123 | 8194781 |
| 660                 | 8198728 | 8199386 | 8200043 | 8200700 | 8201358 |
| 661                 | 8205298 | 8205955 | 8206611 | 8207268 | 8207924 |
| 662                 | 8211859 | 8212514 | 8213170 | 8213825 | 8214480 |



*Artificial Numbers: Or,*

| Natural<br>Numbers. | 0       | 1       | 2       | 3       | 4       |
|---------------------|---------|---------|---------|---------|---------|
| 663                 | 8215135 | 8215790 | 8216445 | 8217100 | 8217755 |
| 664                 | 8221681 | 8222335 | 8222989 | 8223643 | 8224296 |
| 665                 | 8228216 | 8228869 | 8229522 | 8230175 | 8230828 |
| 666                 | 8234742 | 8235394 | 8236046 | 8236698 | 8237350 |
| 667                 | 8241258 | 8241909 | 8242560 | 8243211 | 8243862 |
| 668                 | 8247765 | 8248415 | 8249065 | 8249715 | 8250364 |
| 669                 | 8254261 | 8254910 | 8255559 | 8256208 | 8256857 |
| 670                 | 8260748 | 8261396 | 8262044 | 8262692 | 8263340 |
| 671                 | 8267225 | 8267872 | 8268519 | 8269166 | 8269813 |
| 672                 | 8273693 | 8274339 | 8274985 | 8275631 | 8276277 |
| 673                 | 8280151 | 8280796 | 8281441 | 8282086 | 8282731 |
| 674                 | 8286599 | 8287243 | 8287887 | 8288532 | 8289176 |
| 675                 | 8293038 | 8293681 | 8294324 | 8294967 | 8295611 |
| 676                 | 8299467 | 8300109 | 8300752 | 8301394 | 8302036 |
| 677                 | 8305887 | 8306528 | 8307169 | 8307811 | 8308452 |
| 678                 | 8312297 | 8312937 | 8313578 | 8314218 | 8314858 |
| 679                 | 8318698 | 8319337 | 8319977 | 8320616 | 8321255 |
| 680                 | 8325089 | 8325728 | 8326366 | 8327005 | 8327643 |
| 681                 | 8331471 | 8332109 | 8332746 | 8333384 | 8334021 |
| 682                 | 8337844 | 8338480 | 8339117 | 8339754 | 8340390 |
| 683                 | 8344207 | 8344843 | 8345479 | 8346114 | 8346750 |
| 684                 | 8350561 | 8351196 | 8351831 | 8352465 | 8353100 |
| 685                 | 8356906 | 8357540 | 8358174 | 8358807 | 8359441 |
| 686                 | 8363241 | 8363874 | 8364507 | 8365140 | 8365773 |
| 687                 | 8369567 | 8370199 | 8370832 | 8371463 | 8372095 |
| 688                 | 8375884 | 8376516 | 8377147 | 8377778 | 8378409 |
| 689                 | 8382192 | 8382822 | 8383453 | 8384083 | 8384713 |
| 690                 | 8388491 | 8389120 | 8389750 | 8390379 | 8391008 |
| 691                 | 8394780 | 8395409 | 8396037 | 8396666 | 8397294 |
| 692                 | 8401061 | 8401688 | 8402316 | 8402943 | 8403571 |
| 693                 | 8407332 | 8407959 | 8408586 | 8409212 | 8409838 |
| 694                 | 8413595 | 8414220 | 8414846 | 8415472 | 8416097 |
| 695                 | 8419848 | 8420473 | 8421098 | 8421722 | 8422347 |
| 696                 | 8426092 | 8426716 | 8427340 | 8427964 | 8428588 |
| 697                 | 8432328 | 8432951 | 8433574 | 8434197 | 8434819 |
| 698                 | 8438554 | 8439176 | 8439798 | 8440420 | 8441042 |
| 699                 | 8444772 | 8445393 | 8446014 | 8446635 | 8447256 |
| 700                 | 8450980 | 8451601 | 8452221 | 8452841 | 8453461 |
| 701                 | 8457180 | 8457800 | 8458419 | 8459038 | 8459658 |
| 702                 | 8463371 | 8463990 | 8464608 | 8465227 | 8465845 |
| 703                 | 8469553 | 8470171 | 8470789 | 8471406 | 8472024 |
| 704                 | 8475727 | 8476343 | 8476960 | 8477577 | 8478193 |
| 705                 | 8481891 | 8482507 | 8483123 | 8483739 | 8484355 |
| 706                 | 8488047 | 8488662 | 8489277 | 8489892 | 8490507 |

*Logarithms (to 7069.)*

| Natural<br>Numbers. | 5       | 6       | 7       | 8       | 9       |
|---------------------|---------|---------|---------|---------|---------|
| 663                 | 8218409 | 8219064 | 8219718 | 8220372 | 8221027 |
| 664                 | 8224950 | 8225603 | 8226257 | 8226910 | 8227563 |
| 665                 | 8231481 | 8232133 | 8232786 | 8233438 | 8234090 |
| 666                 | 8238002 | 8238653 | 8239305 | 8239956 | 8240607 |
| 667                 | 8244513 | 8245163 | 8245814 | 8246464 | 8247114 |
| 668                 | 8251014 | 8251664 | 8252313 | 8252963 | 8253612 |
| 669                 | 8257506 | 8258154 | 8258803 | 8259451 | 8260100 |
| 670                 | 8263988 | 8264635 | 8265283 | 8265931 | 8266578 |
| 671                 | 8270460 | 8271107 | 8271753 | 8272400 | 8273046 |
| 672                 | 8276923 | 8277569 | 8278214 | 8278860 | 8279505 |
| 673                 | 8283376 | 8284021 | 8284665 | 8285310 | 8285955 |
| 674                 | 8289820 | 8290463 | 8291107 | 8291751 | 8292394 |
| 675                 | 8296254 | 8296896 | 8297539 | 8298182 | 8298824 |
| 676                 | 8302678 | 8303320 | 8303962 | 8304603 | 8305245 |
| 677                 | 8309093 | 8309734 | 8310375 | 8311016 | 8311656 |
| 678                 | 8315499 | 8316139 | 8316778 | 8317418 | 8318058 |
| 679                 | 8321895 | 8322534 | 8323173 | 8323812 | 8324450 |
| 680                 | 8328281 | 8328919 | 8329558 | 8330195 | 8330833 |
| 681                 | 8334659 | 8335296 | 8335933 | 8336570 | 8337207 |
| 682                 | 8341027 | 8341663 | 8342299 | 8342937 | 8343571 |
| 683                 | 8347385 | 8348021 | 8348656 | 8349291 | 8349926 |
| 684                 | 8353735 | 8354369 | 8355003 | 8355638 | 8356272 |
| 685                 | 8360075 | 8360708 | 8361341 | 8361975 | 8362608 |
| 686                 | 8366405 | 8367038 | 8367670 | 8368303 | 8368935 |
| 687                 | 8372727 | 8373359 | 8373990 | 8374622 | 8375253 |
| 688                 | 8379039 | 8379670 | 8380301 | 8380931 | 8381562 |
| 689                 | 8385343 | 8385973 | 8386602 | 8387232 | 8387861 |
| 690                 | 8391637 | 8392266 | 8392895 | 8393523 | 8394152 |
| 691                 | 8397922 | 8398550 | 8399178 | 8399806 | 8400433 |
| 692                 | 8404198 | 8404825 | 8405452 | 8406079 | 8406706 |
| 693                 | 8410465 | 8411091 | 8411717 | 8412343 | 8412969 |
| 694                 | 8416722 | 8417348 | 8417973 | 8418598 | 8419223 |
| 695                 | 8422971 | 8423596 | 8424220 | 8424844 | 8425468 |
| 696                 | 8429211 | 8429735 | 8430458 | 8431081 | 8431705 |
| 697                 | 8435442 | 8436065 | 8436687 | 8437310 | 8437932 |
| 698                 | 8441664 | 8442286 | 8442907 | 8443529 | 8444150 |
| 699                 | 8447877 | 8448498 | 8449119 | 8449739 | 8450360 |
| 700                 | 8454081 | 8454701 | 8455321 | 8455941 | 8456561 |
| 701                 | 8460277 | 8460896 | 8461515 | 8462134 | 8462752 |
| 702                 | 8466463 | 8467081 | 8467700 | 8468318 | 8468935 |
| 703                 | 8472641 | 8473258 | 8473876 | 8474493 | 8475110 |
| 704                 | 8478810 | 8479426 | 8480043 | 8480659 | 8481275 |
| 705                 | 8484970 | 8485586 | 8486201 | 8486817 | 8487432 |
| 706                 | 8491122 | 8491736 | 8492351 | 8492965 | 8493580 |

*Artificial Numbers: Or,*

| Natural<br>Numbers. | 0       | 1       | 2       | 3       | 4       |
|---------------------|---------|---------|---------|---------|---------|
| 707                 | 8494194 | 8494808 | 8495423 | 8496037 | 8496651 |
| 708                 | 8500333 | 8500946 | 8501559 | 8502172 | 8502786 |
| 709                 | 8506462 | 8507075 | 8507687 | 8508300 | 8508912 |
| 710                 | 8512583 | 8513195 | 8513807 | 8514418 | 8515030 |
| 711                 | 8518696 | 8519307 | 8519917 | 8520528 | 8521139 |
| 712                 | 8524800 | 8525410 | 8526020 | 8526629 | 8527239 |
| 713                 | 8530895 | 8531504 | 8532113 | 8532722 | 8533331 |
| 714                 | 8536982 | 8537590 | 8538198 | 8538806 | 8539414 |
| 715                 | 8543060 | 8543668 | 8544275 | 8544882 | 8545489 |
| 716                 | 8549130 | 8549737 | 8550343 | 8550949 | 8551556 |
| 717                 | 8555192 | 8555797 | 8556403 | 8557008 | 8557614 |
| 718                 | 8561244 | 8561849 | 8562454 | 8563059 | 8563663 |
| 719                 | 8567289 | 8567893 | 8568497 | 8569101 | 8569704 |
| 720                 | 8573325 | 8573928 | 8574531 | 8575134 | 8575737 |
| 721                 | 8579353 | 8579955 | 8580557 | 8581159 | 8581761 |
| 722                 | 8585372 | 8585973 | 8586575 | 8587176 | 8587777 |
| 723                 | 8591383 | 8591984 | 8592584 | 8593185 | 8593785 |
| 724                 | 8597386 | 8597985 | 8598585 | 8599185 | 8599784 |
| 725                 | 8603380 | 8603979 | 8604578 | 8605177 | 8605776 |
| 726                 | 8609366 | 8609964 | 8610562 | 8611160 | 8611758 |
| 727                 | 8615344 | 8615941 | 8616539 | 8617136 | 8617733 |
| 728                 | 8621314 | 8621910 | 8622507 | 8623103 | 8623699 |
| 729                 | 8627275 | 8627871 | 8628467 | 8629062 | 8629658 |
| 730                 | 8633229 | 8633823 | 8634418 | 8635013 | 8635608 |
| 731                 | 8639174 | 8639768 | 8640362 | 8640956 | 8641550 |
| 732                 | 8645111 | 8645704 | 8646297 | 8646890 | 8647483 |
| 733                 | 8651040 | 8651632 | 8652225 | 8652817 | 8653409 |
| 734                 | 8656961 | 8657552 | 8658144 | 8658735 | 8659327 |
| 735                 | 8662873 | 8663464 | 8664055 | 8664646 | 8665236 |
| 736                 | 8668778 | 8669368 | 8669958 | 8670548 | 8671138 |
| 737                 | 8674675 | 8675264 | 8675853 | 8676442 | 8677031 |
| 738                 | 8680564 | 8681152 | 8681740 | 8682329 | 8682917 |
| 739                 | 8686444 | 8687032 | 8687620 | 8688207 | 8688794 |
| 740                 | 8692317 | 8692904 | 8693491 | 8694077 | 8694664 |
| 741                 | 8698182 | 8698768 | 8699354 | 8699940 | 8700526 |
| 742                 | 8704039 | 8704624 | 8705209 | 8705795 | 8706380 |
| 743                 | 8709888 | 8710473 | 8711057 | 8711641 | 8712226 |
| 744                 | 8715729 | 8716313 | 8716897 | 8717480 | 8718064 |
| 745                 | 8721563 | 8722146 | 8722728 | 8723311 | 8723894 |
| 746                 | 8727388 | 8727970 | 8728552 | 8729134 | 8729716 |
| 747                 | 8733206 | 8733788 | 8734369 | 8734950 | 8735531 |
| 748                 | 8739016 | 8739597 | 8740177 | 8740757 | 8741338 |
| 749                 | 8744818 | 8745398 | 8745978 | 8746557 | 8747137 |
| 750                 | 8750613 | 8751192 | 8751771 | 8752349 | 8752928 |



*Logarithms (to 7509.)*

| Natural<br>Numbers. | 5       | 6       | 7       | 8       | 9       |
|---------------------|---------|---------|---------|---------|---------|
| 707                 | 8497264 | 8497878 | 8498492 | 8499106 | 8499719 |
| 708                 | 8503399 | 8504011 | 8504624 | 8505237 | 8505850 |
| 709                 | 8509524 | 8510136 | 8510748 | 8511360 | 8511972 |
| 710                 | 8515641 | 8516252 | 8516863 | 8517474 | 8518085 |
| 711                 | 8521749 | 8522359 | 8522970 | 8523580 | 8524190 |
| 712                 | 8527849 | 8528458 | 8529068 | 8529677 | 8530286 |
| 713                 | 8533940 | 8534548 | 8535157 | 8535765 | 8536374 |
| 714                 | 8540022 | 8540630 | 8541238 | 8541845 | 8542453 |
| 715                 | 8546096 | 8546703 | 8547310 | 8547917 | 8548524 |
| 716                 | 8552162 | 8552768 | 8553374 | 8553980 | 8554586 |
| 717                 | 8558219 | 8558824 | 8559429 | 8560035 | 8560640 |
| 718                 | 8564268 | 8564872 | 8565476 | 8566081 | 8566685 |
| 719                 | 8570308 | 8570912 | 8571515 | 8572118 | 8572722 |
| 720                 | 8576340 | 8576943 | 8577545 | 8578148 | 8578750 |
| 721                 | 8582363 | 8582965 | 8583567 | 8584169 | 8584770 |
| 722                 | 8588379 | 8588980 | 8589581 | 8590181 | 8590782 |
| 723                 | 8594385 | 8594986 | 8595586 | 8596186 | 8596786 |
| 724                 | 8600384 | 8600983 | 8601583 | 8602182 | 8602781 |
| 725                 | 8606374 | 8606973 | 8607571 | 8608170 | 8608768 |
| 726                 | 8612356 | 8612954 | 8613552 | 8614149 | 8614747 |
| 727                 | 8618330 | 8618927 | 8619524 | 8620120 | 8620717 |
| 728                 | 8624296 | 8624892 | 8625488 | 8626084 | 8626679 |
| 729                 | 8630253 | 8630848 | 8631443 | 8632039 | 8632634 |
| 730                 | 8636202 | 8636797 | 8637391 | 8637985 | 8638580 |
| 731                 | 8642143 | 8642737 | 8643331 | 8643924 | 8644517 |
| 732                 | 8648076 | 8648669 | 8649262 | 8649855 | 8650447 |
| 733                 | 8654001 | 8654593 | 8655185 | 8655777 | 8656369 |
| 734                 | 8659918 | 8660509 | 8661100 | 8661691 | 8662282 |
| 735                 | 8665827 | 8666417 | 8667008 | 8667598 | 8668188 |
| 736                 | 8671728 | 8672317 | 8672907 | 8673496 | 8674086 |
| 737                 | 8677620 | 8678209 | 8678798 | 8679387 | 8679975 |
| 738                 | 8683505 | 8684093 | 8684681 | 8685269 | 8685857 |
| 739                 | 8689382 | 8689969 | 8690556 | 8691143 | 8691730 |
| 740                 | 8695251 | 8695837 | 8696423 | 8697010 | 8697596 |
| 741                 | 8701112 | 8701697 | 8702283 | 8702868 | 8703454 |
| 742                 | 8706965 | 8707549 | 8708134 | 8708719 | 8709304 |
| 743                 | 8712810 | 8713394 | 8713978 | 8714562 | 8715146 |
| 744                 | 8718647 | 8719230 | 8719814 | 8720397 | 8720980 |
| 745                 | 8724476 | 8725059 | 8725641 | 8726224 | 8726806 |
| 746                 | 8730298 | 8730880 | 8731461 | 8732043 | 8732625 |
| 747                 | 8736112 | 8736693 | 8737274 | 8737855 | 8738435 |
| 748                 | 8741918 | 8742498 | 8743078 | 8743658 | 8744238 |
| 749                 | 8747716 | 8748296 | 8748875 | 8749454 | 8750034 |
| 750                 | 8753507 | 8754086 | 8754664 | 8755243 | 8755821 |

*Artificial Numbers: Or,*

| Natural<br>Numbers. | 0       | 1       | 2       | 3       | 4       |
|---------------------|---------|---------|---------|---------|---------|
| 751                 | 8756399 | 8756978 | 8757556 | 8758134 | 8758712 |
| 752                 | 8762178 | 8762756 | 8763333 | 8763911 | 8764488 |
| 753                 | 8767950 | 8768526 | 8769103 | 8769680 | 8770256 |
| 754                 | 8773713 | 8774289 | 8774865 | 8775441 | 8776017 |
| 755                 | 8779469 | 8780045 | 8780620 | 8781195 | 8781770 |
| 756                 | 8785218 | 8785792 | 8786367 | 8786941 | 8787515 |
| 757                 | 8790959 | 8791532 | 8792106 | 8792680 | 8793253 |
| 758                 | 8796692 | 8797265 | 8797838 | 8798411 | 8798983 |
| 759                 | 8802418 | 8802990 | 8803562 | 8804134 | 8804706 |
| 760                 | 8808136 | 8808707 | 8809279 | 8809850 | 8810421 |
| 761                 | 8813847 | 8814417 | 8814988 | 8815558 | 8816129 |
| 762                 | 8819550 | 8820120 | 8820689 | 8821259 | 8821829 |
| 763                 | 8825245 | 8825815 | 8826384 | 8826953 | 8827522 |
| 764                 | 8830934 | 8831502 | 8832070 | 8832639 | 8833207 |
| 765                 | 8836614 | 8837182 | 8837750 | 8838317 | 8838885 |
| 766                 | 8842288 | 8842855 | 8843421 | 8843988 | 8844555 |
| 767                 | 8847954 | 8848520 | 8849086 | 8849652 | 8850218 |
| 768                 | 8853612 | 8854178 | 8854743 | 8855308 | 8855874 |
| 769                 | 8859263 | 8859828 | 8860393 | 8860957 | 8861522 |
| 770                 | 8864907 | 8865471 | 8866035 | 8866599 | 8867163 |
| 771                 | 8870544 | 8871107 | 8871670 | 8872233 | 8872796 |
| 772                 | 8876173 | 8876736 | 8877298 | 8877860 | 8878423 |
| 773                 | 8881795 | 8882357 | 8882918 | 8883480 | 8884042 |
| 774                 | 8887410 | 8887971 | 8888532 | 8889093 | 8889653 |
| 775                 | 8893017 | 8893577 | 8894138 | 8894698 | 8895258 |
| 776                 | 8898617 | 8899177 | 8899736 | 8900296 | 8900855 |
| 777                 | 8904210 | 8904769 | 8905328 | 8905887 | 8906445 |
| 778                 | 8909796 | 8910354 | 8910912 | 8911470 | 8912028 |
| 779                 | 8915375 | 8915932 | 8916489 | 8917047 | 8917604 |
| 780                 | 8920946 | 8921503 | 8922059 | 8922616 | 8923173 |
| 781                 | 8926510 | 8927066 | 8927622 | 8928178 | 8928734 |
| 782                 | 8932068 | 8932623 | 8933178 | 8933733 | 8934288 |
| 783                 | 8937618 | 8938172 | 8938727 | 8939281 | 8939836 |
| 784                 | 8943161 | 8943715 | 8944268 | 8944822 | 8945376 |
| 785                 | 8948697 | 8949250 | 8949803 | 8950356 | 8950909 |
| 786                 | 8954225 | 8954778 | 8955330 | 8955883 | 8956435 |
| 787                 | 8959747 | 8960299 | 8960851 | 8961403 | 8961954 |
| 788                 | 8965262 | 8965813 | 8966364 | 8966915 | 8967466 |
| 789                 | 8970770 | 8971320 | 8971871 | 8972421 | 8972971 |
| 790                 | 8976271 | 8976821 | 8977370 | 8977920 | 8978469 |
| 791                 | 8981765 | 8982314 | 8982863 | 8983412 | 8983960 |
| 792                 | 8987252 | 8987800 | 8988348 | 8988897 | 8989445 |
| 793                 | 8992732 | 8993279 | 8993827 | 8994375 | 8994922 |
| 794                 | 8998205 | 8998752 | 8999299 | 8999846 | 9000392 |

*Logarithms (to 7949.)*

| Natural<br>Numbers. | 5       | 6       | 7       | 8       | 9       |
|---------------------|---------|---------|---------|---------|---------|
| 751                 | 8759290 | 8759868 | 8760445 | 8761023 | 8761601 |
| 752                 | 8765065 | 8765642 | 8766219 | 8766796 | 8767373 |
| 753                 | 8770833 | 8771409 | 8771985 | 8772561 | 8773137 |
| 754                 | 8776592 | 8777168 | 8777743 | 8778319 | 8778894 |
| 755                 | 8782345 | 8782919 | 8783494 | 8784069 | 8784643 |
| 756                 | 8788089 | 8788663 | 8789237 | 8789811 | 8790385 |
| 757                 | 8793826 | 8794400 | 8794973 | 8795546 | 8796119 |
| 758                 | 8799556 | 8800128 | 8800701 | 8801273 | 8801846 |
| 759                 | 8805278 | 8805850 | 8806421 | 8806993 | 8807564 |
| 760                 | 8810992 | 8811563 | 8812134 | 8812705 | 8813276 |
| 761                 | 8816699 | 8817269 | 8817840 | 8818410 | 8818980 |
| 762                 | 8822398 | 8822968 | 8823537 | 8824107 | 8824676 |
| 763                 | 8828090 | 8828659 | 8829228 | 8829797 | 8830365 |
| 764                 | 8833775 | 8834343 | 8834911 | 8835479 | 8836047 |
| 765                 | 8839452 | 8840019 | 8840586 | 8841154 | 8841721 |
| 766                 | 8845122 | 8845688 | 8846255 | 8846821 | 8847387 |
| 767                 | 8850784 | 8851350 | 8851915 | 8852481 | 8853047 |
| 768                 | 8856439 | 8857004 | 8857569 | 8858134 | 8858699 |
| 769                 | 8862086 | 8862651 | 8863215 | 8863779 | 8864343 |
| 770                 | 8867726 | 8868290 | 8868854 | 8869417 | 8869980 |
| 771                 | 8873359 | 8873922 | 8874485 | 8875048 | 8875610 |
| 772                 | 8878985 | 8879547 | 8880109 | 8880671 | 8881233 |
| 773                 | 8884603 | 8885165 | 8885726 | 8886287 | 8886848 |
| 774                 | 8890214 | 8890775 | 8891336 | 8891896 | 8892457 |
| 775                 | 8895818 | 8896378 | 8896935 | 8897498 | 8898058 |
| 776                 | 8901415 | 8901974 | 8902533 | 8903092 | 8903651 |
| 777                 | 8907004 | 8907562 | 8908121 | 8908679 | 8909238 |
| 778                 | 8912586 | 8913144 | 8913702 | 8914259 | 8914817 |
| 779                 | 8918161 | 8918718 | 8919275 | 8919832 | 8920389 |
| 780                 | 8923729 | 8924285 | 8924842 | 8925398 | 8925954 |
| 781                 | 8929290 | 8929846 | 8930401 | 8930957 | 8931512 |
| 782                 | 8934843 | 8935398 | 8935953 | 8936508 | 8937063 |
| 783                 | 8940390 | 8940944 | 8941498 | 8942053 | 8942607 |
| 784                 | 8945929 | 8946483 | 8947037 | 8947590 | 8948143 |
| 785                 | 8951462 | 8952015 | 8952567 | 8953120 | 8953673 |
| 786                 | 8956987 | 8957539 | 8958092 | 8958644 | 8959195 |
| 787                 | 8962506 | 8963057 | 8963608 | 8964160 | 8964711 |
| 788                 | 8968017 | 8968568 | 8969118 | 8969669 | 8970219 |
| 789                 | 8973521 | 8974071 | 8974621 | 8975171 | 8975721 |
| 790                 | 8979019 | 8979568 | 8980117 | 8980667 | 8981216 |
| 791                 | 8984509 | 8985058 | 8985606 | 8986155 | 8986703 |
| 792                 | 8989993 | 8990541 | 8991089 | 8991636 | 8992184 |
| 793                 | 8995469 | 8996017 | 8996564 | 8997111 | 8997658 |
| 794                 | 9000939 | 9001486 | 9002032 | 9002579 | 9003125 |



*Artificial Numbers: Or,*

| Natural<br>Numbers. | 0       | 1       | 2       | 3       | 4       |
|---------------------|---------|---------|---------|---------|---------|
| 795                 | 9003671 | 9004218 | 9004764 | 9005310 | 9005856 |
| 796                 | 9009131 | 9009676 | 9010222 | 9010767 | 9011313 |
| 797                 | 9014583 | 9015128 | 9015673 | 9016218 | 9016762 |
| 798                 | 9020029 | 9020573 | 9021117 | 9021661 | 9022205 |
| 799                 | 9025468 | 9026011 | 9026555 | 9027098 | 9027641 |
| 800                 | 9030900 | 9031443 | 9031985 | 9032528 | 9033071 |
| 801                 | 9036325 | 9036867 | 9037409 | 9037951 | 9038493 |
| 802                 | 9041744 | 9042285 | 9042827 | 9043368 | 9043909 |
| 803                 | 9047155 | 9047696 | 9048237 | 9048778 | 9049318 |
| 804                 | 9052560 | 9053101 | 9053641 | 9054181 | 9054721 |
| 805                 | 9057960 | 9058498 | 9059038 | 9059577 | 9060116 |
| 806                 | 9063351 | 9063889 | 9064428 | 9064967 | 9065505 |
| 807                 | 9068735 | 9069273 | 9069812 | 9070350 | 9070887 |
| 808                 | 9074114 | 9074651 | 9075188 | 9075726 | 9076263 |
| 809                 | 9079485 | 9080022 | 9080559 | 9081095 | 9081632 |
| 810                 | 9084850 | 9085386 | 9085922 | 9086458 | 9086994 |
| 811                 | 9090209 | 9090744 | 9091279 | 9091815 | 9092350 |
| 812                 | 9095560 | 9096095 | 9096630 | 9097165 | 9097699 |
| 813                 | 9100905 | 9101440 | 9101974 | 9102508 | 9103042 |
| 814                 | 9106244 | 9106778 | 9107311 | 9107844 | 9108378 |
| 815                 | 9111576 | 9112109 | 9112642 | 9113174 | 9113707 |
| 816                 | 9116902 | 9117434 | 9117966 | 9118498 | 9119030 |
| 817                 | 9122220 | 9122752 | 9123284 | 9123815 | 9124346 |
| 818                 | 9127533 | 9128064 | 9128595 | 9129126 | 9129656 |
| 819                 | 9132839 | 9133369 | 9133899 | 9134430 | 9134960 |
| 820                 | 9138139 | 9138668 | 9139198 | 9139727 | 9140257 |
| 821                 | 9143432 | 9143961 | 9144489 | 9145018 | 9145547 |
| 822                 | 9148718 | 9149246 | 9149775 | 9150303 | 9150831 |
| 823                 | 9153998 | 9154526 | 9155054 | 9155581 | 9156109 |
| 824                 | 9159272 | 9159799 | 9160326 | 9160853 | 9161380 |
| 825                 | 9164539 | 9165066 | 9165592 | 9166118 | 9166645 |
| 826                 | 9169800 | 9170326 | 9170852 | 9171378 | 9171903 |
| 827                 | 9175055 | 9175580 | 9176105 | 9176630 | 9177155 |
| 828                 | 9180303 | 9180828 | 9181352 | 9181877 | 9182401 |
| 829                 | 9185545 | 9186069 | 9186593 | 9187117 | 9187640 |
| 830                 | 9190781 | 9191304 | 9191827 | 9192350 | 9192873 |
| 831                 | 9196010 | 9196533 | 9197055 | 9197578 | 9198100 |
| 832                 | 9201233 | 9201755 | 9202277 | 9202799 | 9203321 |
| 833                 | 9206450 | 9206971 | 9207493 | 9208014 | 9208535 |
| 834                 | 9211661 | 9212181 | 9212702 | 9213222 | 9213743 |
| 835                 | 9216865 | 9217385 | 9217905 | 9218425 | 9218945 |
| 836                 | 9222063 | 9222582 | 9223102 | 9223621 | 9224140 |
| 837                 | 9227255 | 9227773 | 9228292 | 9228811 | 9229330 |
| 838                 | 9232440 | 9232958 | 9233477 | 9233995 | 9234513 |

*Logarithms (to 8389.)*

| Natural<br>Numbers. | 5       | 6       | 7       | 8       | 9       |
|---------------------|---------|---------|---------|---------|---------|
| 795                 | 9006402 | 9006948 | 9007494 | 9008039 | 9008585 |
| 796                 | 9011858 | 9012403 | 9012948 | 9013493 | 9014038 |
| 797                 | 9017307 | 9017851 | 9018396 | 9018940 | 9019485 |
| 798                 | 9022749 | 9023293 | 9023837 | 9024381 | 9024924 |
| 799                 | 9028185 | 9028728 | 9029271 | 9029814 | 9030357 |
| 800                 | 9033613 | 9034156 | 9034698 | 9035241 | 9035783 |
| 801                 | 9039035 | 9039577 | 9040119 | 9040661 | 9041202 |
| 802                 | 9044450 | 9044992 | 9045533 | 9046073 | 9046615 |
| 803                 | 9049859 | 9050399 | 9050940 | 9051480 | 9052020 |
| 804                 | 9055261 | 9055800 | 9056340 | 9056880 | 9057419 |
| 805                 | 9060655 | 9061195 | 9061734 | 9062274 | 9062812 |
| 806                 | 9066044 | 9066582 | 9067121 | 9067659 | 9068197 |
| 807                 | 9071425 | 9071963 | 9072501 | 9073038 | 9073576 |
| 808                 | 9076800 | 9077337 | 9077874 | 9078411 | 9078948 |
| 809                 | 9082169 | 9082705 | 9083241 | 9083778 | 9084314 |
| 810                 | 9087530 | 9088066 | 9088602 | 9089137 | 9089673 |
| 811                 | 9092885 | 9093420 | 9093955 | 9094490 | 9095025 |
| 812                 | 9098234 | 9098768 | 9099303 | 9099837 | 9100371 |
| 813                 | 9103576 | 9104109 | 9104643 | 9105177 | 9105710 |
| 814                 | 9108911 | 9109444 | 9109977 | 9110510 | 9111043 |
| 815                 | 9114240 | 9114772 | 9115305 | 9115837 | 9116369 |
| 816                 | 9119562 | 9120094 | 9120626 | 9121157 | 9121689 |
| 817                 | 9124878 | 9125409 | 9125940 | 9126471 | 9127002 |
| 818                 | 9130187 | 9130717 | 9131248 | 9131778 | 9132309 |
| 819                 | 9135490 | 9136019 | 9136549 | 9137079 | 9137609 |
| 820                 | 9140786 | 9141315 | 9141844 | 9142373 | 9142903 |
| 821                 | 9146076 | 9146604 | 9147133 | 9147661 | 9148190 |
| 822                 | 9151359 | 9151887 | 9152415 | 9152943 | 9153471 |
| 823                 | 9156636 | 9157163 | 9157691 | 9158218 | 9158745 |
| 824                 | 9161907 | 9162433 | 9162960 | 9163487 | 9164013 |
| 825                 | 9167171 | 9167697 | 9168223 | 9168749 | 9169275 |
| 826                 | 9172429 | 9172954 | 9173479 | 9174005 | 9174530 |
| 827                 | 9177680 | 9178205 | 9178730 | 9179254 | 9179779 |
| 828                 | 9182925 | 9183449 | 9183973 | 9184497 | 9185021 |
| 829                 | 9188164 | 9188687 | 9189211 | 9189734 | 9190258 |
| 830                 | 9193396 | 9193919 | 9194442 | 9194965 | 9195488 |
| 831                 | 9198623 | 9199145 | 9199667 | 9200189 | 9200711 |
| 832                 | 9203842 | 9204364 | 9204886 | 9205407 | 9205929 |
| 833                 | 9209056 | 9209577 | 9210098 | 9210619 | 9211140 |
| 834                 | 9214263 | 9214784 | 9215304 | 9215824 | 9216345 |
| 835                 | 9219465 | 9219984 | 9220504 | 9221024 | 9221543 |
| 836                 | 9224659 | 9225179 | 9225698 | 9226217 | 9226736 |
| 837                 | 9229848 | 9230367 | 9230885 | 9231404 | 9231922 |
| 838                 | 9235031 | 9235549 | 9236066 | 9236584 | 9237102 |

*Artificial Numbers: Or,*

| Natural<br>Numbers. | 0       | 1       | 2       | 3       | 4       |
|---------------------|---------|---------|---------|---------|---------|
| 839                 | 9237620 | 9238137 | 9238655 | 9239172 | 9239690 |
| 840                 | 9242793 | 9243310 | 9243827 | 9244344 | 9244860 |
| 841                 | 9247960 | 9248476 | 9248993 | 9249509 | 9250025 |
| 842                 | 9253121 | 9253637 | 9254152 | 9254668 | 9255184 |
| 843                 | 9258276 | 9258791 | 9259306 | 9259821 | 9260336 |
| 844                 | 9263424 | 9263939 | 9264453 | 9264968 | 9265482 |
| 845                 | 9268567 | 9269081 | 9269595 | 9270109 | 9270622 |
| 846                 | 9273704 | 9274217 | 9274730 | 9275243 | 9275757 |
| 847                 | 9278834 | 9279347 | 9279859 | 9280372 | 9280885 |
| 848                 | 9283959 | 9284471 | 9284983 | 9285495 | 9286007 |
| 849                 | 9289077 | 9289588 | 9290100 | 9290611 | 9291123 |
| 850                 | 9294189 | 9294700 | 9295211 | 9295722 | 9296233 |
| 851                 | 9299296 | 9299806 | 9300316 | 9300826 | 9301336 |
| 852                 | 9304396 | 9304906 | 9305415 | 9305925 | 9306434 |
| 853                 | 9309490 | 9309999 | 9310508 | 9311017 | 9311526 |
| 854                 | 9314579 | 9315087 | 9315596 | 9316104 | 9316612 |
| 855                 | 9319661 | 9320169 | 9320677 | 9321185 | 9321692 |
| 856                 | 9324738 | 9325245 | 9325752 | 9326259 | 9326767 |
| 857                 | 9329808 | 9330315 | 9330822 | 9331328 | 9331835 |
| 858                 | 9334873 | 9335379 | 9335885 | 9336391 | 9336897 |
| 859                 | 9339932 | 9340437 | 9340943 | 9341448 | 9341953 |
| 860                 | 9344984 | 9345489 | 9345994 | 9346499 | 9347004 |
| 861                 | 9350032 | 9350536 | 9351040 | 9351544 | 9352049 |
| 862                 | 9355073 | 9355576 | 9356080 | 9356584 | 9357087 |
| 863                 | 9360108 | 9360611 | 9361114 | 9361617 | 9362120 |
| 864                 | 9365137 | 9365640 | 9366143 | 9366645 | 9367148 |
| 865                 | 9370161 | 9370663 | 9371165 | 9371667 | 9372169 |
| 866                 | 9375179 | 9375680 | 9376182 | 9376683 | 9377184 |
| 867                 | 9380191 | 9380692 | 9381193 | 9381693 | 9382194 |
| 868                 | 9385197 | 9385697 | 9386198 | 9386698 | 9387198 |
| 869                 | 9390198 | 9390697 | 9391197 | 9391697 | 9392196 |
| 870                 | 9395193 | 9395692 | 9396191 | 9396690 | 9397189 |
| 871                 | 9400182 | 9400680 | 9401179 | 9401677 | 9402176 |
| 872                 | 9405165 | 9405663 | 9406161 | 9406659 | 9407157 |
| 873                 | 9410142 | 9410640 | 9411137 | 9411635 | 9412132 |
| 874                 | 9415114 | 9415611 | 9416108 | 9416605 | 9417101 |
| 875                 | 9420081 | 9420577 | 9421073 | 9421569 | 9422065 |
| 876                 | 9425041 | 9425537 | 9426032 | 9426528 | 9427024 |
| 877                 | 9429996 | 9430491 | 9430986 | 9431481 | 9431976 |
| 878                 | 9434945 | 9435440 | 9435934 | 9436429 | 9436923 |
| 879                 | 9439889 | 9440383 | 9440877 | 9441371 | 9441865 |
| 880                 | 9444827 | 9445320 | 9445814 | 9446307 | 9446800 |
| 881                 | 9449759 | 9450252 | 9450745 | 9451238 | 9451730 |
| 882                 | 9454686 | 9455178 | 9455671 | 9456163 | 9456655 |



*Logarithms (to 8829.)*

| Natural<br>Numbers. | 5       | 6       | 7       | 8       | 9       |
|---------------------|---------|---------|---------|---------|---------|
| 839                 | 9240208 | 9240724 | 9241246 | 9241759 | 9242276 |
| 840                 | 9245377 | 9245894 | 9246410 | 9246927 | 9247444 |
| 841                 | 9250541 | 9251057 | 9251573 | 9252689 | 9252605 |
| 842                 | 9255699 | 9256215 | 9256730 | 9257245 | 9257761 |
| 843                 | 9260851 | 9261366 | 9261880 | 9262395 | 9262910 |
| 844                 | 9265995 | 9266511 | 9267025 | 9267539 | 9268053 |
| 845                 | 9271136 | 9271650 | 9272163 | 9272677 | 9273190 |
| 846                 | 9276270 | 9276783 | 9277296 | 9277808 | 9278321 |
| 847                 | 9281397 | 9281909 | 9282422 | 9282934 | 9283446 |
| 848                 | 9286518 | 9287030 | 9287542 | 9288054 | 9288565 |
| 849                 | 9291634 | 9292145 | 9292656 | 9293167 | 9293678 |
| 850                 | 9296743 | 9297254 | 9297764 | 9298275 | 9298785 |
| 851                 | 9301847 | 9302357 | 9302866 | 9303376 | 9303886 |
| 852                 | 9306944 | 9307453 | 9307963 | 9308472 | 9308981 |
| 853                 | 9312035 | 9312544 | 9313053 | 9313561 | 9314070 |
| 854                 | 9317121 | 9317629 | 9318137 | 9318645 | 9319153 |
| 855                 | 9322200 | 9322708 | 9323215 | 9323723 | 9324230 |
| 856                 | 9327274 | 9327781 | 9328288 | 9328795 | 9329301 |
| 857                 | 9332341 | 9332848 | 9333354 | 9333860 | 9334367 |
| 858                 | 9337403 | 9337909 | 9338415 | 9338920 | 9339426 |
| 859                 | 9342459 | 9342964 | 9343469 | 9343974 | 9344479 |
| 860                 | 9347509 | 9348013 | 9348518 | 9349022 | 9349527 |
| 861                 | 9352553 | 9353057 | 9353561 | 9354065 | 9354569 |
| 862                 | 9357591 | 9358095 | 9358598 | 9359101 | 9359605 |
| 863                 | 9362623 | 9363126 | 9363629 | 9364132 | 9364635 |
| 864                 | 9367650 | 9368152 | 9368655 | 9369157 | 9369659 |
| 865                 | 9372671 | 9373172 | 9373674 | 9374176 | 9374677 |
| 866                 | 9377686 | 9378187 | 9378688 | 9379189 | 9379690 |
| 867                 | 9382695 | 9383195 | 9383696 | 9384196 | 9384697 |
| 868                 | 9387698 | 9388198 | 9388698 | 9389198 | 9389698 |
| 869                 | 9392696 | 9393195 | 9393695 | 9394194 | 9394693 |
| 870                 | 9397688 | 9398187 | 9398685 | 9399184 | 9399683 |
| 871                 | 9402674 | 9403172 | 9403670 | 9404169 | 9404667 |
| 872                 | 9407654 | 9408152 | 9408650 | 9409147 | 9409645 |
| 873                 | 9412629 | 9413126 | 9413623 | 9414120 | 9414617 |
| 874                 | 9417598 | 9418095 | 9418591 | 9419088 | 9419584 |
| 875                 | 9422561 | 9423058 | 9423554 | 9424049 | 9424545 |
| 876                 | 9427519 | 9428015 | 9428510 | 9429005 | 9429501 |
| 877                 | 9432471 | 9432966 | 9433461 | 9433956 | 9434450 |
| 878                 | 9437418 | 9437912 | 9438406 | 9438900 | 9439395 |
| 879                 | 9442358 | 9442852 | 9443346 | 9443840 | 9444333 |
| 880                 | 9447294 | 9447787 | 9448280 | 9448773 | 9449266 |
| 881                 | 9452223 | 9452716 | 9453208 | 9453701 | 9454193 |
| 882                 | 9457147 | 9457639 | 9458131 | 9458623 | 9459115 |

*Artificial Numbers: Or,*

| Natural<br>Numbers. | 0       | 1       | 2       | 3       | 4       |
|---------------------|---------|---------|---------|---------|---------|
| 883                 | 9459607 | 9460099 | 9460591 | 9461082 | 9461574 |
| 884                 | 9464523 | 9465014 | 9465505 | 9465996 | 9466487 |
| 885                 | 9469433 | 9469923 | 9470414 | 9470905 | 9471395 |
| 886                 | 9474337 | 9474827 | 9475317 | 9475807 | 9476297 |
| 887                 | 9479236 | 9479726 | 9480215 | 9480705 | 9481194 |
| 888                 | 9484130 | 9484619 | 9485108 | 9485597 | 9486085 |
| 889                 | 9489018 | 9489506 | 9489994 | 9490483 | 9490971 |
| 890                 | 9493900 | 9494388 | 9494876 | 9495364 | 9495852 |
| 891                 | 9498777 | 9499264 | 9499752 | 9500239 | 9500726 |
| 892                 | 9503649 | 9504135 | 9504622 | 9505109 | 9505596 |
| 893                 | 9508515 | 9509001 | 9509487 | 9509973 | 9510459 |
| 894                 | 9513375 | 9513861 | 9514347 | 9514832 | 9515318 |
| 895                 | 9518230 | 9518716 | 9519201 | 9519686 | 9520171 |
| 896                 | 9523080 | 9523565 | 9524049 | 9524534 | 9525018 |
| 897                 | 9527924 | 9528409 | 9528893 | 9529377 | 9529861 |
| 898                 | 9532763 | 9533247 | 9533730 | 9534214 | 9534697 |
| 899                 | 9537597 | 9538080 | 9538563 | 9539046 | 9539529 |
| 900                 | 9542425 | 9542908 | 9543390 | 9543872 | 9544355 |
| 901                 | 9547248 | 9547730 | 9548212 | 9548694 | 9549176 |
| 902                 | 9552065 | 9552547 | 9553028 | 9553510 | 9553991 |
| 903                 | 9556877 | 9557358 | 9557839 | 9558320 | 9558801 |
| 904                 | 9561684 | 9562165 | 9562645 | 9563125 | 9563605 |
| 905                 | 9566486 | 9566966 | 9567445 | 9567925 | 9568405 |
| 906                 | 9571282 | 9571761 | 9572241 | 9572720 | 9573199 |
| 907                 | 9576073 | 9576552 | 9577030 | 9577509 | 9577988 |
| 908                 | 9580858 | 9581337 | 9581815 | 9582293 | 9582771 |
| 909                 | 9585639 | 9586117 | 9586594 | 9587072 | 9587549 |
| 910                 | 9590414 | 9590891 | 9591368 | 9591845 | 9592322 |
| 911                 | 9595184 | 9595660 | 9596137 | 9596614 | 9597090 |
| 912                 | 9599948 | 9600425 | 9600901 | 9601377 | 9601853 |
| 913                 | 9604708 | 9605183 | 9605659 | 9606135 | 9606610 |
| 914                 | 9609462 | 9609937 | 9610412 | 9610887 | 9611362 |
| 915                 | 9614211 | 9614686 | 9615160 | 9615635 | 9616109 |
| 916                 | 9618955 | 9619429 | 9619903 | 9620377 | 9620851 |
| 917                 | 9623693 | 9624167 | 9624640 | 9625114 | 9625587 |
| 918                 | 9628427 | 9628900 | 9629373 | 9629846 | 9630319 |
| 919                 | 9633155 | 9633628 | 9634100 | 9634573 | 9635045 |
| 920                 | 9637878 | 9638350 | 9638822 | 9639294 | 9639766 |
| 921                 | 9642596 | 9643068 | 9643539 | 9644011 | 9644482 |
| 922                 | 9647309 | 9647780 | 9648251 | 9648722 | 9649193 |
| 923                 | 9652017 | 9652488 | 9652958 | 9653428 | 9653899 |
| 924                 | 9656720 | 9657190 | 9657660 | 9658130 | 9658599 |
| 925                 | 9661417 | 9661887 | 9662356 | 9662826 | 9663295 |
| 926                 | 9666110 | 9667579 | 9667048 | 9667517 | 9667985 |

*Logarithms (to 9269.)*

| Natural<br>Numbers. | 5       | 6       | 7       | 8       | 9       |
|---------------------|---------|---------|---------|---------|---------|
| 883                 | 9462066 | 9462557 | 9463048 | 9463540 | 9464031 |
| 884                 | 9466978 | 9467469 | 9467960 | 9468451 | 9468942 |
| 885                 | 9471886 | 9472376 | 9472866 | 9473357 | 9473847 |
| 886                 | 9476787 | 9477277 | 9477767 | 9478257 | 9478747 |
| 887                 | 9481684 | 9482173 | 9482662 | 9483151 | 9483641 |
| 888                 | 9486574 | 9487063 | 9487552 | 9488040 | 9488529 |
| 889                 | 9491460 | 9491948 | 9492436 | 9492924 | 9493412 |
| 890                 | 9496330 | 9496827 | 9497314 | 9497802 | 9498290 |
| 891                 | 9501213 | 9501701 | 9502188 | 9502675 | 9503162 |
| 892                 | 9506082 | 9506569 | 9507055 | 9507542 | 9508028 |
| 893                 | 9510946 | 9511432 | 9511918 | 9512404 | 9512889 |
| 894                 | 9515803 | 9516289 | 9516774 | 9517260 | 9517745 |
| 895                 | 9520656 | 9521141 | 9521626 | 9522111 | 9522595 |
| 896                 | 9525503 | 9525987 | 9526472 | 9526956 | 9527440 |
| 897                 | 9530345 | 9530828 | 9531312 | 9531796 | 9532280 |
| 898                 | 9535181 | 9535664 | 9536147 | 9536631 | 9537114 |
| 899                 | 9540012 | 9540494 | 9540977 | 9541460 | 9541943 |
| 900                 | 9544837 | 9545319 | 9545802 | 9546284 | 9546766 |
| 901                 | 9549657 | 9550139 | 9550621 | 9551102 | 9551584 |
| 902                 | 9554472 | 9554953 | 9555434 | 9555915 | 9556397 |
| 903                 | 9559282 | 9559762 | 9560243 | 9560723 | 9561204 |
| 904                 | 9564086 | 9564566 | 9565046 | 9565526 | 9566006 |
| 905                 | 9568885 | 9569364 | 9569844 | 9570323 | 9570803 |
| 906                 | 9573678 | 9574157 | 9574636 | 9575115 | 9575594 |
| 907                 | 9578466 | 9578945 | 9579423 | 9579902 | 9580380 |
| 908                 | 9583249 | 9583727 | 9584205 | 9584683 | 9585161 |
| 909                 | 9588027 | 9588505 | 9588982 | 9589459 | 9589937 |
| 910                 | 9592799 | 9593276 | 9593754 | 9594230 | 9594707 |
| 911                 | 9597567 | 9598043 | 9598520 | 9598996 | 9599472 |
| 912                 | 9602329 | 9602805 | 9603280 | 9603756 | 9604232 |
| 913                 | 9607086 | 9607561 | 9608036 | 9608511 | 9608987 |
| 914                 | 9611837 | 9612312 | 9612787 | 9613261 | 9613736 |
| 915                 | 9616583 | 9617058 | 9617532 | 9618006 | 9618481 |
| 916                 | 9621325 | 9621799 | 9622272 | 9622746 | 9623220 |
| 917                 | 9626061 | 9626534 | 9627007 | 9627481 | 9627954 |
| 918                 | 9630792 | 9631264 | 9631737 | 9632210 | 9632683 |
| 919                 | 9635517 | 9635990 | 9636462 | 9636934 | 9637406 |
| 920                 | 9640238 | 9640710 | 9641181 | 9641653 | 9642125 |
| 921                 | 9644953 | 9645425 | 9645896 | 9646367 | 9646838 |
| 922                 | 9649664 | 9650134 | 9650605 | 9651076 | 9651546 |
| 923                 | 9654369 | 9654839 | 9655309 | 9655780 | 9656250 |
| 924                 | 9659069 | 9659539 | 9660009 | 9660478 | 9660948 |
| 925                 | 9663764 | 9664233 | 9664703 | 9665172 | 9665641 |
| 926                 | 9668454 | 9668923 | 9669392 | 9669860 | 9670329 |



*Artificial Numbers: Or,*

| Natural<br>Numbers. | 0       | 1       | 2       | 3       | 4       |
|---------------------|---------|---------|---------|---------|---------|
| 927                 | 9670797 | 9671266 | 9671734 | 9672203 | 9672671 |
| 928                 | 9675480 | 9675948 | 9676416 | 9676883 | 9677351 |
| 929                 | 9680157 | 9680625 | 9681092 | 9681559 | 9682027 |
| 930                 | 9684829 | 9685296 | 9685763 | 9686230 | 9686697 |
| 931                 | 9689497 | 9689963 | 9690430 | 9690896 | 9691362 |
| 932                 | 9694159 | 9694625 | 9695091 | 9695557 | 9696023 |
| 933                 | 9698816 | 9699282 | 9699747 | 9700213 | 9700678 |
| 934                 | 9703469 | 9703934 | 9704399 | 9704863 | 9705328 |
| 935                 | 9708116 | 9708581 | 9709045 | 9709509 | 9709974 |
| 936                 | 9712758 | 9713222 | 9713686 | 9714150 | 9714614 |
| 937                 | 9717396 | 9717859 | 9718323 | 9718786 | 9719249 |
| 938                 | 9722028 | 9722491 | 9722954 | 9723417 | 9723880 |
| 939                 | 9726656 | 9727118 | 9727581 | 9728043 | 9728506 |
| 940                 | 9731278 | 9731741 | 9732202 | 9732664 | 9733126 |
| 941                 | 9735896 | 9736358 | 9736819 | 9737281 | 9737742 |
| 942                 | 9740509 | 9740970 | 9741431 | 9741892 | 9742353 |
| 943                 | 9745117 | 9745577 | 9746038 | 9746498 | 9746959 |
| 944                 | 9749720 | 9750180 | 9750640 | 9751100 | 9751560 |
| 945                 | 9754318 | 9754778 | 9755237 | 9755695 | 9756156 |
| 946                 | 9758911 | 9759370 | 9759829 | 9760288 | 9760747 |
| 947                 | 9763500 | 9763958 | 9764417 | 9764875 | 9765334 |
| 948                 | 9768083 | 9768541 | 9768999 | 9769457 | 9769915 |
| 949                 | 9772662 | 9773120 | 9773577 | 9774035 | 9774492 |
| 950                 | 9777236 | 9777693 | 9778150 | 9778607 | 9779064 |
| 951                 | 9781805 | 9782262 | 9782718 | 9783175 | 9783631 |
| 952                 | 9786369 | 9786826 | 9787282 | 9787738 | 9788194 |
| 953                 | 9790929 | 9791385 | 9791840 | 9792296 | 9792751 |
| 954                 | 9795484 | 9795939 | 9796394 | 9796849 | 9797304 |
| 955                 | 9800034 | 9800488 | 9800943 | 9801398 | 9801852 |
| 956                 | 9804579 | 9805033 | 9805487 | 9805942 | 9806396 |
| 957                 | 9809119 | 9809573 | 9810027 | 9810481 | 9810934 |
| 958                 | 9813655 | 9814108 | 9814562 | 9815015 | 9815468 |
| 959                 | 9818186 | 9818639 | 9819092 | 9819544 | 9819997 |
| 960                 | 9822712 | 9823165 | 9823617 | 9824069 | 9824522 |
| 961                 | 9827234 | 9827686 | 9828138 | 9828589 | 9829041 |
| 962                 | 9831751 | 9832202 | 9832654 | 9833105 | 9833556 |
| 963                 | 9836263 | 9836714 | 9837165 | 9837616 | 9838066 |
| 964                 | 9840770 | 9841221 | 9841671 | 9842122 | 9842572 |
| 965                 | 9845273 | 9845723 | 9846173 | 9846623 | 9847073 |
| 966                 | 9849771 | 9850221 | 9850670 | 9851120 | 9851569 |
| 967                 | 9854265 | 9854714 | 9855163 | 9855612 | 9856061 |
| 968                 | 9858754 | 9859202 | 9859651 | 9860099 | 9860548 |
| 969                 | 9863238 | 9863686 | 9864134 | 9864582 | 9865030 |
| 970                 | 9867717 | 9868165 | 9868613 | 9869060 | 9869508 |

*Logarithms (to 9709.)*

| Natural<br>Numbers. | 5       | 6       | 7       | 8       | 9       |
|---------------------|---------|---------|---------|---------|---------|
| 927                 | 9673139 | 9673607 | 9674076 | 9674544 | 9675012 |
| 928                 | 9677819 | 9678287 | 9678754 | 9679222 | 9679690 |
| 929                 | 9682494 | 9682961 | 9683428 | 9683895 | 9684362 |
| 930                 | 9687164 | 9687630 | 9688097 | 9688564 | 9689030 |
| 931                 | 9691829 | 9692295 | 9692761 | 9693227 | 9693693 |
| 932                 | 9696488 | 9696954 | 9697420 | 9697885 | 9698351 |
| 933                 | 9701143 | 9701608 | 9702074 | 9702539 | 9703004 |
| 934                 | 9705793 | 9706258 | 9706722 | 9707187 | 9707652 |
| 935                 | 9710438 | 9710902 | 9711366 | 9711830 | 9712294 |
| 936                 | 9715078 | 9715542 | 9716005 | 9716469 | 9716932 |
| 937                 | 9719713 | 9720176 | 9720639 | 9721102 | 9721565 |
| 938                 | 9724343 | 9724805 | 9725268 | 9725731 | 9726193 |
| 939                 | 9728968 | 9729430 | 9729892 | 9730354 | 9730816 |
| 940                 | 9733588 | 9734050 | 9734511 | 9734973 | 9735435 |
| 941                 | 9738203 | 9738664 | 9739126 | 9739587 | 9740048 |
| 942                 | 9742814 | 9743274 | 9743735 | 9744196 | 9744656 |
| 943                 | 9747419 | 9747879 | 9748340 | 9748800 | 9749260 |
| 944                 | 9752020 | 9752479 | 9752939 | 9753399 | 9753858 |
| 945                 | 9756615 | 9757075 | 9757534 | 9757993 | 9758452 |
| 946                 | 9761206 | 9761665 | 9762124 | 9762582 | 9763041 |
| 947                 | 9765792 | 9766251 | 9766709 | 9767167 | 9767625 |
| 948                 | 9770373 | 9770831 | 9771289 | 9771747 | 9772204 |
| 949                 | 9774950 | 9775407 | 9775864 | 9776322 | 9776779 |
| 950                 | 9779521 | 9779978 | 9780435 | 9780892 | 9781348 |
| 951                 | 9784083 | 9784544 | 9785001 | 9785457 | 9785913 |
| 952                 | 9788650 | 9789106 | 9789562 | 9790017 | 9790473 |
| 953                 | 9793207 | 9793662 | 9794118 | 9794573 | 9795028 |
| 954                 | 9797759 | 9798214 | 9798669 | 9799124 | 9799579 |
| 955                 | 9802307 | 9802761 | 9803216 | 9803670 | 9804125 |
| 956                 | 9806850 | 9807304 | 9807758 | 9808212 | 9808666 |
| 957                 | 9811388 | 9811841 | 9812295 | 9812748 | 9813202 |
| 958                 | 9815921 | 9816374 | 9816827 | 9817280 | 9817733 |
| 959                 | 9820450 | 9820902 | 9821355 | 9821807 | 9822260 |
| 960                 | 9824974 | 9825426 | 9825878 | 9826330 | 9826782 |
| 961                 | 9829493 | 9830945 | 9830396 | 9830848 | 9831299 |
| 962                 | 9834007 | 9834459 | 9834910 | 9835361 | 9835812 |
| 963                 | 9838517 | 9838968 | 9839419 | 9839869 | 9840320 |
| 964                 | 9843022 | 9843473 | 9843923 | 9844373 | 9844823 |
| 965                 | 9847523 | 9847973 | 9848422 | 9848872 | 9849322 |
| 966                 | 9852019 | 9852468 | 9852917 | 9853366 | 9853816 |
| 967                 | 9856510 | 9856959 | 9857407 | 9857856 | 9858305 |
| 968                 | 9860996 | 9861445 | 9861893 | 9862341 | 9862790 |
| 969                 | 9865478 | 9865926 | 9866374 | 9866822 | 9867270 |
| 970                 | 9869955 | 9870403 | 9870850 | 9871298 | 9871745 |

*Artificial Numbers: Or,*

| Natural<br>Numbers. | 0       | 1       | 2       | 3       | 4       |
|---------------------|---------|---------|---------|---------|---------|
| 971                 | 9872192 | 9872640 | 9873087 | 9873534 | 9873981 |
| 972                 | 9876663 | 9877109 | 9877556 | 9878003 | 9878449 |
| 973                 | 9881128 | 9881575 | 9882021 | 9882467 | 9882913 |
| 974                 | 9885590 | 9886035 | 9886481 | 9886927 | 9887373 |
| 975                 | 9890046 | 9890492 | 9890937 | 9891382 | 9891828 |
| 976                 | 9894498 | 9894943 | 9895388 | 9895833 | 9896278 |
| 977                 | 9898946 | 9899390 | 9899835 | 9900279 | 9900723 |
| 978                 | 9903389 | 9903833 | 9904277 | 9904721 | 9905164 |
| 979                 | 9907827 | 9908270 | 9908714 | 9909158 | 9909601 |
| 980                 | 9912261 | 9912704 | 9913147 | 9913590 | 9914033 |
| 981                 | 9916690 | 9917133 | 9917575 | 9918018 | 9918461 |
| 982                 | 9921115 | 9921557 | 9921999 | 9922441 | 9922884 |
| 983                 | 9925535 | 9925977 | 9926419 | 9926860 | 9927302 |
| 984                 | 9929951 | 9930392 | 9930834 | 9931275 | 9931716 |
| 985                 | 9934362 | 9934803 | 9935244 | 9935685 | 9936126 |
| 986                 | 9938769 | 9939210 | 9939650 | 9940090 | 9940531 |
| 987                 | 9943172 | 9943612 | 9944051 | 9944491 | 9944931 |
| 988                 | 9947569 | 9948009 | 9948448 | 9948888 | 9949327 |
| 989                 | 9951963 | 9952402 | 9952841 | 9953280 | 9953719 |
| 990                 | 9956352 | 9956791 | 9957229 | 9957668 | 9958106 |
| 991                 | 9960737 | 9961175 | 9961613 | 9962051 | 9962489 |
| 992                 | 9965117 | 9965554 | 9965992 | 9966430 | 9966868 |
| 993                 | 9969492 | 9969930 | 9970367 | 9970804 | 9971242 |
| 994                 | 9973864 | 9974301 | 9974738 | 9975174 | 9975611 |
| 995                 | 9978231 | 9978667 | 9979104 | 9979540 | 9979976 |
| 996                 | 9982593 | 9983029 | 9983465 | 9983901 | 9984337 |
| 997                 | 9986952 | 9987387 | 9987823 | 9988258 | 9988694 |
| 998                 | 9991305 | 9991740 | 9992176 | 9992611 | 9993046 |
| 999                 | 9995655 | 9996090 | 9996524 | 9996959 | 9997393 |



*Logarithms (to 10000.)*

| Natural<br>Numbers. | 5       | 6       | 7       | 8       | 9       |
|---------------------|---------|---------|---------|---------|---------|
| 971                 | 9874428 | 9874875 | 9875322 | 9875769 | 9876216 |
| 972                 | 9878896 | 9879343 | 9879789 | 9880236 | 9880682 |
| 973                 | 9883360 | 9883806 | 9884252 | 9884698 | 9885144 |
| 974                 | 9887818 | 9888264 | 9888710 | 9889155 | 9889601 |
| 975                 | 9892273 | 9892718 | 9893163 | 9893608 | 9894050 |
| 976                 | 9896722 | 9897167 | 9897612 | 9898056 | 9898501 |
| 977                 | 9901168 | 9901612 | 9902056 | 9902500 | 9902944 |
| 978                 | 9905608 | 9906052 | 9906496 | 9906940 | 9907383 |
| 979                 | 9910044 | 9910488 | 9910931 | 9911374 | 9911818 |
| 980                 | 9914476 | 9914919 | 9915362 | 9915805 | 9916247 |
| 981                 | 9918903 | 9919345 | 9919788 | 9920230 | 9920673 |
| 982                 | 9923326 | 9923768 | 9924210 | 9924651 | 9925093 |
| 983                 | 9927744 | 9928185 | 9928627 | 9929068 | 9929510 |
| 984                 | 9932157 | 9932598 | 9933039 | 9933480 | 9933921 |
| 985                 | 9936566 | 9937007 | 9937448 | 9937888 | 9938329 |
| 986                 | 9940971 | 9941411 | 9941851 | 9942291 | 9942731 |
| 987                 | 9945371 | 9945811 | 9946251 | 9946690 | 9947130 |
| 988                 | 9949767 | 9950206 | 9950645 | 9951085 | 9951524 |
| 989                 | 9954158 | 9954597 | 9955036 | 9955474 | 9955913 |
| 990                 | 9958545 | 9958983 | 9959422 | 9959860 | 9960298 |
| 991                 | 9962927 | 9963365 | 9963803 | 9964241 | 9964679 |
| 992                 | 9967305 | 9967743 | 9968180 | 9968618 | 9969055 |
| 993                 | 9971679 | 9972116 | 9972553 | 9972990 | 9973427 |
| 994                 | 9976048 | 9976485 | 9976921 | 9977358 | 9977794 |
| 995                 | 9980413 | 9980849 | 9981285 | 9981721 | 9982157 |
| 996                 | 9984773 | 9985209 | 9985645 | 9986080 | 9986516 |
| 997                 | 9989129 | 9989564 | 9990000 | 9990435 | 9990870 |
| 998                 | 9993481 | 9993916 | 9994350 | 9994785 | 9995220 |
| 999                 | 9997828 | 9998262 | 9998697 | 9999131 | 9999566 |

10000 it's Log.=4.0000000

*The End of the Table of Logarithms.*

(continued from page 2)

| NAME                | DATE | TIME  | PLACE    | REMARKS  |
|---------------------|------|-------|----------|----------|
| JOHN A. BROWN       | 1965 | 10:30 | NEW YORK | ARRIVED  |
| MARY K. SMITH       | 1965 | 11:00 | NEW YORK | DEPARTED |
| ROBERT L. JONES     | 1965 | 11:30 | NEW YORK | ARRIVED  |
| ELIZABETH C. WHITE  | 1965 | 12:00 | NEW YORK | DEPARTED |
| WILLIAM D. BLACK    | 1965 | 12:30 | NEW YORK | ARRIVED  |
| JANE E. GREEN       | 1965 | 13:00 | NEW YORK | DEPARTED |
| CHARLES F. HARRIS   | 1965 | 13:30 | NEW YORK | ARRIVED  |
| SARAH G. KELLER     | 1965 | 14:00 | NEW YORK | DEPARTED |
| EDWARD H. LYNN      | 1965 | 14:30 | NEW YORK | ARRIVED  |
| MARGARET I. MORGAN  | 1965 | 15:00 | NEW YORK | DEPARTED |
| FRANK J. NICHOLS    | 1965 | 15:30 | NEW YORK | ARRIVED  |
| HELEN K. OLSON      | 1965 | 16:00 | NEW YORK | DEPARTED |
| ALFRED L. PERKINS   | 1965 | 16:30 | NEW YORK | ARRIVED  |
| BEATRICE M. ROSS    | 1965 | 17:00 | NEW YORK | DEPARTED |
| CLAUDE N. STEVENSON | 1965 | 17:30 | NEW YORK | ARRIVED  |
| IRVING O. THOMPSON  | 1965 | 18:00 | NEW YORK | DEPARTED |
| JOSEPH P. WATSON    | 1965 | 18:30 | NEW YORK | ARRIVED  |
| LOUISE Q. YOUNG     | 1965 | 19:00 | NEW YORK | DEPARTED |
| WALTER R. ZIMMERMAN | 1965 | 19:30 | NEW YORK | ARRIVED  |

3 JY 65

1000 H. L. G. 1000

The End of the Table of Logarithms

A  
**T A B L E**  
 OF THE  
**Proportional Parts**  
 OF THE  
**DIFFERENCES**  
 OF  
**LOGARITHMS.**



By which the foregoing TABLE is made to extend to  
 the Logarithm of 100000, &c.

| Differences<br>of<br>Logarithms. |    | <i>Tenth Parts of those Differences.</i><br>(VIZ.) |     |     |     |     |     |     |     |   |
|----------------------------------|----|----------------------------------------------------|-----|-----|-----|-----|-----|-----|-----|---|
|                                  |    | 1                                                  | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9 |
| 435                              | 44 | 87                                                 | 130 | 174 | 218 | 261 | 305 | 348 | 392 |   |
| 436                              | 44 | 87                                                 | 131 | 174 | 218 | 262 | 305 | 349 | 392 |   |
| 438                              | 44 | 88                                                 | 131 | 175 | 219 | 263 | 307 | 350 | 394 |   |
| 440                              | 44 | 88                                                 | 132 | 176 | 220 | 264 | 308 | 352 | 396 |   |
| 442                              | 44 | 88                                                 | 133 | 177 | 221 | 265 | 309 | 354 | 398 |   |
| 444                              | 44 | 89                                                 | 133 | 178 | 222 | 266 | 311 | 355 | 400 |   |
| 446                              | 45 | 89                                                 | 134 | 178 | 223 | 268 | 312 | 357 | 401 |   |
| 448                              | 45 | 90                                                 | 134 | 179 | 224 | 269 | 314 | 358 | 403 |   |
| 450                              | 45 | 90                                                 | 135 | 180 | 225 | 270 | 315 | 360 | 405 |   |
| 452                              | 45 | 90                                                 | 136 | 181 | 226 | 271 | 316 | 362 | 407 |   |
| 454                              | 45 | 91                                                 | 136 | 182 | 227 | 272 | 318 | 363 | 409 |   |
| 456                              | 46 | 91                                                 | 137 | 182 | 228 | 274 | 319 | 365 | 410 |   |
| 458                              | 46 | 92                                                 | 137 | 183 | 229 | 275 | 321 | 366 | 412 |   |



# A TABLE of Proportional Parts.

| Tenth Parts of those Differences. |    |     |     |     |     |     |     |     |     |
|-----------------------------------|----|-----|-----|-----|-----|-----|-----|-----|-----|
| Diff. of<br>Logar.                | 1  | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   |
| 460                               | 46 | 92  | 138 | 184 | 230 | 276 | 322 | 368 | 414 |
| 462                               | 46 | 92  | 139 | 185 | 231 | 277 | 323 | 370 | 416 |
| 464                               | 46 | 93  | 139 | 186 | 232 | 278 | 325 | 371 | 418 |
| 466                               | 47 | 93  | 140 | 186 | 233 | 280 | 326 | 373 | 419 |
| 468                               | 47 | 94  | 140 | 187 | 234 | 281 | 328 | 374 | 421 |
| 470                               | 47 | 94  | 141 | 188 | 235 | 282 | 329 | 376 | 423 |
| 472                               | 47 | 94  | 142 | 189 | 236 | 283 | 330 | 378 | 425 |
| 474                               | 47 | 95  | 142 | 190 | 237 | 284 | 332 | 379 | 427 |
| 476                               | 48 | 95  | 143 | 190 | 238 | 286 | 333 | 381 | 428 |
| 478                               | 48 | 96  | 143 | 191 | 239 | 287 | 335 | 382 | 430 |
| 480                               | 48 | 96  | 144 | 192 | 240 | 288 | 336 | 384 | 432 |
| 482                               | 48 | 96  | 145 | 193 | 241 | 289 | 337 | 386 | 434 |
| 484                               | 48 | 97  | 145 | 194 | 242 | 290 | 339 | 387 | 436 |
| 486                               | 49 | 97  | 146 | 194 | 243 | 292 | 340 | 389 | 437 |
| 488                               | 49 | 98  | 146 | 195 | 244 | 293 | 342 | 390 | 439 |
| 490                               | 49 | 98  | 147 | 196 | 245 | 294 | 343 | 392 | 441 |
| 492                               | 49 | 98  | 148 | 197 | 246 | 295 | 344 | 394 | 443 |
| 494                               | 49 | 99  | 148 | 198 | 247 | 296 | 346 | 395 | 445 |
| 496                               | 50 | 99  | 149 | 198 | 248 | 298 | 347 | 397 | 446 |
| 498                               | 50 | 100 | 149 | 199 | 249 | 299 | 348 | 398 | 448 |
| 500                               | 50 | 100 | 150 | 200 | 250 | 300 | 350 | 400 | 450 |
| 502                               | 50 | 100 | 151 | 201 | 251 | 301 | 351 | 402 | 452 |
| 504                               | 50 | 101 | 151 | 202 | 252 | 302 | 353 | 403 | 454 |
| 506                               | 51 | 101 | 152 | 202 | 253 | 304 | 354 | 405 | 455 |
| 508                               | 51 | 102 | 152 | 203 | 254 | 305 | 356 | 406 | 457 |
| 510                               | 51 | 102 | 153 | 204 | 255 | 306 | 357 | 408 | 459 |
| 512                               | 51 | 102 | 154 | 205 | 256 | 307 | 358 | 410 | 461 |
| 514                               | 51 | 103 | 154 | 206 | 257 | 308 | 360 | 411 | 463 |
| 516                               | 52 | 103 | 155 | 206 | 258 | 310 | 361 | 413 | 464 |
| 518                               | 52 | 104 | 155 | 207 | 259 | 311 | 363 | 414 | 466 |
| 520                               | 52 | 104 | 156 | 208 | 260 | 312 | 364 | 416 | 468 |
| 522                               | 52 | 104 | 157 | 209 | 261 | 313 | 365 | 418 | 470 |
| 524                               | 52 | 105 | 157 | 210 | 262 | 314 | 367 | 419 | 472 |
| 526                               | 53 | 105 | 158 | 210 | 263 | 316 | 368 | 421 | 473 |
| 528                               | 53 | 106 | 158 | 211 | 264 | 317 | 370 | 422 | 475 |
| 530                               | 53 | 106 | 159 | 212 | 265 | 318 | 371 | 424 | 477 |
| 532                               | 53 | 106 | 160 | 213 | 266 | 319 | 372 | 426 | 479 |
| 534                               | 53 | 107 | 160 | 214 | 267 | 320 | 374 | 427 | 481 |
| 536                               | 54 | 107 | 161 | 214 | 268 | 322 | 375 | 429 | 482 |
| 538                               | 54 | 108 | 161 | 215 | 269 | 323 | 377 | 430 | 484 |

# *TABLE of Proportional Parts.*

| Tenth Parts of those Differences. |    |     |     |     |     |     |     |     |     |
|-----------------------------------|----|-----|-----|-----|-----|-----|-----|-----|-----|
| Diff. of<br>Logar.                | 1  | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   |
| 540                               | 54 | 108 | 162 | 216 | 270 | 324 | 378 | 432 | 486 |
| 542                               | 54 | 108 | 163 | 217 | 271 | 325 | 379 | 434 | 488 |
| 544                               | 54 | 109 | 163 | 218 | 272 | 326 | 381 | 435 | 490 |
| 546                               | 55 | 109 | 164 | 218 | 273 | 328 | 382 | 437 | 491 |
| 548                               | 55 | 110 | 164 | 219 | 274 | 329 | 384 | 438 | 493 |
| 550                               | 55 | 110 | 165 | 220 | 275 | 330 | 385 | 440 | 495 |
| 552                               | 55 | 110 | 166 | 221 | 276 | 331 | 386 | 442 | 497 |
| 554                               | 55 | 111 | 166 | 222 | 277 | 332 | 388 | 443 | 499 |
| 556                               | 56 | 111 | 167 | 222 | 278 | 334 | 389 | 445 | 500 |
| 558                               | 56 | 112 | 167 | 223 | 279 | 335 | 391 | 446 | 502 |
| 560                               | 56 | 112 | 168 | 224 | 280 | 336 | 392 | 448 | 504 |
| 562                               | 56 | 112 | 169 | 225 | 281 | 337 | 393 | 450 | 506 |
| 564                               | 56 | 113 | 169 | 226 | 282 | 338 | 395 | 451 | 508 |
| 566                               | 57 | 113 | 170 | 226 | 283 | 340 | 396 | 453 | 509 |
| 568                               | 57 | 114 | 170 | 227 | 284 | 341 | 398 | 454 | 511 |
| 570                               | 57 | 114 | 171 | 228 | 285 | 342 | 399 | 456 | 513 |
| 572                               | 57 | 114 | 172 | 229 | 286 | 343 | 400 | 458 | 515 |
| 574                               | 57 | 115 | 172 | 230 | 287 | 344 | 401 | 459 | 516 |
| 576                               | 58 | 115 | 173 | 230 | 288 | 346 | 403 | 461 | 518 |
| 578                               | 58 | 116 | 173 | 231 | 289 | 347 | 405 | 462 | 520 |
| 580                               | 58 | 116 | 174 | 232 | 290 | 348 | 406 | 464 | 522 |
| 582                               | 58 | 116 | 175 | 233 | 291 | 349 | 407 | 466 | 524 |
| 584                               | 58 | 117 | 175 | 234 | 292 | 350 | 409 | 467 | 526 |
| 586                               | 59 | 117 | 176 | 234 | 293 | 352 | 410 | 469 | 527 |
| 588                               | 59 | 118 | 176 | 235 | 294 | 353 | 412 | 470 | 529 |
| 590                               | 59 | 118 | 177 | 236 | 295 | 354 | 413 | 472 | 531 |
| 592                               | 59 | 118 | 178 | 237 | 295 | 355 | 414 | 474 | 533 |
| 594                               | 59 | 119 | 178 | 238 | 297 | 356 | 416 | 475 | 535 |
| 596                               | 60 | 119 | 179 | 238 | 298 | 358 | 417 | 477 | 536 |
| 598                               | 60 | 120 | 179 | 239 | 299 | 359 | 419 | 478 | 538 |
| 600                               | 60 | 120 | 180 | 240 | 300 | 360 | 420 | 480 | 540 |
| 602                               | 60 | 120 | 181 | 241 | 301 | 361 | 421 | 482 | 542 |
| 604                               | 60 | 121 | 181 | 242 | 302 | 362 | 423 | 483 | 544 |
| 606                               | 61 | 121 | 182 | 242 | 303 | 364 | 424 | 485 | 545 |
| 608                               | 61 | 122 | 182 | 243 | 304 | 365 | 426 | 486 | 547 |
| 610                               | 61 | 122 | 183 | 244 | 305 | 366 | 427 | 488 | 549 |
| 612                               | 61 | 122 | 184 | 245 | 306 | 367 | 428 | 490 | 551 |
| 614                               | 61 | 123 | 184 | 246 | 307 | 368 | 430 | 492 | 553 |
| 616                               | 62 | 123 | 185 | 246 | 308 | 370 | 431 | 493 | 554 |
| 618                               | 62 | 124 | 185 | 247 | 309 | 371 | 433 | 494 | 556 |

# A TABLE of Proportional Parts.

| Tenth Parts of those Differences. |    |     |     |     |     |     |     |     |     |
|-----------------------------------|----|-----|-----|-----|-----|-----|-----|-----|-----|
| Diff. of<br>Logar.                | 1  | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   |
| 620                               | 62 | 124 | 186 | 248 | 310 | 372 | 434 | 496 | 558 |
| 622                               | 62 | 124 | 187 | 249 | 311 | 373 | 435 | 498 | 560 |
| 624                               | 62 | 125 | 187 | 250 | 312 | 374 | 437 | 499 | 562 |
| 626                               | 63 | 125 | 188 | 250 | 313 | 376 | 438 | 501 | 563 |
| 628                               | 63 | 126 | 188 | 251 | 314 | 377 | 440 | 502 | 565 |
| 630                               | 63 | 126 | 189 | 252 | 315 | 378 | 441 | 504 | 567 |
| 632                               | 63 | 126 | 190 | 253 | 316 | 379 | 442 | 506 | 569 |
| 634                               | 63 | 127 | 190 | 254 | 317 | 380 | 444 | 507 | 571 |
| 636                               | 64 | 127 | 191 | 254 | 318 | 382 | 445 | 509 | 572 |
| 638                               | 64 | 128 | 191 | 255 | 319 | 383 | 447 | 510 | 574 |
| 640                               | 64 | 128 | 192 | 256 | 320 | 384 | 448 | 512 | 576 |
| 642                               | 64 | 128 | 193 | 257 | 321 | 385 | 449 | 514 | 578 |
| 644                               | 64 | 129 | 193 | 258 | 322 | 386 | 451 | 515 | 580 |
| 646                               | 65 | 129 | 194 | 258 | 323 | 388 | 452 | 517 | 581 |
| 648                               | 65 | 130 | 194 | 259 | 324 | 389 | 454 | 518 | 583 |
| 650                               | 65 | 130 | 195 | 260 | 325 | 390 | 455 | 520 | 585 |
| 652                               | 65 | 130 | 196 | 261 | 326 | 391 | 456 | 522 | 587 |
| 654                               | 65 | 131 | 196 | 262 | 327 | 392 | 458 | 523 | 589 |
| 656                               | 66 | 131 | 197 | 262 | 328 | 394 | 459 | 525 | 590 |
| 658                               | 66 | 132 | 197 | 263 | 329 | 395 | 461 | 526 | 592 |
| 660                               | 66 | 132 | 198 | 264 | 330 | 396 | 462 | 528 | 594 |
| 662                               | 66 | 132 | 199 | 265 | 331 | 397 | 463 | 530 | 596 |
| 664                               | 66 | 133 | 199 | 266 | 332 | 398 | 465 | 531 | 598 |
| 666                               | 67 | 133 | 200 | 266 | 333 | 400 | 466 | 533 | 599 |
| 668                               | 67 | 134 | 200 | 267 | 334 | 401 | 468 | 534 | 601 |
| 670                               | 67 | 134 | 201 | 268 | 335 | 402 | 469 | 536 | 603 |
| 672                               | 67 | 134 | 202 | 269 | 336 | 403 | 470 | 538 | 605 |
| 674                               | 67 | 135 | 202 | 270 | 337 | 404 | 472 | 539 | 607 |
| 676                               | 68 | 135 | 203 | 270 | 338 | 406 | 473 | 541 | 608 |
| 678                               | 68 | 136 | 203 | 271 | 339 | 407 | 475 | 542 | 610 |
| 680                               | 68 | 136 | 204 | 272 | 340 | 408 | 476 | 544 | 612 |
| 682                               | 68 | 136 | 205 | 273 | 341 | 409 | 477 | 546 | 614 |
| 684                               | 68 | 137 | 205 | 274 | 342 | 410 | 479 | 547 | 616 |
| 686                               | 69 | 137 | 206 | 274 | 343 | 412 | 480 | 549 | 617 |
| 688                               | 69 | 138 | 206 | 275 | 344 | 413 | 482 | 550 | 619 |
| 690                               | 69 | 138 | 207 | 276 | 345 | 414 | 483 | 552 | 621 |
| 692                               | 69 | 138 | 208 | 277 | 346 | 415 | 484 | 554 | 623 |
| 694                               | 69 | 139 | 208 | 278 | 347 | 416 | 486 | 555 | 625 |
| 696                               | 70 | 139 | 209 | 278 | 348 | 417 | 487 | 557 | 626 |
| 698                               | 70 | 140 | 209 | 279 | 349 | 419 | 489 | 558 | 628 |



# *A TABLE of Proportional Parts.*

| Tenth Parts of those Differences. |    |     |     |     |     |     |     |     |     |
|-----------------------------------|----|-----|-----|-----|-----|-----|-----|-----|-----|
| Diff. of<br>Logar.                | 1  | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   |
| 700                               | 70 | 140 | 210 | 280 | 350 | 420 | 490 | 560 | 630 |
| 702                               | 70 | 140 | 211 | 281 | 351 | 421 | 491 | 562 | 632 |
| 704                               | 70 | 141 | 211 | 282 | 352 | 422 | 492 | 563 | 634 |
| 706                               | 71 | 141 | 212 | 282 | 353 | 424 | 494 | 565 | 635 |
| 708                               | 71 | 142 | 212 | 283 | 354 | 425 | 496 | 566 | 637 |
| 710                               | 71 | 142 | 213 | 284 | 355 | 426 | 497 | 568 | 639 |
| 712                               | 71 | 142 | 214 | 285 | 356 | 427 | 498 | 570 | 641 |
| 714                               | 71 | 143 | 214 | 286 | 357 | 428 | 500 | 571 | 643 |
| 716                               | 72 | 143 | 215 | 286 | 358 | 430 | 501 | 573 | 644 |
| 718                               | 72 | 144 | 215 | 287 | 359 | 431 | 503 | 574 | 646 |
| 720                               | 72 | 144 | 216 | 288 | 360 | 432 | 504 | 576 | 648 |
| 722                               | 72 | 144 | 217 | 289 | 361 | 433 | 505 | 578 | 650 |
| 724                               | 72 | 145 | 217 | 290 | 362 | 434 | 507 | 579 | 652 |
| 726                               | 73 | 145 | 218 | 290 | 363 | 436 | 508 | 581 | 653 |
| 728                               | 73 | 146 | 218 | 291 | 364 | 437 | 510 | 582 | 655 |
| 730                               | 73 | 146 | 219 | 292 | 365 | 438 | 511 | 584 | 657 |
| 732                               | 73 | 146 | 220 | 293 | 366 | 439 | 512 | 586 | 659 |
| 734                               | 73 | 147 | 220 | 294 | 367 | 440 | 514 | 587 | 661 |
| 736                               | 74 | 147 | 221 | 294 | 368 | 442 | 515 | 589 | 662 |
| 738                               | 74 | 148 | 221 | 295 | 369 | 443 | 517 | 590 | 664 |
| 740                               | 74 | 148 | 222 | 296 | 370 | 444 | 518 | 592 | 666 |
| 742                               | 74 | 148 | 223 | 297 | 371 | 445 | 519 | 594 | 668 |
| 744                               | 74 | 149 | 223 | 298 | 372 | 446 | 521 | 595 | 670 |
| 746                               | 75 | 149 | 224 | 298 | 373 | 448 | 522 | 597 | 671 |
| 748                               | 75 | 150 | 224 | 299 | 374 | 449 | 524 | 598 | 673 |
| 750                               | 75 | 150 | 225 | 300 | 375 | 450 | 525 | 600 | 675 |
| 752                               | 75 | 150 | 226 | 301 | 376 | 451 | 526 | 602 | 677 |
| 754                               | 75 | 151 | 226 | 302 | 377 | 452 | 528 | 603 | 679 |
| 756                               | 76 | 151 | 227 | 302 | 378 | 454 | 529 | 605 | 680 |
| 758                               | 76 | 152 | 227 | 303 | 379 | 455 | 531 | 606 | 682 |
| 760                               | 76 | 152 | 228 | 304 | 380 | 456 | 532 | 608 | 684 |
| 762                               | 76 | 152 | 229 | 305 | 381 | 457 | 533 | 610 | 686 |
| 764                               | 76 | 153 | 229 | 306 | 382 | 458 | 535 | 611 | 688 |
| 766                               | 77 | 153 | 230 | 306 | 383 | 460 | 536 | 613 | 689 |
| 768                               | 77 | 154 | 230 | 307 | 384 | 461 | 538 | 614 | 691 |
| 770                               | 77 | 154 | 231 | 308 | 385 | 462 | 539 | 616 | 693 |
| 772                               | 77 | 154 | 232 | 309 | 386 | 463 | 540 | 618 | 695 |
| 774                               | 77 | 155 | 232 | 310 | 387 | 464 | 542 | 619 | 697 |
| 776                               | 78 | 155 | 233 | 310 | 388 | 466 | 543 | 621 | 699 |
| 778                               | 78 | 156 | 233 | 311 | 389 | 467 | 545 | 622 | 700 |

# *A TABLE of Proportional Parts.*

| Tenth Parts of those Differences. |    |     |     |     |     |     |     |     |     |
|-----------------------------------|----|-----|-----|-----|-----|-----|-----|-----|-----|
| Diff. of<br>Logar.                | 1  | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   |
| 780                               | 78 | 156 | 234 | 312 | 390 | 468 | 546 | 624 | 702 |
| 782                               | 78 | 156 | 235 | 313 | 391 | 469 | 547 | 626 | 704 |
| 784                               | 78 | 157 | 235 | 314 | 392 | 470 | 549 | 627 | 706 |
| 786                               | 79 | 157 | 236 | 314 | 393 | 472 | 550 | 629 | 707 |
| 788                               | 79 | 158 | 236 | 315 | 394 | 473 | 552 | 630 | 709 |
| 790                               | 79 | 158 | 237 | 316 | 395 | 474 | 553 | 632 | 711 |
| 792                               | 79 | 158 | 238 | 317 | 396 | 475 | 554 | 634 | 713 |
| 794                               | 79 | 159 | 238 | 318 | 397 | 477 | 556 | 635 | 715 |
| 796                               | 80 | 159 | 239 | 318 | 398 | 478 | 557 | 637 | 717 |
| 798                               | 80 | 160 | 239 | 319 | 399 | 479 | 559 | 638 | 718 |
| 800                               | 80 | 160 | 240 | 320 | 400 | 480 | 560 | 640 | 720 |
| 802                               | 80 | 160 | 241 | 321 | 401 | 481 | 561 | 642 | 722 |
| 804                               | 80 | 161 | 241 | 322 | 402 | 482 | 563 | 643 | 724 |
| 806                               | 81 | 161 | 242 | 322 | 403 | 484 | 564 | 645 | 725 |
| 808                               | 81 | 162 | 242 | 323 | 404 | 485 | 566 | 646 | 727 |
| 810                               | 81 | 162 | 243 | 324 | 405 | 486 | 567 | 648 | 729 |
| 812                               | 81 | 162 | 244 | 325 | 406 | 487 | 568 | 650 | 731 |
| 814                               | 81 | 163 | 244 | 326 | 407 | 488 | 570 | 651 | 733 |
| 816                               | 82 | 163 | 245 | 326 | 408 | 490 | 571 | 653 | 734 |
| 818                               | 82 | 164 | 245 | 327 | 409 | 491 | 572 | 654 | 736 |
| 820                               | 82 | 164 | 246 | 328 | 410 | 492 | 574 | 656 | 738 |
| 822                               | 82 | 164 | 247 | 329 | 411 | 493 | 575 | 658 | 740 |
| 824                               | 82 | 165 | 247 | 330 | 412 | 494 | 577 | 659 | 742 |
| 826                               | 83 | 165 | 248 | 330 | 413 | 496 | 578 | 661 | 743 |
| 828                               | 83 | 166 | 248 | 331 | 414 | 497 | 580 | 662 | 745 |
| 830                               | 83 | 166 | 249 | 332 | 415 | 498 | 581 | 664 | 747 |
| 832                               | 83 | 166 | 250 | 333 | 416 | 499 | 582 | 666 | 749 |
| 834                               | 83 | 167 | 250 | 334 | 417 | 500 | 584 | 667 | 751 |
| 836                               | 84 | 167 | 251 | 334 | 418 | 502 | 585 | 669 | 753 |
| 838                               | 84 | 168 | 251 | 335 | 419 | 503 | 587 | 670 | 754 |
| 840                               | 84 | 168 | 252 | 336 | 420 | 504 | 588 | 672 | 756 |
| 842                               | 84 | 168 | 253 | 337 | 421 | 505 | 589 | 674 | 758 |
| 844                               | 84 | 169 | 253 | 338 | 422 | 506 | 591 | 675 | 760 |
| 846                               | 85 | 169 | 254 | 338 | 423 | 508 | 592 | 677 | 761 |
| 848                               | 85 | 170 | 254 | 339 | 424 | 509 | 594 | 678 | 763 |
| 850                               | 85 | 170 | 255 | 340 | 425 | 510 | 595 | 680 | 765 |
| 852                               | 85 | 170 | 256 | 341 | 426 | 511 | 596 | 682 | 767 |
| 854                               | 85 | 171 | 256 | 342 | 427 | 512 | 598 | 683 | 769 |
| 856                               | 86 | 171 | 257 | 342 | 428 | 514 | 599 | 685 | 771 |
| 858                               | 86 | 172 | 257 | 343 | 429 | 515 | 601 | 686 | 773 |

# A TABLE of Proportional Parts.

| Tenth Parts of those Differences. |    |     |     |     |     |     |     |     |     |
|-----------------------------------|----|-----|-----|-----|-----|-----|-----|-----|-----|
| Diff. of<br>Logar.                | 1  | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   |
| 850                               | 86 | 172 | 258 | 344 | 430 | 516 | 602 | 688 | 774 |
| 862                               | 86 | 172 | 259 | 345 | 431 | 517 | 603 | 690 | 776 |
| 864                               | 86 | 173 | 259 | 346 | 432 | 518 | 605 | 691 | 778 |
| 866                               | 87 | 173 | 260 | 346 | 433 | 520 | 606 | 693 | 779 |
| 868                               | 87 | 174 | 260 | 347 | 434 | 521 | 608 | 694 | 781 |
| 870                               | 87 | 174 | 261 | 348 | 435 | 522 | 609 | 696 | 783 |
| 872                               | 87 | 174 | 262 | 349 | 436 | 523 | 611 | 698 | 785 |
| 874                               | 87 | 175 | 262 | 350 | 437 | 524 | 612 | 699 | 787 |
| 876                               | 88 | 175 | 263 | 350 | 438 | 526 | 613 | 701 | 789 |
| 878                               | 88 | 176 | 263 | 351 | 439 | 527 | 615 | 702 | 791 |
| 880                               | 88 | 176 | 264 | 352 | 440 | 528 | 616 | 704 | 793 |
| 882                               | 88 | 176 | 265 | 353 | 441 | 529 | 617 | 706 | 795 |
| 884                               | 88 | 177 | 265 | 354 | 442 | 530 | 619 | 707 | 796 |
| 886                               | 89 | 177 | 266 | 354 | 443 | 532 | 620 | 709 | 797 |
| 888                               | 89 | 178 | 266 | 355 | 444 | 533 | 622 | 710 | 799 |
| 890                               | 89 | 178 | 267 | 356 | 445 | 534 | 623 | 712 | 801 |
| 892                               | 89 | 178 | 268 | 357 | 446 | 535 | 624 | 714 | 803 |
| 894                               | 89 | 179 | 268 | 358 | 447 | 536 | 626 | 715 | 805 |
| 896                               | 90 | 179 | 269 | 358 | 448 | 538 | 627 | 717 | 806 |
| 898                               | 90 | 180 | 269 | 359 | 449 | 539 | 629 | 718 | 808 |
| 900                               | 90 | 180 | 270 | 360 | 450 | 540 | 630 | 720 | 810 |
| 903                               | 90 | 181 | 271 | 361 | 451 | 542 | 632 | 722 | 813 |
| 906                               | 91 | 181 | 272 | 363 | 453 | 544 | 634 | 725 | 815 |
| 909                               | 91 | 182 | 273 | 364 | 454 | 545 | 636 | 727 | 818 |
| 912                               | 91 | 182 | 274 | 365 | 456 | 547 | 638 | 730 | 821 |
| 915                               | 91 | 183 | 274 | 366 | 457 | 549 | 640 | 732 | 823 |
| 918                               | 92 | 184 | 275 | 367 | 459 | 551 | 643 | 734 | 826 |
| 921                               | 92 | 184 | 276 | 368 | 460 | 553 | 645 | 737 | 829 |
| 924                               | 92 | 185 | 277 | 370 | 462 | 554 | 647 | 739 | 832 |
| 927                               | 93 | 185 | 278 | 371 | 463 | 556 | 649 | 742 | 834 |
| 930                               | 93 | 186 | 279 | 372 | 465 | 558 | 651 | 744 | 837 |
| 933                               | 93 | 187 | 280 | 373 | 466 | 560 | 653 | 746 | 839 |
| 936                               | 94 | 187 | 281 | 374 | 468 | 562 | 655 | 749 | 842 |
| 939                               | 94 | 188 | 282 | 376 | 469 | 563 | 657 | 751 | 845 |
| 942                               | 94 | 188 | 283 | 377 | 471 | 565 | 659 | 754 | 848 |
| 945                               | 94 | 189 | 283 | 378 | 472 | 567 | 661 | 756 | 850 |
| 948                               | 95 | 190 | 284 | 379 | 474 | 569 | 664 | 758 | 853 |
| 951                               | 95 | 190 | 285 | 380 | 475 | 571 | 666 | 761 | 856 |
| 954                               | 95 | 191 | 286 | 382 | 477 | 572 | 668 | 763 | 859 |
| 957                               | 96 | 191 | 287 | 383 | 478 | 574 | 670 | 766 | 861 |



# A TABLE of Proportional Parts.

| Diff. of<br>Logar. | Tenth Parts of those Differences. |     |     |     |     |     |     |     |     |
|--------------------|-----------------------------------|-----|-----|-----|-----|-----|-----|-----|-----|
|                    | 1                                 | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   |
| 960                | 96                                | 192 | 288 | 384 | 480 | 576 | 672 | 768 | 864 |
| 963                | 96                                | 193 | 289 | 385 | 481 | 578 | 674 | 770 | 867 |
| 966                | 97                                | 193 | 290 | 386 | 483 | 580 | 676 | 773 | 869 |
| 969                | 97                                | 194 | 291 | 388 | 484 | 581 | 678 | 775 | 872 |
| 972                | 97                                | 194 | 292 | 389 | 486 | 583 | 680 | 778 | 875 |
| 975                | 98                                | 195 | 292 | 390 | 487 | 585 | 682 | 780 | 878 |
| 978                | 98                                | 196 | 293 | 391 | 489 | 587 | 685 | 782 | 880 |
| 981                | 98                                | 196 | 294 | 392 | 490 | 589 | 687 | 785 | 883 |
| 984                | 98                                | 197 | 295 | 394 | 492 | 590 | 689 | 787 | 886 |
| 987                | 99                                | 197 | 296 | 395 | 493 | 592 | 691 | 790 | 888 |
| 990                | 99                                | 198 | 297 | 396 | 495 | 594 | 693 | 792 | 891 |
| 993                | 99                                | 199 | 298 | 397 | 496 | 596 | 695 | 794 | 894 |
| 996                | 100                               | 199 | 299 | 398 | 498 | 598 | 697 | 797 | 897 |
| 999                | 100                               | 200 | 300 | 400 | 499 | 599 | 699 | 799 | 899 |
| 1005               | 101                               | 201 | 301 | 402 | 502 | 603 | 703 | 804 | 904 |
| 1009               | 101                               | 202 | 303 | 404 | 504 | 605 | 706 | 807 | 908 |
| 1013               | 101                               | 203 | 304 | 405 | 506 | 608 | 709 | 810 | 912 |
| 1017               | 102                               | 203 | 305 | 407 | 508 | 610 | 712 | 814 | 915 |
| 1021               | 102                               | 204 | 306 | 408 | 510 | 613 | 715 | 817 | 919 |
| 1025               | 103                               | 205 | 307 | 410 | 512 | 615 | 717 | 820 | 923 |
| 1029               | 103                               | 206 | 309 | 412 | 514 | 617 | 720 | 823 | 926 |
| 1033               | 103                               | 207 | 310 | 413 | 516 | 620 | 723 | 826 | 930 |
| 1037               | 104                               | 207 | 311 | 415 | 518 | 622 | 726 | 830 | 933 |
| 1041               | 104                               | 208 | 312 | 416 | 520 | 625 | 729 | 833 | 937 |
| 1045               | 105                               | 209 | 313 | 418 | 522 | 627 | 731 | 836 | 941 |
| 1049               | 105                               | 210 | 315 | 420 | 524 | 629 | 734 | 839 | 944 |
| 1053               | 105                               | 211 | 316 | 421 | 526 | 632 | 737 | 842 | 948 |
| 1057               | 106                               | 211 | 317 | 423 | 528 | 634 | 740 | 846 | 951 |
| 1061               | 106                               | 212 | 318 | 424 | 530 | 637 | 743 | 849 | 955 |
| 1065               | 107                               | 213 | 319 | 426 | 532 | 639 | 745 | 852 | 959 |
| 1069               | 107                               | 214 | 321 | 428 | 534 | 641 | 748 | 855 | 962 |
| 1073               | 107                               | 215 | 322 | 429 | 536 | 644 | 751 | 858 | 966 |
| 1077               | 108                               | 215 | 323 | 431 | 538 | 646 | 754 | 862 | 969 |
| 1081               | 108                               | 216 | 324 | 432 | 540 | 649 | 757 | 865 | 973 |
| 1085               | 109                               | 217 | 325 | 434 | 542 | 651 | 759 | 868 | 977 |
| 1089               | 109                               | 218 | 327 | 436 | 544 | 653 | 762 | 871 | 980 |
| 1093               | 110                               | 219 | 328 | 437 | 546 | 656 | 765 | 874 | 984 |
| 1097               | 110                               | 219 | 329 | 439 | 548 | 658 | 768 | 878 | 987 |
| 1101               | 110                               | 220 | 330 | 440 | 550 | 661 | 771 | 881 | 991 |
| 1105               | 111                               | 221 | 331 | 442 | 552 | 663 | 773 | 884 | 994 |

# A TABLE of Proportional Parts.

| Tenth Parts of those Differences. |     |     |     |     |     |     |     |      |      |
|-----------------------------------|-----|-----|-----|-----|-----|-----|-----|------|------|
| Diff. of<br>Logar.                | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8    | 9    |
| 1109                              | 111 | 222 | 333 | 444 | 554 | 665 | 776 | 887  | 998  |
| 1113                              | 111 | 223 | 334 | 445 | 556 | 668 | 779 | 890  | 1002 |
| 1117                              | 112 | 223 | 335 | 447 | 558 | 670 | 782 | 894  | 1005 |
| 1121                              | 112 | 224 | 336 | 448 | 560 | 673 | 785 | 897  | 1009 |
| 1125                              | 113 | 225 | 337 | 450 | 562 | 675 | 787 | 900  | 1013 |
| 1129                              | 113 | 226 | 339 | 452 | 564 | 677 | 790 | 903  | 1016 |
| 1133                              | 113 | 227 | 340 | 453 | 566 | 680 | 793 | 906  | 1020 |
| 1137                              | 114 | 227 | 341 | 455 | 568 | 682 | 796 | 910  | 1023 |
| 1141                              | 114 | 228 | 342 | 456 | 570 | 685 | 799 | 913  | 1027 |
| 1145                              | 115 | 229 | 343 | 458 | 572 | 687 | 801 | 916  | 1031 |
| 1149                              | 115 | 230 | 345 | 460 | 574 | 689 | 804 | 919  | 1034 |
| 1153                              | 115 | 231 | 346 | 461 | 576 | 692 | 807 | 922  | 1038 |
| 1157                              | 116 | 231 | 347 | 463 | 578 | 694 | 810 | 926  | 1041 |
| 1161                              | 116 | 232 | 348 | 464 | 580 | 697 | 813 | 929  | 1045 |
| 1165                              | 117 | 233 | 349 | 466 | 582 | 699 | 815 | 932  | 1049 |
| 1169                              | 117 | 234 | 351 | 468 | 584 | 701 | 818 | 935  | 1052 |
| 1173                              | 117 | 235 | 352 | 469 | 586 | 704 | 821 | 938  | 1056 |
| 1177                              | 118 | 235 | 353 | 471 | 588 | 706 | 824 | 942  | 1059 |
| 1181                              | 118 | 236 | 354 | 472 | 590 | 709 | 827 | 945  | 1063 |
| 1185                              | 118 | 237 | 355 | 474 | 592 | 711 | 829 | 948  | 1067 |
| 1189                              | 119 | 238 | 357 | 476 | 594 | 713 | 832 | 951  | 1070 |
| 1193                              | 119 | 239 | 358 | 477 | 596 | 716 | 835 | 954  | 1074 |
| 1197                              | 120 | 239 | 359 | 479 | 598 | 718 | 838 | 958  | 1077 |
| 1201                              | 120 | 240 | 360 | 480 | 600 | 721 | 841 | 961  | 1081 |
| 1205                              | 120 | 241 | 361 | 482 | 602 | 723 | 843 | 964  | 1084 |
| 1209                              | 121 | 242 | 363 | 484 | 604 | 725 | 846 | 967  | 1088 |
| 1213                              | 121 | 243 | 364 | 485 | 606 | 728 | 849 | 970  | 1092 |
| 1217                              | 122 | 243 | 365 | 487 | 608 | 730 | 852 | 974  | 1095 |
| 1221                              | 122 | 244 | 366 | 488 | 610 | 732 | 855 | 977  | 1099 |
| 1225                              | 122 | 245 | 367 | 490 | 612 | 735 | 857 | 980  | 1102 |
| 1229                              | 123 | 246 | 369 | 492 | 614 | 737 | 860 | 983  | 1106 |
| 1233                              | 123 | 247 | 370 | 493 | 616 | 740 | 863 | 986  | 1110 |
| 1237                              | 124 | 247 | 371 | 495 | 618 | 742 | 866 | 990  | 1113 |
| 1241                              | 124 | 248 | 372 | 496 | 620 | 745 | 869 | 993  | 1117 |
| 1245                              | 124 | 249 | 373 | 498 | 622 | 747 | 871 | 996  | 1120 |
| 1249                              | 125 | 250 | 375 | 500 | 624 | 749 | 874 | 999  | 1124 |
| 1253                              | 125 | 251 | 376 | 501 | 626 | 752 | 877 | 1002 | 1128 |
| 1257                              | 126 | 251 | 377 | 503 | 628 | 754 | 880 | 1006 | 1131 |
| 1260                              | 126 | 252 | 378 | 504 | 630 | 756 | 882 | 1008 | 1134 |
| 1263                              | 126 | 253 | 379 | 505 | 631 | 758 | 884 | 1010 | 1137 |

# A TABLE of Proportional Parts.

| Tenth Parts of those Differences. |     |     |     |     |     |     |     |      |      |
|-----------------------------------|-----|-----|-----|-----|-----|-----|-----|------|------|
| Diff. of<br>Logar.                | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8    | 9    |
| 1267                              | 127 | 253 | 380 | 507 | 633 | 760 | 887 | 1014 | 1140 |
| 1271                              | 127 | 254 | 381 | 508 | 635 | 763 | 890 | 1017 | 1144 |
| 1275                              | 127 | 255 | 382 | 510 | 637 | 765 | 892 | 1020 | 1147 |
| 1279                              | 128 | 256 | 384 | 512 | 639 | 767 | 895 | 1023 | 1151 |
| 1283                              | 128 | 257 | 385 | 513 | 641 | 770 | 898 | 1026 | 1155 |
| 1287                              | 129 | 257 | 386 | 515 | 643 | 772 | 901 | 1030 | 1158 |
| 1291                              | 129 | 258 | 387 | 516 | 645 | 775 | 904 | 1033 | 1162 |
| 1295                              | 129 | 259 | 388 | 518 | 647 | 777 | 906 | 1036 | 1165 |
| 1299                              | 130 | 260 | 389 | 520 | 649 | 779 | 909 | 1039 | 1169 |
| 1303                              | 130 | 261 | 391 | 521 | 651 | 782 | 912 | 1042 | 1173 |
| 1307                              | 131 | 261 | 392 | 523 | 653 | 784 | 915 | 1046 | 1176 |
| 1311                              | 131 | 262 | 393 | 524 | 655 | 787 | 918 | 1049 | 1180 |
| 1315                              | 131 | 263 | 394 | 526 | 657 | 789 | 920 | 1052 | 1183 |
| 1319                              | 132 | 264 | 396 | 528 | 659 | 791 | 923 | 1055 | 1187 |
| 1323                              | 132 | 265 | 397 | 529 | 661 | 794 | 926 | 1058 | 1191 |
| 1327                              | 133 | 265 | 398 | 531 | 663 | 796 | 929 | 1062 | 1194 |
| 1331                              | 133 | 266 | 399 | 532 | 665 | 799 | 932 | 1065 | 1198 |
| 1335                              | 133 | 267 | 400 | 534 | 667 | 801 | 934 | 1068 | 1201 |
| 1339                              | 134 | 268 | 402 | 536 | 669 | 803 | 937 | 1071 | 1205 |
| 1343                              | 134 | 269 | 403 | 537 | 671 | 806 | 940 | 1074 | 1209 |
| 1347                              | 135 | 269 | 404 | 539 | 673 | 808 | 943 | 1078 | 1212 |
| 1351                              | 135 | 270 | 405 | 540 | 675 | 811 | 946 | 1081 | 1216 |
| 1355                              | 135 | 271 | 406 | 542 | 677 | 813 | 948 | 1084 | 1219 |
| 1359                              | 136 | 272 | 408 | 544 | 679 | 815 | 951 | 1087 | 1223 |
| 1363                              | 136 | 273 | 409 | 545 | 681 | 818 | 954 | 1090 | 1227 |
| 1367                              | 137 | 273 | 410 | 547 | 683 | 820 | 957 | 1094 | 1230 |
| 1371                              | 137 | 274 | 411 | 548 | 685 | 823 | 960 | 1097 | 1234 |
| 1375                              | 137 | 275 | 412 | 550 | 687 | 825 | 962 | 1100 | 1237 |
| 1379                              | 138 | 276 | 414 | 552 | 689 | 827 | 965 | 1103 | 1241 |
| 1383                              | 138 | 277 | 415 | 553 | 691 | 830 | 968 | 1106 | 1245 |
| 1387                              | 139 | 277 | 416 | 555 | 693 | 832 | 971 | 1110 | 1248 |
| 1391                              | 139 | 278 | 417 | 556 | 695 | 835 | 974 | 1113 | 1252 |
| 1395                              | 140 | 279 | 418 | 558 | 697 | 837 | 976 | 1116 | 1255 |
| 1399                              | 140 | 280 | 420 | 559 | 699 | 839 | 979 | 1119 | 1259 |
| 1403                              | 140 | 281 | 421 | 561 | 701 | 842 | 982 | 1122 | 1263 |
| 1407                              | 141 | 281 | 422 | 563 | 703 | 844 | 985 | 1126 | 1266 |
| 1411                              | 141 | 282 | 423 | 564 | 705 | 847 | 988 | 1129 | 1270 |
| 1415                              | 141 | 283 | 424 | 566 | 707 | 849 | 990 | 1132 | 1273 |
| 1420                              | 142 | 284 | 426 | 568 | 710 | 852 | 994 | 1136 | 1278 |
| 1425                              | 142 | 285 | 427 | 570 | 712 | 855 | 997 | 1140 | 1282 |



# A TABLE of Proportional Parts.

| Tenth Parts of those Differences. |     |     |     |     |     |     |      |      |      |
|-----------------------------------|-----|-----|-----|-----|-----|-----|------|------|------|
| Diff <sup>o</sup><br>Logar.       | 1   | 2   | 3   | 4   | 5   | 6   | 7    | 8    | 9    |
| 1430                              | 143 | 286 | 429 | 572 | 715 | 858 | 1001 | 1144 | 1287 |
| 1435                              | 143 | 287 | 430 | 574 | 717 | 861 | 1004 | 1148 | 1291 |
| 1440                              | 144 | 288 | 432 | 576 | 720 | 864 | 1008 | 1152 | 1296 |
| 1445                              | 144 | 289 | 433 | 578 | 722 | 867 | 1011 | 1156 | 1300 |
| 1450                              | 145 | 290 | 435 | 580 | 725 | 870 | 1015 | 1160 | 1305 |
| 1455                              | 145 | 291 | 436 | 582 | 727 | 873 | 1018 | 1164 | 1309 |
| 1460                              | 146 | 292 | 438 | 584 | 730 | 876 | 1022 | 1168 | 1314 |
| 1465                              | 146 | 293 | 439 | 586 | 732 | 879 | 1025 | 1172 | 1318 |
| 1470                              | 147 | 294 | 441 | 588 | 735 | 882 | 1029 | 1176 | 1323 |
| 1475                              | 147 | 295 | 442 | 590 | 737 | 885 | 1032 | 1180 | 1327 |
| 1480                              | 148 | 296 | 444 | 592 | 740 | 888 | 1036 | 1184 | 1332 |
| 1485                              | 148 | 297 | 445 | 594 | 742 | 891 | 1039 | 1188 | 1336 |
| 1490                              | 149 | 298 | 447 | 596 | 745 | 894 | 1043 | 1192 | 1341 |
| 1495                              | 149 | 299 | 448 | 598 | 747 | 897 | 1046 | 1196 | 1345 |
| 1500                              | 150 | 300 | 450 | 600 | 750 | 900 | 1050 | 1200 | 1350 |
| 1505                              | 150 | 301 | 451 | 602 | 752 | 903 | 1053 | 1204 | 1354 |
| 1510                              | 151 | 302 | 453 | 604 | 755 | 906 | 1057 | 1208 | 1359 |
| 1515                              | 151 | 303 | 454 | 606 | 757 | 909 | 1060 | 1212 | 1363 |
| 1520                              | 152 | 304 | 456 | 608 | 760 | 912 | 1064 | 1216 | 1368 |
| 1525                              | 152 | 305 | 457 | 610 | 762 | 915 | 1067 | 1220 | 1372 |
| 1530                              | 153 | 306 | 459 | 612 | 765 | 918 | 1071 | 1224 | 1377 |
| 1535                              | 153 | 307 | 460 | 614 | 767 | 921 | 1074 | 1228 | 1381 |
| 1540                              | 154 | 308 | 462 | 616 | 770 | 924 | 1078 | 1232 | 1386 |
| 1545                              | 154 | 309 | 463 | 618 | 772 | 927 | 1081 | 1236 | 1390 |
| 1550                              | 155 | 310 | 465 | 620 | 775 | 930 | 1085 | 1240 | 1395 |
| 1555                              | 155 | 311 | 466 | 622 | 777 | 933 | 1088 | 1244 | 1399 |
| 1560                              | 156 | 312 | 468 | 624 | 780 | 936 | 1092 | 1248 | 1404 |
| 1565                              | 156 | 313 | 469 | 626 | 782 | 939 | 1095 | 1252 | 1408 |
| 1570                              | 157 | 314 | 471 | 628 | 785 | 942 | 1099 | 1256 | 1413 |
| 1575                              | 157 | 315 | 472 | 630 | 787 | 945 | 1102 | 1260 | 1417 |
| 1580                              | 158 | 316 | 474 | 632 | 790 | 948 | 1106 | 1264 | 1422 |
| 1585                              | 158 | 317 | 475 | 634 | 792 | 951 | 1109 | 1268 | 1426 |
| 1590                              | 159 | 318 | 477 | 636 | 795 | 954 | 1113 | 1272 | 1431 |
| 1595                              | 159 | 319 | 478 | 638 | 797 | 957 | 1116 | 1276 | 1435 |
| 1600                              | 160 | 320 | 480 | 640 | 800 | 960 | 1120 | 1280 | 1440 |
| 1605                              | 160 | 321 | 481 | 642 | 802 | 963 | 1123 | 1284 | 1444 |
| 1610                              | 161 | 322 | 483 | 644 | 805 | 966 | 1127 | 1288 | 1449 |
| 1615                              | 161 | 323 | 484 | 646 | 807 | 969 | 1130 | 1292 | 1453 |
| 1620                              | 162 | 324 | 486 | 648 | 810 | 972 | 1134 | 1296 | 1458 |
| 1625                              | 162 | 325 | 487 | 650 | 812 | 975 | 1137 | 1300 | 1462 |

# A TABLE of Proportional Parts.

| Tenth Parts of those Differences. |     |     |     |     |     |      |      |      |      |
|-----------------------------------|-----|-----|-----|-----|-----|------|------|------|------|
| Diff. of<br>Logar.                | 1   | 2   | 3   | 4   | 5   | 6    | 7    | 8    | 9    |
| 1630                              | 163 | 326 | 489 | 652 | 815 | 978  | 1141 | 1304 | 1467 |
| 1635                              | 163 | 327 | 490 | 654 | 817 | 981  | 1144 | 1308 | 1471 |
| 1640                              | 164 | 328 | 492 | 656 | 820 | 984  | 1148 | 1312 | 1476 |
| 1645                              | 164 | 329 | 493 | 658 | 822 | 987  | 1151 | 1316 | 1480 |
| 1650                              | 165 | 330 | 495 | 660 | 825 | 990  | 1155 | 1320 | 1485 |
| 1655                              | 165 | 331 | 496 | 662 | 827 | 993  | 1158 | 1324 | 1489 |
| 1660                              | 166 | 332 | 498 | 664 | 830 | 996  | 1162 | 1328 | 1494 |
| 1665                              | 167 | 333 | 499 | 666 | 832 | 999  | 1166 | 1332 | 1498 |
| 1670                              | 167 | 334 | 501 | 668 | 835 | 1002 | 1169 | 1336 | 1503 |
| 1675                              | 167 | 335 | 502 | 670 | 837 | 1005 | 1172 | 1340 | 1507 |
| 1680                              | 168 | 336 | 504 | 672 | 840 | 1008 | 1176 | 1344 | 1512 |
| 1685                              | 168 | 337 | 505 | 674 | 842 | 1011 | 1179 | 1348 | 1516 |
| 1690                              | 169 | 338 | 507 | 676 | 845 | 1014 | 1183 | 1352 | 1521 |
| 1695                              | 169 | 339 | 508 | 678 | 847 | 1017 | 1186 | 1356 | 1525 |
| 1700                              | 170 | 340 | 510 | 680 | 850 | 1020 | 1190 | 1360 | 1530 |
| 1705                              | 170 | 341 | 511 | 682 | 852 | 1023 | 1193 | 1364 | 1534 |
| 1710                              | 171 | 342 | 513 | 684 | 855 | 1026 | 1197 | 1368 | 1539 |
| 1715                              | 171 | 343 | 514 | 686 | 857 | 1029 | 1200 | 1372 | 1543 |
| 1720                              | 172 | 344 | 516 | 688 | 860 | 1032 | 1204 | 1376 | 1548 |
| 1725                              | 172 | 345 | 517 | 690 | 862 | 1035 | 1207 | 1380 | 1552 |
| 1730                              | 173 | 346 | 519 | 692 | 865 | 1038 | 1211 | 1384 | 1557 |
| 1735                              | 173 | 347 | 520 | 694 | 867 | 1041 | 1214 | 1388 | 1561 |
| 1740                              | 174 | 348 | 522 | 696 | 870 | 1044 | 1218 | 1392 | 1566 |
| 1745                              | 174 | 349 | 523 | 698 | 872 | 1047 | 1221 | 1396 | 1570 |
| 1750                              | 175 | 350 | 525 | 700 | 875 | 1050 | 1225 | 1400 | 1575 |
| 1755                              | 175 | 351 | 526 | 702 | 877 | 1053 | 1228 | 1404 | 1579 |
| 1760                              | 176 | 352 | 528 | 704 | 880 | 1056 | 1232 | 1408 | 1584 |
| 1765                              | 176 | 353 | 529 | 706 | 882 | 1059 | 1235 | 1412 | 1588 |
| 1770                              | 177 | 354 | 531 | 708 | 885 | 1062 | 1239 | 1416 | 1593 |
| 1775                              | 178 | 355 | 532 | 710 | 887 | 1065 | 1242 | 1420 | 1597 |
| 1780                              | 178 | 356 | 534 | 712 | 890 | 1068 | 1246 | 1424 | 1602 |
| 1785                              | 179 | 357 | 535 | 714 | 892 | 1071 | 1249 | 1428 | 1606 |
| 1790                              | 179 | 358 | 537 | 716 | 895 | 1074 | 1253 | 1432 | 1611 |
| 1795                              | 180 | 359 | 538 | 718 | 897 | 1077 | 1256 | 1436 | 1615 |
| 1800                              | 180 | 360 | 540 | 720 | 900 | 1080 | 1260 | 1440 | 1620 |
| 1805                              | 181 | 361 | 541 | 722 | 902 | 1083 | 1263 | 1444 | 1624 |
| 1810                              | 181 | 362 | 543 | 724 | 905 | 1086 | 1267 | 1448 | 1629 |
| 1815                              | 182 | 363 | 544 | 726 | 907 | 1089 | 1270 | 1452 | 1633 |
| 1820                              | 182 | 364 | 546 | 728 | 910 | 1092 | 1274 | 1456 | 1638 |
| 1825                              | 183 | 365 | 547 | 730 | 912 | 1095 | 1277 | 1460 | 1642 |

# A TABLE of Proportional Parts.

| Tenth Parts of those Differences. |     |     |     |     |      |      |      |      |      |
|-----------------------------------|-----|-----|-----|-----|------|------|------|------|------|
| Diff. of<br>Logar.                | 1   | 2   | 3   | 4   | 5    | 6    | 7    | 8    | 9    |
| 1830                              | 183 | 366 | 549 | 732 | 915  | 1098 | 1281 | 1464 | 1647 |
| 1835                              | 184 | 367 | 550 | 734 | 917  | 1101 | 1284 | 1468 | 1651 |
| 1840                              | 184 | 368 | 552 | 736 | 920  | 1104 | 1288 | 1472 | 1656 |
| 1845                              | 185 | 369 | 553 | 738 | 922  | 1107 | 1291 | 1476 | 1660 |
| 1850                              | 185 | 370 | 555 | 740 | 925  | 1110 | 1295 | 1480 | 1665 |
| 1855                              | 186 | 371 | 556 | 742 | 927  | 1113 | 1298 | 1484 | 1669 |
| 1860                              | 186 | 372 | 558 | 744 | 930  | 1116 | 1302 | 1488 | 1674 |
| 1865                              | 187 | 373 | 559 | 746 | 932  | 1119 | 1305 | 1492 | 1678 |
| 1870                              | 187 | 374 | 561 | 748 | 935  | 1122 | 1309 | 1496 | 1683 |
| 1875                              | 188 | 375 | 562 | 750 | 937  | 1125 | 1312 | 1500 | 1687 |
| 1880                              | 188 | 376 | 564 | 752 | 940  | 1128 | 1316 | 1504 | 1692 |
| 1885                              | 189 | 377 | 565 | 754 | 942  | 1131 | 1319 | 1508 | 1696 |
| 1890                              | 189 | 378 | 567 | 756 | 945  | 1134 | 1323 | 1512 | 1701 |
| 1895                              | 190 | 379 | 568 | 758 | 947  | 1137 | 1326 | 1516 | 1705 |
| 1900                              | 190 | 380 | 570 | 760 | 950  | 1140 | 1330 | 1520 | 1710 |
| 1905                              | 191 | 381 | 571 | 762 | 952  | 1143 | 1333 | 1524 | 1714 |
| 1910                              | 191 | 382 | 573 | 764 | 955  | 1146 | 1337 | 1528 | 1719 |
| 1915                              | 192 | 383 | 574 | 766 | 957  | 1149 | 1340 | 1532 | 1723 |
| 1920                              | 192 | 384 | 576 | 768 | 960  | 1152 | 1344 | 1536 | 1728 |
| 1925                              | 193 | 385 | 577 | 770 | 962  | 1155 | 1347 | 1540 | 1732 |
| 1930                              | 193 | 386 | 579 | 772 | 965  | 1158 | 1351 | 1544 | 1737 |
| 1935                              | 194 | 387 | 580 | 774 | 967  | 1161 | 1354 | 1548 | 1742 |
| 1940                              | 194 | 388 | 582 | 776 | 970  | 1164 | 1358 | 1552 | 1746 |
| 1945                              | 195 | 389 | 583 | 778 | 972  | 1167 | 1361 | 1556 | 1750 |
| 1950                              | 195 | 390 | 585 | 780 | 975  | 1170 | 1365 | 1560 | 1755 |
| 1955                              | 196 | 391 | 586 | 782 | 977  | 1173 | 1368 | 1564 | 1759 |
| 1960                              | 196 | 392 | 588 | 784 | 980  | 1176 | 1372 | 1568 | 1764 |
| 1965                              | 197 | 393 | 589 | 786 | 982  | 1179 | 1375 | 1572 | 1768 |
| 1970                              | 197 | 394 | 591 | 788 | 985  | 1182 | 1379 | 1576 | 1773 |
| 1975                              | 198 | 395 | 592 | 790 | 987  | 1185 | 1382 | 1580 | 1777 |
| 1980                              | 198 | 396 | 594 | 792 | 990  | 1188 | 1386 | 1584 | 1782 |
| 1985                              | 199 | 397 | 595 | 794 | 992  | 1191 | 1389 | 1588 | 1786 |
| 1990                              | 199 | 398 | 597 | 796 | 995  | 1194 | 1393 | 1592 | 1791 |
| 1995                              | 200 | 399 | 598 | 798 | 997  | 1197 | 1396 | 1596 | 1795 |
| 2000                              | 200 | 400 | 600 | 800 | 1000 | 1200 | 1400 | 1600 | 1800 |
| 2010                              | 201 | 402 | 603 | 804 | 1005 | 1206 | 1407 | 1608 | 1809 |
| 2020                              | 202 | 404 | 606 | 808 | 1010 | 1212 | 1414 | 1616 | 1818 |
| 2030                              | 203 | 406 | 609 | 812 | 1015 | 1218 | 1421 | 1624 | 1827 |
| 2040                              | 204 | 408 | 612 | 816 | 1020 | 1224 | 1428 | 1632 | 1836 |
| 2050                              | 205 | 410 | 615 | 820 | 1025 | 1230 | 1435 | 1640 | 1845 |



# A TABLE of Proportional Parts.

| Tenth Parts of those Differences. |     |     |     |     |      |      |      |      |      |
|-----------------------------------|-----|-----|-----|-----|------|------|------|------|------|
| Diff. of<br>Logar.                | 1   | 2   | 3   | 4   | 5    | 6    | 7    | 8    | 9    |
| 2060                              | 206 | 412 | 618 | 824 | 1030 | 1236 | 1442 | 1648 | 1854 |
| 2070                              | 207 | 414 | 621 | 828 | 1035 | 1242 | 1449 | 1656 | 1863 |
| 2080                              | 208 | 416 | 624 | 832 | 1040 | 1248 | 1456 | 1664 | 1872 |
| 2090                              | 209 | 418 | 627 | 836 | 1045 | 1254 | 1463 | 1672 | 1881 |
| 2100                              | 210 | 420 | 630 | 840 | 1050 | 1260 | 1470 | 1680 | 1890 |
| 2110                              | 211 | 422 | 633 | 844 | 1055 | 1266 | 1477 | 1688 | 1899 |
| 2120                              | 212 | 424 | 636 | 848 | 1060 | 1272 | 1484 | 1696 | 1908 |
| 2130                              | 213 | 426 | 639 | 852 | 1065 | 1278 | 1491 | 1704 | 1917 |
| 2140                              | 214 | 428 | 642 | 856 | 1070 | 1284 | 1498 | 1712 | 1926 |
| 2150                              | 215 | 430 | 645 | 860 | 1075 | 1290 | 1505 | 1720 | 1935 |
| 2160                              | 216 | 432 | 648 | 864 | 1080 | 1296 | 1512 | 1728 | 1944 |
| 2170                              | 217 | 434 | 651 | 868 | 1085 | 1302 | 1519 | 1736 | 1953 |
| 2180                              | 218 | 436 | 654 | 872 | 1090 | 1308 | 1526 | 1744 | 1962 |
| 2190                              | 219 | 438 | 657 | 876 | 1095 | 1314 | 1533 | 1752 | 1971 |
| 2200                              | 220 | 440 | 660 | 880 | 1100 | 1320 | 1540 | 1760 | 1980 |
| 2210                              | 221 | 442 | 663 | 884 | 1105 | 1326 | 1547 | 1768 | 1989 |
| 2220                              | 222 | 444 | 666 | 888 | 1110 | 1332 | 1554 | 1776 | 1998 |
| 2230                              | 223 | 446 | 669 | 892 | 1115 | 1338 | 1561 | 1784 | 2007 |
| 2240                              | 224 | 448 | 672 | 896 | 1120 | 1344 | 1568 | 1792 | 2016 |
| 2250                              | 225 | 450 | 675 | 900 | 1125 | 1350 | 1575 | 1800 | 2025 |
| 2260                              | 226 | 452 | 678 | 904 | 1130 | 1356 | 1582 | 1808 | 2034 |
| 2270                              | 227 | 454 | 681 | 908 | 1135 | 1362 | 1589 | 1816 | 2043 |
| 2280                              | 228 | 456 | 684 | 912 | 1140 | 1368 | 1596 | 1824 | 2052 |
| 2290                              | 229 | 458 | 687 | 916 | 1145 | 1374 | 1603 | 1832 | 2061 |
| 2300                              | 230 | 460 | 690 | 920 | 1150 | 1380 | 1610 | 1840 | 2070 |
| 2310                              | 231 | 462 | 693 | 924 | 1155 | 1386 | 1617 | 1848 | 2079 |
| 2320                              | 232 | 464 | 696 | 928 | 1160 | 1392 | 1624 | 1856 | 2088 |
| 2330                              | 233 | 466 | 699 | 932 | 1165 | 1398 | 1631 | 1864 | 2097 |
| 2340                              | 234 | 468 | 702 | 936 | 1170 | 1404 | 1638 | 1872 | 2106 |
| 2350                              | 235 | 470 | 705 | 940 | 1175 | 1410 | 1645 | 1880 | 2115 |
| 2360                              | 236 | 472 | 708 | 944 | 1180 | 1416 | 1652 | 1888 | 2124 |
| 2370                              | 237 | 474 | 711 | 948 | 1185 | 1422 | 1659 | 1896 | 2133 |
| 2380                              | 238 | 476 | 714 | 952 | 1190 | 1428 | 1666 | 1904 | 2142 |
| 2390                              | 239 | 478 | 717 | 956 | 1195 | 1434 | 1673 | 1912 | 2151 |
| 2400                              | 240 | 480 | 720 | 960 | 1200 | 1440 | 1680 | 1920 | 2160 |
| 2410                              | 241 | 482 | 723 | 964 | 1205 | 1446 | 1687 | 1928 | 2169 |
| 2420                              | 242 | 484 | 726 | 968 | 1210 | 1452 | 1694 | 1936 | 2178 |
| 2430                              | 243 | 486 | 729 | 972 | 1215 | 1458 | 1701 | 1944 | 2187 |
| 2440                              | 244 | 488 | 732 | 976 | 1220 | 1464 | 1708 | 1952 | 2196 |
| 2450                              | 245 | 490 | 735 | 980 | 1225 | 1470 | 1715 | 1960 | 2205 |

# A TABLE of Proportional Parts.

| Tenth Parts of those Differences. |     |     |     |      |      |      |      |      |      |
|-----------------------------------|-----|-----|-----|------|------|------|------|------|------|
| Diff. of<br>Logar.                | 1   | 2   | 3   | 4    | 5    | 6    | 7    | 8    | 9    |
| 2460                              | 246 | 492 | 738 | 984  | 1230 | 1476 | 1722 | 1968 | 2214 |
| 2470                              | 247 | 494 | 741 | 988  | 1235 | 1482 | 1729 | 1976 | 2223 |
| 2480                              | 248 | 496 | 744 | 992  | 1240 | 1488 | 1736 | 1984 | 2232 |
| 2490                              | 249 | 498 | 747 | 996  | 1245 | 1494 | 1743 | 1992 | 2241 |
| 2500                              | 250 | 500 | 750 | 1000 | 1250 | 1500 | 1750 | 2000 | 2250 |
| 2510                              | 251 | 502 | 753 | 1004 | 1255 | 1506 | 1757 | 2008 | 2259 |
| 2520                              | 252 | 504 | 756 | 1008 | 1260 | 1512 | 1764 | 2016 | 2268 |
| 2530                              | 253 | 506 | 759 | 1012 | 1265 | 1518 | 1771 | 2024 | 2277 |
| 2540                              | 254 | 508 | 762 | 1016 | 1270 | 1524 | 1778 | 2032 | 2286 |
| 2550                              | 255 | 510 | 765 | 1020 | 1275 | 1530 | 1785 | 2040 | 2295 |
| 2560                              | 256 | 512 | 768 | 1024 | 1280 | 1536 | 1792 | 2048 | 2304 |
| 2570                              | 257 | 514 | 771 | 1028 | 1285 | 1542 | 1799 | 2056 | 2313 |
| 2580                              | 258 | 516 | 774 | 1032 | 1290 | 1548 | 1806 | 2064 | 2322 |
| 2590                              | 259 | 518 | 777 | 1036 | 1295 | 1554 | 1813 | 2072 | 2331 |
| 2600                              | 260 | 520 | 780 | 1040 | 1300 | 1560 | 1820 | 2080 | 2340 |
| 2610                              | 261 | 522 | 783 | 1044 | 1305 | 1566 | 1827 | 2088 | 2349 |
| 2620                              | 262 | 524 | 786 | 1048 | 1310 | 1572 | 1834 | 2096 | 2358 |
| 2630                              | 263 | 526 | 789 | 1052 | 1315 | 1578 | 1841 | 2104 | 2367 |
| 2640                              | 264 | 528 | 792 | 1056 | 1320 | 1584 | 1848 | 2112 | 2376 |
| 2650                              | 265 | 530 | 795 | 1060 | 1325 | 1590 | 1855 | 2120 | 2385 |
| 2660                              | 266 | 532 | 798 | 1064 | 1330 | 1596 | 1862 | 2128 | 2394 |
| 2670                              | 267 | 534 | 801 | 1068 | 1335 | 1602 | 1869 | 2136 | 2403 |
| 2680                              | 268 | 536 | 804 | 1072 | 1340 | 1608 | 1876 | 2144 | 2412 |
| 2690                              | 269 | 538 | 807 | 1076 | 1345 | 1614 | 1883 | 2152 | 2421 |
| 2700                              | 270 | 540 | 810 | 1080 | 1350 | 1620 | 1890 | 2160 | 2430 |
| 2710                              | 271 | 542 | 813 | 1084 | 1355 | 1626 | 1897 | 2168 | 2439 |
| 2720                              | 272 | 544 | 816 | 1088 | 1360 | 1632 | 1904 | 2176 | 2448 |
| 2730                              | 273 | 546 | 819 | 1092 | 1365 | 1638 | 1911 | 2184 | 2457 |
| 2740                              | 274 | 548 | 822 | 1096 | 1370 | 1644 | 1918 | 2192 | 2466 |
| 2750                              | 275 | 550 | 825 | 1100 | 1375 | 1650 | 1925 | 2200 | 2475 |
| 2760                              | 276 | 552 | 828 | 1104 | 1380 | 1656 | 1932 | 2208 | 2484 |
| 2770                              | 277 | 554 | 831 | 1108 | 1385 | 1662 | 1939 | 2216 | 2493 |
| 2780                              | 278 | 556 | 834 | 1112 | 1390 | 1668 | 1946 | 2224 | 2502 |
| 2790                              | 279 | 558 | 837 | 1116 | 1395 | 1674 | 1953 | 2232 | 2511 |
| 2800                              | 280 | 560 | 840 | 1120 | 1400 | 1680 | 1960 | 2240 | 2520 |
| 2810                              | 281 | 562 | 843 | 1124 | 1405 | 1686 | 1967 | 2248 | 2529 |
| 2820                              | 282 | 564 | 846 | 1128 | 1410 | 1692 | 1974 | 2256 | 2538 |
| 2830                              | 283 | 566 | 849 | 1132 | 1415 | 1698 | 1981 | 2264 | 2547 |
| 2840                              | 284 | 568 | 852 | 1136 | 1420 | 1704 | 1988 | 2272 | 2556 |
| 2850                              | 285 | 570 | 855 | 1140 | 1425 | 1710 | 1995 | 2280 | 2565 |

# A TABLE of Proportional Parts.

| Tenth Parts of those Differences. |     |     |     |      |      |      |      |      |      |
|-----------------------------------|-----|-----|-----|------|------|------|------|------|------|
| Diff. of<br>Logar.                | 1   | 2   | 3   | 4    | 5    | 6    | 7    | 8    | 9    |
| 2860                              | 286 | 572 | 858 | 1144 | 1430 | 1716 | 2002 | 2288 | 2574 |
| 2870                              | 287 | 574 | 861 | 1148 | 1435 | 1722 | 2009 | 2296 | 2583 |
| 2880                              | 288 | 576 | 864 | 1152 | 1440 | 1728 | 2016 | 2304 | 2592 |
| 2890                              | 289 | 578 | 867 | 1156 | 1445 | 1734 | 2023 | 2312 | 2601 |
| 2900                              | 290 | 580 | 870 | 1160 | 1450 | 1740 | 2030 | 2320 | 2610 |
| 2910                              | 291 | 582 | 873 | 1164 | 1455 | 1746 | 2037 | 2328 | 2619 |
| 2920                              | 292 | 584 | 876 | 1168 | 1460 | 1752 | 2044 | 2336 | 2628 |
| 2930                              | 293 | 586 | 879 | 1172 | 1465 | 1758 | 2051 | 2344 | 2637 |
| 2940                              | 294 | 588 | 882 | 1176 | 1470 | 1764 | 2058 | 2352 | 2646 |
| 2950                              | 295 | 590 | 885 | 1180 | 1475 | 1770 | 2065 | 2360 | 2655 |
| 2960                              | 296 | 592 | 888 | 1184 | 1480 | 1776 | 2072 | 2368 | 2664 |
| 2970                              | 297 | 594 | 891 | 1188 | 1485 | 1782 | 2079 | 2376 | 2673 |
| 2980                              | 298 | 596 | 894 | 1192 | 1490 | 1788 | 2086 | 2384 | 2682 |
| 2990                              | 299 | 598 | 897 | 1196 | 1495 | 1794 | 2093 | 2392 | 2691 |
| 3000                              | 300 | 600 | 900 | 1200 | 1500 | 1800 | 2100 | 2400 | 2700 |
| 3010                              | 301 | 602 | 903 | 1204 | 1505 | 1806 | 2107 | 2408 | 2709 |
| 3020                              | 302 | 604 | 906 | 1208 | 1510 | 1812 | 2114 | 2416 | 2718 |
| 3030                              | 303 | 606 | 909 | 1212 | 1515 | 1818 | 2121 | 2424 | 2727 |
| 3040                              | 304 | 608 | 912 | 1216 | 1520 | 1824 | 2128 | 2432 | 2736 |
| 3050                              | 305 | 610 | 915 | 1220 | 1525 | 1830 | 2135 | 2440 | 2745 |
| 3060                              | 306 | 612 | 918 | 1224 | 1530 | 1836 | 2142 | 2448 | 2754 |
| 3070                              | 307 | 614 | 921 | 1228 | 1535 | 1842 | 2149 | 2456 | 2763 |
| 3080                              | 308 | 616 | 924 | 1232 | 1540 | 1848 | 2156 | 2464 | 2772 |
| 3090                              | 309 | 618 | 927 | 1236 | 1545 | 1854 | 2163 | 2472 | 2781 |
| 3100                              | 310 | 620 | 930 | 1240 | 1550 | 1860 | 2170 | 2480 | 2790 |
| 3110                              | 311 | 622 | 933 | 1244 | 1555 | 1866 | 2177 | 2488 | 2799 |
| 3120                              | 312 | 624 | 936 | 1248 | 1560 | 1872 | 2184 | 2496 | 2808 |
| 3130                              | 313 | 626 | 939 | 1252 | 1565 | 1878 | 2191 | 2504 | 2817 |
| 3140                              | 314 | 628 | 942 | 1256 | 1570 | 1884 | 2198 | 2512 | 2826 |
| 3150                              | 315 | 630 | 945 | 1260 | 1575 | 1890 | 2205 | 2520 | 2835 |
| 3160                              | 316 | 632 | 948 | 1264 | 1580 | 1896 | 2212 | 2528 | 2844 |
| 3170                              | 317 | 634 | 951 | 1268 | 1585 | 1902 | 2219 | 2536 | 2853 |
| 3180                              | 318 | 636 | 954 | 1272 | 1590 | 1908 | 2226 | 2544 | 2862 |
| 3190                              | 319 | 638 | 957 | 1276 | 1595 | 1914 | 2233 | 2552 | 2871 |
| 3200                              | 320 | 640 | 960 | 1280 | 1600 | 1920 | 2240 | 2560 | 2880 |
| 3210                              | 321 | 642 | 963 | 1284 | 1605 | 1926 | 2247 | 2568 | 2889 |
| 3220                              | 322 | 644 | 966 | 1288 | 1610 | 1932 | 2254 | 2576 | 2898 |
| 3230                              | 323 | 646 | 969 | 1292 | 1615 | 1938 | 2261 | 2584 | 2907 |
| 3240                              | 324 | 648 | 972 | 1296 | 1620 | 1944 | 2268 | 2592 | 2916 |
| 3250                              | 325 | 650 | 975 | 1300 | 1625 | 1950 | 2275 | 2600 | 2925 |



# A TABLE of Proportional Parts.

| Tenth Parts of those Differences. |     |     |      |      |      |      |      |      |      |
|-----------------------------------|-----|-----|------|------|------|------|------|------|------|
| Diff. of<br>Logar.                | 1   | 2   | 3    | 4    | 5    | 6    | 7    | 8    | 9    |
| 3260                              | 326 | 652 | 978  | 1304 | 1630 | 1956 | 2282 | 2608 | 2934 |
| 3270                              | 327 | 654 | 981  | 1308 | 1635 | 1962 | 2289 | 2616 | 2943 |
| 3280                              | 328 | 656 | 984  | 1312 | 1640 | 1968 | 2296 | 2624 | 2952 |
| 3290                              | 329 | 658 | 987  | 1316 | 1645 | 1974 | 2303 | 2632 | 2961 |
| 3300                              | 330 | 660 | 990  | 1320 | 1650 | 1980 | 2310 | 2640 | 2970 |
| 3310                              | 331 | 662 | 993  | 1324 | 1655 | 1986 | 2317 | 2648 | 2979 |
| 3320                              | 332 | 664 | 996  | 1328 | 1660 | 1992 | 2324 | 2656 | 2988 |
| 3330                              | 333 | 666 | 999  | 1332 | 1665 | 1998 | 2331 | 2664 | 2997 |
| 3340                              | 334 | 668 | 1002 | 1336 | 1670 | 2004 | 2338 | 2672 | 3006 |
| 3350                              | 335 | 670 | 1005 | 1340 | 1675 | 2010 | 2345 | 2680 | 3015 |
| 3360                              | 336 | 672 | 1008 | 1344 | 1680 | 2016 | 2352 | 2688 | 3024 |
| 3370                              | 337 | 674 | 1011 | 1348 | 1685 | 2022 | 2359 | 2696 | 3033 |
| 3380                              | 338 | 676 | 1014 | 1352 | 1690 | 2028 | 2366 | 2704 | 3042 |
| 3390                              | 339 | 678 | 1017 | 1356 | 1695 | 2034 | 2373 | 2712 | 3051 |
| 3400                              | 340 | 680 | 1020 | 1360 | 1700 | 2040 | 2380 | 2720 | 3060 |
| 3410                              | 341 | 682 | 1023 | 1364 | 1705 | 2046 | 2387 | 2728 | 3069 |
| 3420                              | 342 | 684 | 1026 | 1368 | 1710 | 2052 | 2394 | 2736 | 3078 |
| 3430                              | 343 | 686 | 1029 | 1372 | 1715 | 2058 | 2401 | 2744 | 3087 |
| 3440                              | 344 | 688 | 1032 | 1376 | 1720 | 2064 | 2408 | 2752 | 3096 |
| 3450                              | 345 | 690 | 1035 | 1380 | 1725 | 2070 | 2415 | 2760 | 3105 |
| 3460                              | 346 | 692 | 1038 | 1384 | 1730 | 2076 | 2422 | 2768 | 3114 |
| 3470                              | 347 | 694 | 1041 | 1388 | 1735 | 2082 | 2429 | 2776 | 3123 |
| 3480                              | 348 | 696 | 1044 | 1392 | 1740 | 2088 | 2436 | 2784 | 3132 |
| 3490                              | 349 | 698 | 1047 | 1396 | 1745 | 2094 | 2443 | 2792 | 3141 |
| 3500                              | 350 | 700 | 1050 | 1400 | 1750 | 2100 | 2450 | 2800 | 3150 |
| 3510                              | 351 | 702 | 1053 | 1404 | 1755 | 2106 | 2457 | 2808 | 3159 |
| 3520                              | 352 | 704 | 1056 | 1408 | 1760 | 2112 | 2464 | 2816 | 3168 |
| 3530                              | 353 | 706 | 1059 | 1412 | 1765 | 2118 | 2471 | 2824 | 3177 |
| 3540                              | 354 | 708 | 1062 | 1416 | 1770 | 2124 | 2478 | 2832 | 3186 |
| 3550                              | 355 | 710 | 1065 | 1420 | 1775 | 2130 | 2485 | 2840 | 3195 |
| 3560                              | 356 | 712 | 1068 | 1424 | 1780 | 2136 | 2492 | 2848 | 3204 |
| 3570                              | 357 | 714 | 1071 | 1428 | 1785 | 2142 | 2499 | 2856 | 3213 |
| 3580                              | 358 | 716 | 1074 | 1432 | 1790 | 2148 | 2506 | 2864 | 3222 |
| 3590                              | 359 | 718 | 1077 | 1436 | 1795 | 2154 | 2513 | 2872 | 3231 |
| 3600                              | 360 | 720 | 1080 | 1440 | 1800 | 2160 | 2520 | 2880 | 3240 |
| 3610                              | 361 | 722 | 1083 | 1444 | 1805 | 2166 | 2527 | 2888 | 3249 |
| 3620                              | 362 | 724 | 1086 | 1448 | 1810 | 2172 | 2534 | 2896 | 3258 |
| 3630                              | 363 | 726 | 1089 | 1452 | 1815 | 2178 | 2541 | 2904 | 3267 |
| 3640                              | 364 | 728 | 1092 | 1456 | 1820 | 2184 | 2548 | 2912 | 3276 |
| 3650                              | 365 | 730 | 1095 | 1460 | 1825 | 2190 | 2555 | 2920 | 3285 |

# A TABLE of Proportional Parts.

| Tenth Parts of those Differences. |     |     |      |      |      |      |      |      |      |
|-----------------------------------|-----|-----|------|------|------|------|------|------|------|
| Diff. of<br>Logar.                | 1   | 2   | 3    | 4    | 5    | 6    | 7    | 8    | 9    |
| 3660                              | 366 | 732 | 1098 | 1464 | 1830 | 2196 | 2562 | 2928 | 3294 |
| 3670                              | 367 | 734 | 1101 | 1468 | 1835 | 2202 | 2569 | 2936 | 3303 |
| 3680                              | 368 | 736 | 1104 | 1472 | 1840 | 2208 | 2576 | 2944 | 3312 |
| 3690                              | 369 | 738 | 1107 | 1476 | 1845 | 2214 | 2583 | 2952 | 3321 |
| 3700                              | 370 | 740 | 1110 | 1480 | 1850 | 2220 | 2590 | 2960 | 3330 |
| 3710                              | 371 | 742 | 1113 | 1484 | 1855 | 2226 | 2597 | 2968 | 3339 |
| 3720                              | 372 | 744 | 1116 | 1488 | 1860 | 2232 | 2604 | 2976 | 3348 |
| 3730                              | 373 | 746 | 1119 | 1492 | 1865 | 2238 | 2611 | 2984 | 3357 |
| 3740                              | 374 | 748 | 1122 | 1496 | 1870 | 2244 | 2618 | 2992 | 3366 |
| 3750                              | 375 | 750 | 1125 | 1500 | 1875 | 2250 | 2625 | 3000 | 3375 |
| 3760                              | 376 | 752 | 1128 | 1504 | 1880 | 2256 | 2632 | 3008 | 3384 |
| 3770                              | 377 | 754 | 1131 | 1508 | 1885 | 2262 | 2639 | 3016 | 3393 |
| 3780                              | 378 | 756 | 1134 | 1512 | 1890 | 2268 | 2646 | 3024 | 3402 |
| 3790                              | 379 | 758 | 1137 | 1516 | 1895 | 2274 | 2653 | 3032 | 3411 |
| 3800                              | 380 | 760 | 1140 | 1520 | 1900 | 2280 | 2660 | 3040 | 3420 |
| 3810                              | 381 | 762 | 1143 | 1524 | 1905 | 2286 | 2667 | 3048 | 3429 |
| 3820                              | 382 | 764 | 1146 | 1528 | 1910 | 2292 | 2674 | 3056 | 3438 |
| 3830                              | 383 | 766 | 1149 | 1532 | 1915 | 2298 | 2681 | 3064 | 3447 |
| 3840                              | 384 | 768 | 1152 | 1536 | 1920 | 2304 | 2688 | 3072 | 3456 |
| 3850                              | 385 | 770 | 1155 | 1540 | 1925 | 2310 | 2695 | 3080 | 3465 |
| 3860                              | 386 | 772 | 1158 | 1544 | 1930 | 2316 | 2702 | 3088 | 3474 |
| 3870                              | 387 | 774 | 1161 | 1548 | 1935 | 2322 | 2709 | 3096 | 3483 |
| 3880                              | 388 | 776 | 1164 | 1552 | 1940 | 2328 | 2716 | 3104 | 3492 |
| 3890                              | 389 | 778 | 1167 | 1556 | 1945 | 2334 | 2723 | 3112 | 3501 |
| 3900                              | 390 | 780 | 1170 | 1560 | 1950 | 2340 | 2730 | 3120 | 3510 |
| 3910                              | 391 | 782 | 1173 | 1564 | 1955 | 2346 | 2737 | 3128 | 3519 |
| 3920                              | 392 | 784 | 1176 | 1568 | 1960 | 2352 | 2744 | 3136 | 3528 |
| 3930                              | 393 | 786 | 1179 | 1572 | 1965 | 2358 | 2751 | 3144 | 3537 |
| 3940                              | 394 | 788 | 1182 | 1576 | 1970 | 2364 | 2758 | 3152 | 3546 |
| 3950                              | 395 | 790 | 1185 | 1580 | 1975 | 2370 | 2765 | 3160 | 3555 |
| 3960                              | 396 | 792 | 1188 | 1584 | 1980 | 2376 | 2772 | 3168 | 3564 |
| 3970                              | 397 | 794 | 1191 | 1588 | 1985 | 2382 | 2779 | 3176 | 3573 |
| 3980                              | 398 | 796 | 1194 | 1592 | 1990 | 2388 | 2786 | 3184 | 3582 |
| 3990                              | 399 | 798 | 1197 | 1596 | 1995 | 2394 | 2793 | 3192 | 3591 |
| 4000                              | 400 | 800 | 1200 | 1600 | 2000 | 2400 | 2800 | 3200 | 3600 |
| 4010                              | 401 | 802 | 1203 | 1604 | 2005 | 2406 | 2807 | 3208 | 3609 |
| 4020                              | 402 | 804 | 1206 | 1608 | 2010 | 2412 | 2814 | 3216 | 3618 |
| 4030                              | 403 | 806 | 1209 | 1612 | 2015 | 2418 | 2821 | 3224 | 3627 |
| 4040                              | 404 | 808 | 1212 | 1616 | 2020 | 2424 | 2828 | 3232 | 3636 |
| 4050                              | 405 | 810 | 1215 | 1620 | 2025 | 2430 | 2835 | 3240 | 3645 |

# *A TABLE of Proportional Parts.*

| Tenth Parts of those Differences. |     |     |      |      |      |      |      |      |      |
|-----------------------------------|-----|-----|------|------|------|------|------|------|------|
| Diff. of<br>Logar.                | 1   | 2   | 3    | 4    | 5    | 6    | 7    | 8    | 9    |
| 4060                              | 406 | 812 | 1218 | 1624 | 2030 | 2436 | 2842 | 3248 | 3654 |
| 4070                              | 407 | 814 | 1221 | 1628 | 2035 | 2442 | 2849 | 3256 | 3663 |
| 4080                              | 408 | 816 | 1224 | 1632 | 2040 | 2448 | 2856 | 3264 | 3672 |
| 4090                              | 409 | 818 | 1227 | 1636 | 2045 | 2454 | 2863 | 3272 | 3681 |
| 4100                              | 410 | 820 | 1230 | 1640 | 2050 | 2460 | 2870 | 3280 | 3690 |
| 4110                              | 411 | 822 | 1233 | 1644 | 2055 | 2466 | 2877 | 3288 | 3699 |
| 4120                              | 412 | 824 | 1236 | 1648 | 2060 | 2472 | 2884 | 3296 | 3708 |
| 4130                              | 413 | 826 | 1239 | 1652 | 2065 | 2478 | 2891 | 3304 | 3717 |
| 4140                              | 414 | 828 | 1242 | 1656 | 2070 | 2484 | 2898 | 3312 | 3726 |
| 4150                              | 415 | 830 | 1245 | 1660 | 2075 | 2490 | 2905 | 3320 | 3735 |
| 4160                              | 416 | 832 | 1248 | 1664 | 2080 | 2496 | 2912 | 3328 | 3744 |
| 4170                              | 417 | 834 | 1251 | 1668 | 2085 | 2502 | 2919 | 3336 | 3753 |
| 4180                              | 418 | 836 | 1254 | 1672 | 2090 | 2508 | 2926 | 3344 | 3762 |
| 4190                              | 419 | 838 | 1257 | 1676 | 2095 | 2514 | 2933 | 3352 | 3771 |
| 4200                              | 420 | 840 | 1260 | 1680 | 2100 | 2520 | 2940 | 3360 | 3780 |
| 4210                              | 421 | 842 | 1263 | 1684 | 2105 | 2526 | 2947 | 3368 | 3789 |
| 4220                              | 422 | 844 | 1266 | 1688 | 2110 | 2532 | 2954 | 3376 | 3798 |
| 4230                              | 423 | 846 | 1269 | 1692 | 2115 | 2538 | 2961 | 3384 | 3807 |
| 4240                              | 424 | 848 | 1272 | 1696 | 2120 | 2544 | 2968 | 3392 | 3816 |
| 4250                              | 425 | 850 | 1275 | 1700 | 2125 | 2550 | 2975 | 3400 | 3825 |
| 4260                              | 426 | 852 | 1278 | 1704 | 2130 | 2556 | 2982 | 3408 | 3834 |
| 4270                              | 427 | 854 | 1281 | 1708 | 2135 | 2562 | 2989 | 3416 | 3843 |
| 4280                              | 428 | 856 | 1284 | 1712 | 2140 | 2568 | 2996 | 3424 | 3852 |
| 4290                              | 429 | 858 | 1287 | 1716 | 2145 | 2574 | 3003 | 3432 | 3861 |
| 4300                              | 430 | 860 | 1290 | 1720 | 2150 | 2580 | 3010 | 3440 | 3870 |
| 4310                              | 431 | 862 | 1293 | 1724 | 2155 | 2586 | 3017 | 3448 | 3879 |
| 4320                              | 432 | 864 | 1296 | 1728 | 2160 | 2592 | 3024 | 3456 | 3888 |

*The End of the Table of Parts Proportional.*





CHAP. VIII.

*Lineal Arithmetic.* (See Plate A.)

**T**HIS fourth Species of Arithmetic is performed by Right Lines, so that Magnitude being here considered, it may be called *Geometrical Arithmetic*.

2. A Right Line is properly defined to be the nearest Distance between two Points, as the Line *a b* (Fig. 1.) is the nearest Distance between the Points *a* and *b*.

3. The Disposition of the two principal Lines, by which this kind of Arithmetic is performed, are either in a Right Angle or an Oblique.

4. A Right Angle is when one Line falls Perpendicular on another, as the Line *a b* (Fig. 2.) does on *b c*; but when two Lines meet not so, they form an Angle called *Oblique*, as the Angle *d e f* (Fig. 3) by both which, Questions are answered.

As in Logarithms those Artificial Numbers are not employed about the most easy and self-evident Rules of *Addition* and *Subtraction* of Natural Numbers: So here by Lines I shall only shew how *Multiplication*, *Division*, the *Rule of Proportion*, with *Extraction of Roots*, are performed thereby.

SECT. II. *Multiplication by Lines, three Ways.*

1. What is the Product of  $17\frac{1}{2}$  by  $12\frac{1}{2}$ ? See Fig. 4. in the folded Sheet (A.)

To answer this, draw a Line at pleasure, and set off  $12\frac{1}{2}$  equal Parts as from *a* to *b*, from the Line of equal Parts.

Then draw a perpendicular Line at pleasure as *b c*, and set off from *b* to *c*  $17\frac{1}{2}$ .

Lastly, Draw Lines between each equal Part, and the  $\frac{1}{2}$ , Parallel to (*a b*) and to (*b c*), then telling the several little Squares each of which is a Unit, you'll find 204 intire, and 17 and 12 Halves, or  $14\frac{1}{2}$  which together, and the  $\frac{1}{4}$  at the Angle (*c*) make the Product  $218\frac{1}{4}$ .

If you would multiply 175 by 125, that is, if you suppose every Unit in the Line (*a b*) to be 10, then  $12\frac{1}{2}$  will be 125, and by

Y y

the

the same Rule the Line  $bc$  will be 175, and by supposing each little Square 100 (or 10 times 10, the Lines  $ab$  and  $bc$  being 10 times as many as before) you'll find 20400 little Squares or Units, and 17 and 12 Fiftie (or 1450) and 25 the 4th of 100 at the Angle ( $c$ ), which added together, makes  $21875 =$  the Product required.

*A second Way to multiply by Lines.*

But before I proceed therein, it will be necessary to shew how to draw a Line Parallel to another, and through any Point assigned: So to draw a Line through the Point ( $a$ ) (See Fig. 5. Plate A.) and Parallel to ( $mn$ ), set the Foot of your Compasses in the Point, and extend the other to the Line  $mn$ , so as to describe the Arc  $cc$  just to touch the Line  $mn$ ; and with that Distance of the Feet set one in the Point ( $e$ ) (or thereabout) and describe the Arc ( $dd$ ); then lay a Ruler from that Arc to the Point  $a$ , and draw the Line  $pq$ , which is the Answer.

To multiply by 2 Lines, including an Oblique Angle as that ( $edn$ .) Fig. 6.

What is the Product of 3.5 by 2.2? For answer, having made an Angle at pleasure, not too acute.

Rule.] Take a Line of equal Parts, and set off 1 from the Angle  $d$  to  $1$ . Then set the Multiplicand from  $d$  to 3.5 on the other Leg ( $dn$ ). 3dly, Set the Multiplier from 1 to 2.2. 4thly, Draw the Line 1 to 3.5, produced to about ( $m$ ), for the better drawing the Line (2.2; 7.7) Parallel to ( $1, m$ ); so is the Distance between 3.5 and 7.7 the Answer, which measured upon the same Line of equal Parts, is found  $7.7 =$  the Product.

2. Or if you set the Multiplicand from 1 to  $b$ ; and the Multiplier from  $d$  to  $a$ , a Line ( $bc$ ) drawn Parallel to ( $1, a$ ) cutteth the Product ( $ac$ ) 7.7 as before.

3. Or if you suppose the Factors 10 times as much, the Answer will be by the same Method 770.

*A third Way to multiply by Lines.*

This shews the Use of a Line of Numbers, or as 'tis often called Gunter's Line; being a Line of Logarithms, as will appear by the Use, Addition performing Multiplication; Substraction, the Work of Division; Division, the Extraction of Roots, &c.

This



### SECT. III. Division by Lines, three Ways.

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This useful line was the Invention of Mr. Edmund Gunter, Professor of *Astronomy* at *Gresham-College* about the Year 1620; and it is a line divided into 10 unequal Parts, and those 10 Parts repeated are called 20, 30, 40, &c. when the former Parts are called 1, 2, 3, &c.

If the first 10 are Tens,

The 2d ten Parts are 100's

If the first 10 are Hundreds,

The 2d Radius are so many

(Thousands.

If you multiply 1 place by 2 places,

The Product is so many 10's

(as the Figure denotes.

As your own Experience will easily discover to you. See the making of this line near the End of *Set.* 4th following.

To multiply any 2 Numbers, as those in the last Case 2.2 by 3.5. See *Fig. 7. Plate A.*

*Rule.]* Extend the Foot of a Pair of Compasses (or other Measure) from 1 to 2.2, and that Extent reaches from 3.5 to 7.7;

2dly, Or the Extent from 1 to 3.5, reaches from 2.2 to 7.7.

3dly, Or if you multiply 35 by 22. In this case, what is called 1, 2, &c. in the 1st part, must be supposed 10, 20, &c. and what are 10th parts must be called Units, and if both Factors be 10 or more each, the Product is Hundreds. So the Extent as before from 10 to 22, reaches from 35 to 770; for here 35 being multiplied by a Number which is above 10, is so many 100's and 10's as the Measure extends to; for your Inspection shews, that 35 by 10 is 350, which is 3 places; and if for any Number of Tens, the like: And if the 2d Extent be beyond 10 on the line, it is still so many Hundreds, as 30 by 50 is 1500, &c.

And *Note.* That whatever you call the larger Divisions, the smaller in the same Radius are a 10th thereof, as is before hinted, and the contrary.

### SECT. III. Divisions by Lines, three Ways.

As I have shewed *Multiplication* by three different lines, and by each two ways; so I intend to shew how to divide.

*Case 1.]* What is the Quotient of 2184 by 174? See *Fig. 4.*

The Dividend is represented by the several small Squares and Parts in the Superficies *a c*, which if you divide by the side (*a* 174) the Quotient will be one of the smaller sides *a b* or (174 *c*).

For having divided a Line ( $a$ ,  $17\frac{1}{2}$ ) into  $17\frac{1}{2}$  parts, your Divisor, and made a little Square upon each part, and half of one for the half; if you add to those Squares at Right Angles so many as will make them up  $218\frac{1}{2}$ , and as many in one Row of Squares as another; you will then find that the Line including a Right Angle with ( $a$   $17\frac{1}{2}$ ) will be ( $a$   $b$ ) which will contain  $12\frac{1}{2}$  of those small Squares, and is consequently the Quotient, because that Quote multiplied in the Divisor ( $a$ ,  $17\frac{1}{2}$ ) produceth the Dividend or Number of small Squares ( $a$   $c$ ) =  $218\frac{1}{2}$ . And the same may be said of other like Superficies and their Sides, whatever they be for Numbers.

*A second Way to divide Lines.*

What is the Quote of  $7.7$  divided by  $3.5$ ? (Fig. 6. for I make use of the same Numbers as in *Multiplication*, to prevent Confusion by a Multitude of prickt Lines.)

- 1<sup>st</sup>, Take 1 from any Line of equal Parts, and set off ( $d$  1).
- 2<sup>dly</sup>, Set the Divisor ( $d$ ,  $3.5$ ) off from the same Line of Parts.
- 3<sup>dly</sup>, Draw the Line ( $1$ ,  $3.5$ ) and produce it to  $m$ , &c.
- 4<sup>tly</sup>, Set your Dividend ( $7.7$ ) from  $3\frac{1}{2}$  towards  $n$ , which extends to  $7.7$ .
- 5<sup>tly</sup>, Through that Point draw a Line parallel to ( $1$   $m$ ) and it will pass through the Point  $2.2$  in the Line ( $d$   $e$ ).
- 6<sup>tly</sup>, Take the Distance from  $1$  to  $2.2$ , and apply it to your equal Parts, and you'll find it  $2.2$  = the Quote.

Or if you were to divide  $7.7$  by  $2.2$ , the Divisor set from  $d$  to  $a$ , and the Dividend from  $a$  to  $c$ : If a Line be drawn ( $1$   $a$ ), and another parallel thereto through the Point  $c$ , that Line will pass thro'  $b$  in the Line  $d$   $e$ , and the distance ( $1$   $b$ ) measured on the same Line of equal Parts, will be  $3.5$  = the Quote.

Or if you divide  $770$  by  $35$ , the Method is the same as the first of these, and  $77$  by  $22$  the same as the second or last, viz.  $35$ . And the like of other Numbers.

*A third Way to divide by Lines.*

Suppose by the Line of Numbers, Fig. 7. Plate A. you would divide  $770$  by  $22$ . Extend the Compasses from  $10$  to  $22$  (calling those  $10$ , &c. in the first Radius, which are  $1$ ,  $2$ , &c. and those  $100$ 's in the second, which are  $10$ ,  $20$ ,  $30$ , &c.) and the same

SECT. IV. *The direct Rule of Proportion by Lines.* 349

same Extent reaches from 770 in the second Radius, backward to 35.

Or to divide 770 by 35, the Extent from 10 to 35, extends backward from 770 to 22. And the like of other Numbers.

SECT. IV. *The direct Rule of Proportion by Lines.*

If 10 lb of Sugar cost 22 s. what will 35 lb cost?

1<sup>st</sup>, By the lines, Fig. 6. set 10 from the Angle to 1 (which suppose 10, as if the 1 was not there;) then set the 22 s. from 1 to 2.2, that is, from 10 to 22. 3<sup>dly</sup>, Set the third Number from the Angle at *d* to 3.5, or to 35. Then draw a line from 10 to 35 = the third Number given, and a line parallel to that through the Point 2.2 (here 22 will pass through, or cut the line *dn* in 7.7) so the Distance 7.7 to 3.5 (measured upon the line whereby *d* 1 is 10) is 77 s. = the value of the 35 lb of Sugar. I have done this (as I said before) by the same parallel lines, to prevent Confusion by a Multiplicity.

2<sup>dly</sup>, By the line of Numbers, Fig. 7. If 12 C. of Sugar cost 25 l. 4 s. what will 1 Ton or 20 C. cost?

Extend the Compasses from 12 C. to 25.2, and the same Extent reaches from 20 C. to 42 l. the Answer.

Quest. 2. If you would know the Interest of 90 l. for a Year at 6 per Cent. extend the Compasses from 100 backward to 6; and the same Extent reaches from 90 to 5.4 l. or l. 5 : 8 :: the Answer.

3<sup>dly</sup>, To Extract the Square Root, take half the Distance between the Square Number given and 1. And 1 third of the Distance from a Cube Number to 1 is the Cube Root, taking the third next to 1.

And now I might end this part of Arithmetic, but before I do I shall shew the making of a line of Chords of Numbers, and a line of half Tangents with something of their Use, and that of equal Parts, which will be a Preparative for those who would know how to perform Trigonometry Geometrically.

The line of Chords is 90 Degrees (or equal Parts in the fourth Part of a Circle) set off upon a straight line, as Fig. 8. Plate A. the Degree (*em*) are set upon the Diameter (*ecn*) where (*e* 60) of the Circle, is equal to Radius or the Semi-diameter of that Circle, and *em* equal to *en*, &c.

Now to measure any Angle by this line extend the Compasses from *e* to 60 in the line (*en*), and with that (suppose in Fig. 9.) set



set one Foot in  $b$ , and describe the Arc ( $ro$ ), then take the Distance ( $ro$ ) in your Compasses, and apply it to your line of Chords, and you'll find it 33 Degrees: That is to say, the line or leg ( $bf$ ) is distant from that  $g$  33 Degrees or equal Parts of the Circle ( $dme$ , &c.) as those are transferred upon the line ( $ed$ ); so the Angle ( $bfg$ ) or at  $f$ , is found 58 Degrees, and the Sum of those two Angles deducted from 180 leaves the Angle ( $fgb$ , or) at  $g$ . And the Sides of this Triangle are measured by applying them between the Compasses to the line of equal Parts ( $y z$ ), Fig. 10.

*The way to make a Line of Numbers or Logarithms (commonly called Gunter's Line).*

The line (1, 10, Plate A) is thus made: First, Make a line of equal Parts according to the Length you intend your line of Numbers, as here (0, 1000) is divided actually into 100 equal Parts, and each Part supposed to be divided into 10, makes 1000; and if your line were designed to be much longer, you might divide your equal Parts into 10000.

2dly, Lay before you a Table of Logarithms, and from that take the Logarithms (omitting the Index) of 2, 3, 4, &c. to 10, the largest Divisions of your line: As, for 2 I find the Logarithm .3010300, but I take Notice only of the three next places to the Point agreeable to my line of equal Parts, as 301 taken from that line, and set off upon the Line (1, 10,) from 1 to 2 gives the first of the first 9 unequal Parts of the line of Numbers.

|                               |                                                                                                                                                                                                                                                                                                                                                                    |                                                           |       |      |   |  |      |   |  |      |   |  |      |   |  |      |   |  |      |   |  |      |                                                                                      |                                                                                                                                                                                                                                                                                                                                                                                            |   |      |                                                           |   |      |   |      |   |      |   |      |   |      |   |      |
|-------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------|-------|------|---|--|------|---|--|------|---|--|------|---|--|------|---|--|------|---|--|------|--------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---|------|-----------------------------------------------------------|---|------|---|------|---|------|---|------|---|------|---|------|
| Then I find the Logarithms of | <table border="0"> <tr> <td>3</td> <td>to be</td> <td>.477</td> </tr> <tr> <td>4</td> <td></td> <td>.602</td> </tr> <tr> <td>5</td> <td></td> <td>.698</td> </tr> <tr> <td>6</td> <td></td> <td>.778</td> </tr> <tr> <td>7</td> <td></td> <td>.845</td> </tr> <tr> <td>8</td> <td></td> <td>.903</td> </tr> <tr> <td>9</td> <td></td> <td>.954</td> </tr> </table> | 3                                                         | to be | .477 | 4 |  | .602 | 5 |  | .698 | 6 |  | .778 | 7 |  | .845 | 8 |  | .903 | 9 |  | .954 | Which Logarithms taken from the line of equal Parts, and set on that of Numbers from | <table border="0"> <tr> <td>1</td> <td>to 3</td> <td rowspan="9">           Gives you the other unequal Parts of the line of Numbers.         </td> </tr> <tr> <td>1</td> <td>to 4</td> </tr> <tr> <td>1</td> <td>to 5</td> </tr> <tr> <td>1</td> <td>to 6</td> </tr> <tr> <td>1</td> <td>to 7</td> </tr> <tr> <td>1</td> <td>to 8</td> </tr> <tr> <td>1</td> <td>to 9</td> </tr> </table> | 1 | to 3 | Gives you the other unequal Parts of the line of Numbers. | 1 | to 4 | 1 | to 5 | 1 | to 6 | 1 | to 7 | 1 | to 8 | 1 | to 9 |
| 3                             | to be                                                                                                                                                                                                                                                                                                                                                              | .477                                                      |       |      |   |  |      |   |  |      |   |  |      |   |  |      |   |  |      |   |  |      |                                                                                      |                                                                                                                                                                                                                                                                                                                                                                                            |   |      |                                                           |   |      |   |      |   |      |   |      |   |      |   |      |
| 4                             |                                                                                                                                                                                                                                                                                                                                                                    | .602                                                      |       |      |   |  |      |   |  |      |   |  |      |   |  |      |   |  |      |   |  |      |                                                                                      |                                                                                                                                                                                                                                                                                                                                                                                            |   |      |                                                           |   |      |   |      |   |      |   |      |   |      |   |      |
| 5                             |                                                                                                                                                                                                                                                                                                                                                                    | .698                                                      |       |      |   |  |      |   |  |      |   |  |      |   |  |      |   |  |      |   |  |      |                                                                                      |                                                                                                                                                                                                                                                                                                                                                                                            |   |      |                                                           |   |      |   |      |   |      |   |      |   |      |   |      |
| 6                             |                                                                                                                                                                                                                                                                                                                                                                    | .778                                                      |       |      |   |  |      |   |  |      |   |  |      |   |  |      |   |  |      |   |  |      |                                                                                      |                                                                                                                                                                                                                                                                                                                                                                                            |   |      |                                                           |   |      |   |      |   |      |   |      |   |      |   |      |
| 7                             |                                                                                                                                                                                                                                                                                                                                                                    | .845                                                      |       |      |   |  |      |   |  |      |   |  |      |   |  |      |   |  |      |   |  |      |                                                                                      |                                                                                                                                                                                                                                                                                                                                                                                            |   |      |                                                           |   |      |   |      |   |      |   |      |   |      |   |      |
| 8                             |                                                                                                                                                                                                                                                                                                                                                                    | .903                                                      |       |      |   |  |      |   |  |      |   |  |      |   |  |      |   |  |      |   |  |      |                                                                                      |                                                                                                                                                                                                                                                                                                                                                                                            |   |      |                                                           |   |      |   |      |   |      |   |      |   |      |   |      |
| 9                             |                                                                                                                                                                                                                                                                                                                                                                    | .954                                                      |       |      |   |  |      |   |  |      |   |  |      |   |  |      |   |  |      |   |  |      |                                                                                      |                                                                                                                                                                                                                                                                                                                                                                                            |   |      |                                                           |   |      |   |      |   |      |   |      |   |      |   |      |
| 1                             | to 3                                                                                                                                                                                                                                                                                                                                                               | Gives you the other unequal Parts of the line of Numbers. |       |      |   |  |      |   |  |      |   |  |      |   |  |      |   |  |      |   |  |      |                                                                                      |                                                                                                                                                                                                                                                                                                                                                                                            |   |      |                                                           |   |      |   |      |   |      |   |      |   |      |   |      |
| 1                             | to 4                                                                                                                                                                                                                                                                                                                                                               |                                                           |       |      |   |  |      |   |  |      |   |  |      |   |  |      |   |  |      |   |  |      |                                                                                      |                                                                                                                                                                                                                                                                                                                                                                                            |   |      |                                                           |   |      |   |      |   |      |   |      |   |      |   |      |
| 1                             | to 5                                                                                                                                                                                                                                                                                                                                                               |                                                           |       |      |   |  |      |   |  |      |   |  |      |   |  |      |   |  |      |   |  |      |                                                                                      |                                                                                                                                                                                                                                                                                                                                                                                            |   |      |                                                           |   |      |   |      |   |      |   |      |   |      |   |      |
| 1                             | to 6                                                                                                                                                                                                                                                                                                                                                               |                                                           |       |      |   |  |      |   |  |      |   |  |      |   |  |      |   |  |      |   |  |      |                                                                                      |                                                                                                                                                                                                                                                                                                                                                                                            |   |      |                                                           |   |      |   |      |   |      |   |      |   |      |   |      |
| 1                             | to 7                                                                                                                                                                                                                                                                                                                                                               |                                                           |       |      |   |  |      |   |  |      |   |  |      |   |  |      |   |  |      |   |  |      |                                                                                      |                                                                                                                                                                                                                                                                                                                                                                                            |   |      |                                                           |   |      |   |      |   |      |   |      |   |      |   |      |
| 1                             | to 8                                                                                                                                                                                                                                                                                                                                                               |                                                           |       |      |   |  |      |   |  |      |   |  |      |   |  |      |   |  |      |   |  |      |                                                                                      |                                                                                                                                                                                                                                                                                                                                                                                            |   |      |                                                           |   |      |   |      |   |      |   |      |   |      |   |      |
| 1                             | to 9                                                                                                                                                                                                                                                                                                                                                               |                                                           |       |      |   |  |      |   |  |      |   |  |      |   |  |      |   |  |      |   |  |      |                                                                                      |                                                                                                                                                                                                                                                                                                                                                                                            |   |      |                                                           |   |      |   |      |   |      |   |      |   |      |   |      |

Then for the Sub-divisions or smaller unequal Parts; I proceed in the very same manner 'till I have set them all off from 1 (the beginning of the line of Numbers): As.

The

|               |           |                         |                         |
|---------------|-----------|-------------------------|-------------------------|
| The Logarithm | .041      | 1 to 1.1                | On the Line of Numbers. |
|               | .079      | 1 to 1.2                |                         |
|               | .113      | 1 to 1.3                |                         |
|               | .146      | Taken from the 1 to 1.4 |                         |
|               | .176      | Line of equal 1 to 1.5  |                         |
|               | .204      | Parts, extends 1 to 1.6 |                         |
|               | .230      | from 1 to 1.7           |                         |
|               | .255      | 1 to 1.8                |                         |
|               | .278      | 1 to 1.9                |                         |
|               | .322      | 1 to 2.1                |                         |
|               | .342, &c. | 1 to 2.2, &c.           |                         |

I have been more particular in this Affair than has been shewed before, because I find most Persons, and some even the Instrument-Makers, ignorant, how this very useful line is made originally to any Radius, which depends on the Magnitude of the equal Parts.

But *Note*, That the line of Numbers consisteth of two such lines, as (1, 10; on *Plate A. Fig. 10.*) next the *Fig. 7.* (or Numbers) and so from 10 is numbered 20, 30, &c. to 100 on the 2d Part, which is made 1 line with the first, and must so be made by the Artist.

*To make a Line of equal Parts.*

To make the line (0, 1000, *Fig. 10.*) imaginarily divided into 1000 Parts, or actually into 100.

1<sup>st</sup>, I divide it into 5 with my Compasses.  
 2<sup>dly</sup>, I divide each 5th into 2, which makes the whole into 10.  
 3<sup>dly</sup>, I divide 1 tenth Part into 10 (as 0, 100).  
 4<sup>thly</sup>, I take (0, 100) between my Compasses, and setting 1 Foot in (i), the other extends to (i), or a 10th of the next 10th, and so forward divides the whole line into 100, and each 100 must be supposed 10, in taking off the 1000th Parts to make the line of Numbers by, or in other Uses.

The line of half Tangents, is the line (c m, *Fig. 8.*) made by laying a Ruler to (d), and the several Degrees in the Quadrant (e m), and is used in measuring such straight lines as pass through the Center of a Circle, and do represent Circles, though they appear Right lines, as (c m), (c e), (c d), &c. The Use of the line of equal Parts has been fully shewn before in measuring Right lines, and

and there is nothing more falls within the Compass of Lineal or Geometrical Arithmetic; which will be of great Use to such as intend to study Geometry, to which I recommend the reading of *Euclid*, lib. 2. I shall therefore proceed to



## CH A P. IX.

### INSTRUMENTAL ARITHMETIC.

**I**NSTRUMENTAL Arithmetic is that Part which is performed by Instruments, accommodated with Figures or Numbers for that purpose. Of this kind, that of *Neper's Bones* justly claims the first Consideration, as being the most plain and easy Instrument for all Operations, whether in Multiplication, Division, or Extraction of Roots.

*A Description of Neper's Bones, (See Fig. 1. Plate B.)* As to their Matter, they are sometimes made of Ivory, and then called *Bones*, sometimes of Box-Wood, and then called *Virgulæ*, or *little Rods*; for as to their *Magnitude and Form*, they may be made as large as you please: But the common Sort are each Rod about 2.5 Inches long, and .32 Inches broad, and are in Figure what Geometricians call a *Parallelopipedon*, which is a long Square solid, bounded with 6 Superficies, (or like 7, 8 &c Dice joined exactly one to another.)

*The Number of these Rods in a Set are 10*, that is, the 9 Digits and Cypher, besides the *Index-Bone* which is fixed to the Box, and hath only one Face divided into 9 equal Parts, whereon the 9 Digits are placed as you see *Fig. 1. Plate B.* But the other 10 have 4 Faces each, whereon are placed the 9 Digits at the top, and an Arithmetical Progression of those Digits added as 2, 4, 6; or 3, 9, 12, &c. And whatever the first Figure of any Rod is, that of the opposite Side is the Complement that makes it up 9. And the 10 times 4 Faces or Sides being 40, contain 4 of each of the 9 Digits, and their Series's Arithmetical, and 4 more Faces are of Cyphers.

*How to add the Figures on the Rods. See Fig. 12. Plate A.*

To reduce any line of Figures on the Rods which stand against any Digit on the Index-Face to a true legible Number, you must



must begin at the right hand: Thus to add the Digits standing against 6 in the Set, *Fig. 1. Plate B.* You see there is nothing to do but to add the two Digits together, which stand between the same Slant or Diagonal lines, as 4 is 4, then 5 and 8 is 13; 1 and 4 is 5, and 2 is 7; 4 and 6 is 10; 1 and 3 is 4; 3 and 4 is 7; 2 and 8 is 10; 1 and 1 is 2, and 2 is 4; 1 and 6 is 7: So that six times 123456789, is found 740740734.

SECT. II. *To multiply by Neper's Rods or Bones.*

*Quest. 1.* What is the Product of 34729 by 23795; See *Fig. 13. Plate A.*

First, tabulate the Multiplicand, (or either Factor, but the larger is most usual;) that is, take so many Rods out of your Set, that the first Figures (or those at the upper End of each Rod) will compose your Multiplicand in order, as you see done, *Fig. 13. of Plate A.*

Then looking for the Figures of your Multiplier severally in the Index-Rod next the left hand, you find against each its respective Product in the Multiplicand, as *Fig. 13. aforesaid.*

*Quest. 2.* What is the Product of 3040999 by 2341?

In this Example there being 3 Nines and 2 Cyphers, on the opposite Sides, 3040999 = the Multiplicand.

Multipled by 1 = 3040999  
by 4 = 12163996  
by 3 = 9122997  
by 2 = 6081998

7118978659 = Product.

not another Set, you may easily supply the Defect by observing the Product of each Figure (as you go on) in the Multiplier, and those other Nines, and so making one of those extraordinary: Thus in Multiplying by the 4 there (as *Fig. 14.*) and if there were another Rod for the third Nine in the Multiplier, I can easily see there would be more (as *Fig. 15. Plate A.*) which therefore, I suppose, and so put down 6, and then two Nines instead of the one Nine in the Rods; so when I multiply by the 3, I find in the Tabuler toward the right hand against 3, (see *Fig. 16. Plate A.*) But I know if I had another Rod of the

same Digit 9, there would have been another, as *Fig. 17. Plate A.* which supposing there, it would make 2997, that is, a 9 more than I find in the *Tabulet*, and so the rest.

### SECT. III. Division by Neper's Bones or Rods.

What is the Quote of 23734.691084 divided by 253.24?

An Example of the whole Process, and of what help you have in dividing from the Rods. See *Fig. 18. Plate A.*

Hence it appears, that the Rods are only useful in exhibiting the Products of the Divisor by each Figure put in the Quotient, which knowing, you proceed in all other Respects as in Division of Integers, *SECT. 5. Chap. I. &c.* first Tabulating the Divisor, and then whatsoever the Quote be, you have each Figure multiplied in the Divisor, although I have only placed in the Index-Rod the Figures that fall in the Quote above, and that in the Order they happen therein, for the more orderly shewing the Nature of the Work, and how it agrees with common Division; so that in doing this Question above, you finding that 227916 is the next Number on the Rods to, and less than, 237346 (the first Part of the Dividend) looking against that in the Index-Rod you find 9; therefore put 9 in the Quote, and deduct as in the Example, and the Remainder (with 9 brought down to it) is 94309; which found in the Rods, or what is next less, I observe 3 to stand against it, which I put in the Quote, and so proceed with the rest as in the Example, which is so plain, that it will serve for all. I have put the Figures of the Quote against their respective Dividuals, which added and placed as above in its proper place, is the general Quotient required 93.7241.

### SECT. IV. The Use of Neper's Bones or Rods in Extraction of the Square Root.

What is the Square Root of 54756? (See *Fig. 3. Plate B.*)

*Rule.*] Having pointed the Square Number given, as taught in intire Numbers; by the three Points, it appears, there will be Hundreds, Tens, and Units in the Root.

2dly, Having put the Rod of the Square of the 9 Digits next towards the right hand of the Index-Rod, you'll find that against 4, the

|       |        |
|-------|--------|
| 54756 | (234   |
| 4     |        |
| 147   |        |
| 129   |        |
| 1856  |        |
| 1856  |        |
| 0     | Refts. |

next

next Square to 5 (the first pointed towards the left hand) there stands its Root upon the said fixed Rod, viz. 2; therefore put 2 in the Root, and deduct the said Square 4 from 5, and there rests 1.

3dly, Having brought as usual the next Branch down, it makes 147. Then take a Rod, whose uppermost Number is the double of the Figure last put in that Root (here 4); and place the Rod of Squares to the right hand, that (as you see Fig. 3. Plate B. in the middle Part over 30) then look in the two Rods next the right hand for 147, and the next less is 129, right against which on the Index-Rod stands 3; therefore put 3 in the Root, and deduct 129 from 147, and the Remainder is 18.

4thly, Having placed the next two Figures (or last Binary of the Square given) toward the right hand of 18, they make 1856. Then place two Rods (between the Rods of the Index, and that of Squares) whose uppermost Digits are the double of the 2 in the Root (as you see done in Fig. 3. aforesaid next the right hand) and look in the 3 Rods next that hand for your 1856, which you find just, and towards the left hand thereof, on the Index-Rod stands 4; therefore put 4 in the Root, and deducting, you see nothing remains, and that 234 is the Root required. *This I have made much more plain than has been done before, by Fig. 3. which shews the Position of the Rods upon each Digit put in the Root, and doubt not but it is sufficiently intelligible.*

But Note, That if the double of the Figure put in the Root be more than 9, you must put two Rods next the Index, which begin with the Digits of that double; and three Rods put when the double of two places is above 99. Thus, if 9 is the first in the Root, the double is 18; so I put two Rods between the Index and the Rod of Squares, beginning with 1 and 8. So also, if 99 were the two first in the Root, the double is 198; I put therefore three Rods, beginning with 1, 9, and 8, between the Index-Rod and that of Squares, &c.

For a farther Exemplification; extract the Square Root by the Bones of 97535376. See the Work in the Margin.



1<sup>st</sup>, Here you find 81 the next less Square to 97, and against it is 9 in the fixed Rod; so you put 9 in the Root, and deducting 81, there rests 16, and the next Branch or Period is 1653.

2<sup>dly</sup>, Doubling 9 is 18, which two Rods placed next that fixed; and that of Squares next the right hand; the top Digits will stand in this Order, see Fig. 19. Plate A. and in the Progression lower you'll find against 1504, the next less, to 1653, stands 8, which put in the Root, and deduct 1504, there rests 149; which with the next Period makes 14953.

3<sup>dly</sup>, Doubling 98 is 196; which three Rods placed next that fixed, and that of Squares towards the right, the first Figures on the Tabulet Rods will stand thus, see Fig. 20. Plate A. And in the Progression thereof lower you'll find 13769; against which is 7; which put in the Root, and the 13769 deducted from 14953, there rests 1184; to which bringing down the next Branch, makes 118476.

4<sup>thly</sup>, Doubling 987 in the Root, makes 1974; therefore taking 6 out of the last Tablet, and putting the Rods beginning with 7 and 4, the top of the six Rods will stand thus, see Fig. 21. And in the Progression lower you'll find on the 5 Rods next the right hand 118476, against which towards the left hand upon the fixed Rod is 6: So there Rests (0), and the Operation is finished.

#### SECT. V. The Use of Neper's Bones in the Extraction of the Cube Root.

The Rods make this (which is reckoned the most difficult Rule) very easy, so soon as you know how to place your *Virgulæ*, which I shall therefore illustrate by an Example of their Position at every Figure put in the Root, as I have done in the preceeding Parts of Arithmetic by the Rods.

The Cube Number given must be prepared for Extraction by pointing over the Units, and every third afterwards, as is taught in extracting Roots of Vulgar intire Numbers, &c. for Example:

What is the Cube Root of 182284628? See Figures 22, 23, 24.

Plate A.

1st, The first Figure in the Root is found by placing your Cube-rod next the Index-rod; for looking on the former for 182 (the first Point next the-left) you find the next less to be 125.

182284628 (567  
125  
57

And right against that 125, stands 5 on the fixed Rod, which I therefore put in the Root, and subtracting 125 from 182, there Rests 57. To which bringing down the next Point or Branch, it makes 57284.

45216  
540  
50616  
6668628

Then for a Divisor to divide the 57284: Take treble the Square of the 5 (in the Root) which is 75, see Fig. 23. Plate A. And having tabulated that, and the Rod of Cubes, as you see, I find on those three Rods 45216 the next less to be taken, against which in the Index is 6, which I put in the Root: To which 45216 I add 540 (which is the Square of 6 in the Root multiplied in three times the 5 there) and the Sum is 50616; which subtracted from the Dividend 57284, there rests 6668, to which I bring down 628 = the third Branch.

9408) 6668628  
6585943  
8232  
6668263  
365

And it makes the last Dividend 6668628; and treble the Square of the Root (56) makes 9408 (a Divisor) which Tabulated with the Rod of Cubes, as you see Fig. 24. Plate A. I look upon the Rods, and find 6585943 to be the next less Number than the 6668628;

6668628; and against it is 7 in the Index, which 7 I put in The Root: Then for the 8232 to be added to the Number next above it, I multiply 3 times 56 (in the Root) by the Square of 7 there; and the Sum is 6668263, which I take from the last Dividend 6668628, and the Remainder is 365; to which, if you desire Decimal Places, you may put three Cyphers towards the Right, and so proceed as before. I have made this Example so demonstrative, that I shall need to give no more, for by the very same Method you may do any other.

*Note*, That in the Division (by 75) the next of all on the Rods, and less than the Dividend 57284 is 52843, against which stands 7 on the Index, but then had you put 7 in the Root instead of 6, you would have found a Number too great to be deducted from the said Dividend or Resolvend 57284; and therefore I take still the next less Number which stands against 6.

*2d Note*, That the 540 and the 8232 are placed 1 Degree towards the left hand, because they would fall so if put under the Tens places of the Cube of the last Figure put in the Root; as is done in the common way of Extraction.

*Note also*, That if you have not the Rods of Squares and Cubes, you may do without, either by placing in the room a Rod, whose first Number is the same with that which falls in the Root (which you may know by comparing the Figures on the Rods toward the left hand with those of the Dividend) or by making them (for present Use) of Paper, according to the many Examples above given of them, till you get the Rods of Squares and Cubes.

#### SECT. VI. *Notation and Numeration of the Diagonal Circular Instrument, (See Plate B.)*

This Instrument was by me contrived chiefly for Reduction of Money, Weight, or Measure, into Decimals; or for finding with Ease and Speed the Value of those Fractions, and for Multiplication, Division, and Evolution. It is marked on *Plate B.* with *Fig. 5.* and consisteth of seven Concentric Circles (besides the two outermost, which are Lines of Numbers) as

1. The outward or largest Circle (save two) is of equal Parts, to which all the rest that are lesser Circles refer in their Use; it is first divided into 10 Parts, each of which being subdivided into 10, makes a 100 Parts; and each 100 Part again divided into 10 by the



the Diagonal Line (1000,  $\sigma$ ) &c. divides the whole Circle into 1000 equal Parts. So that for Example: To find in that Circle 767 thousandth Parts, you have 760 in the Arch of that Circle, and 7 in the Diagonal Line, telling the Parts upward from 760.

2. The second Circle is of *English* Coin, the whole representing 1  $\ell$ . divided into 20  $s$ . each Shilling in 12 Pence, and each Penny by the Diagonal into 4 Farthings. So that to set off 13  $s$ . 7  $d$ .  $\frac{1}{4}$   $q$ . look for 13  $s$ . 7  $d$ . in the Ambit of the Circle, and then from 7  $d$ . telling upwards, you see 1 Farthing in the Diagonal.

3. The next inner Circle (which is the third from those of Numbers) is of Beer Measure, being one Barrel divided into 4 Firkins, each Firkin into 9 Gallons, and each Gallon into Quarts and Pints as you see.

4. The next inward is a Barrel of Ale divided into 4 Firkins, those into each 8 Gallons, and those again into Quarts and Pints.

5. The fifth Circle inward from those of Logarithms is the Pound Averdupois divided into 16 Ounces, and those each Ounce into 16 Drams.

6. The next inward Circle is the Hundred Averdupois divided into four Quarters, and each Quarter into 28  $lb$ . And

7. The innermost Circle is Foot Measure, being 1 Foot divided into 12 Inches, and each Inch into 4 Quarters.

And all these Divisions and Subdivisions are actually done, and not imagined only as some Scales are, which are of little or no Use. And any Quantity in the seven inner Circles is easily pointed to by the same Rule as given for the two first; from those of Numbers, only in these three are no Diagonals, there being no need of them, the Parts being sufficiently expressed without.

8. The two outermost Circles are Lines of Numbers, whose Uses follow after 10th.

*Reduction of Decimals by the Circular Instrument.*

*Quest. 1.]* What Decimal of a Pound is 13  $s$ . 6  $d$ .  $\frac{1}{4}$   $q$ ?

*Rule.]* Lay a Ruler from the Center to 13  $s$ . 6  $d$ .  $\frac{1}{4}$   $q$ . in the second Circle, and that will cut in the Diagonal Circle .676, which is the Answer.

*Quest. 2.]* What is the Value of .876543 of a Pound Sterling?

*Rule.]* Lay a Ruler to the Center, and to 876, and half in the Diagonal Circle. (or equal Parts) and you'll find it to cut in the Circle of Money 17  $s$ . 6  $d$ .  $\frac{1}{4}$   $q$ .

*Quest.*

*Quest. 3.]* What Decimal of a Barrel of Beer is 2 Firkins, 7 Gallons, 3 Quarts, and a Pint?

*Rule.]* Lay the Ruler to the Center, and the Quantity given in the third Circle from the Line of Numbers, and it will shew in the first Circle the Decimal required to be .7187, or apparently more than .718 and  $\frac{1}{2}$ .

*Quest. 4.]* What is the Value of .876543 of a Barrel of Ale?

*Rule.]* Lay a Ruler to the Center, and the Decimal given in the Diagonal Circle (I mean the four first Figures next the Point) and it will cut in the fourth Circle 3 Firkins, 4 Gallons, and about half a Pint.

*Quest. 5.]* In 3 Firkins, 5 Gallons, 3 Quarts of Ale, how much Beer?

*Rule.]* Lay a Ruler from the Center to the Quantity given in the Circle of Ale Measure, and it will give in the Circle of Beer Measure 3 Firkins, 6 Gallons, 2 Quarts Beer. And in like manner Beer is reduced to Ale Measure.

*Quest. 6.]* What Decimal of a Pound is 13 Ounces 11 Drams?

*Rule.]* Lay a Ruler from the Center to 13 Ounces and 11 Drams, and it will cut in the Diagonal Circle .855.

*Quest. 7.]* What is the Value of .876543 of a Pound Averdupois?

*Rule.]* By laying a Ruler to the Center, and the Decimal given, it will shew the Value in the fifth Circle from those of Numbers to be 14 Ounces and near half a Dram.

*Quest. 8.]* What Decimal of a hundred Weight is 2 Quarters 21 Pound?

*Rule.]* Lay a Ruler to the Center, and the given Quantity in the sixth Circle, and it will cut in the Diagonal Circle .687, or .6875.

*Quest. 9.]* What is the Value of .876543 of an hundred Weight?

*Rule.]* Laying a Ruler as in the 7th Question, it will cut in the 6th Circle 3 Quarters 14 Pound.

*Quest. 10.]* What Decimal of a Foot is 10 Inches 3 Quarters?

*Rule.]* Laying a Ruler to the Center, and the Quantity given in the 7th Circle, it will cut in the equal Parts .8958, or visibly .895 and more.

And a Line of Numbers being delineated without the largest or diagonal Circle (as by Rules given for drawing that Line, setting the Parts off from the Center) and another to turn round within it; you may by them do most Questions in Multiplication, Division, and Extraction of Roots very accurately, as by Rules in Chap. VIII. Or thus, Suppose I would do the second Question in

See

*Sect. 4. of Chap. VIII. i. e.* If 12 C. of Sugar cost  $l. 25 : 4 : 0$  what will 1 Ton cost?

Turn 12 in the inner Circle, to 20 in the outer Circle; and then against 25.2 in the inner, will stand 42 in the outer.

Or in Case you have only a Cut of this Instrument pasted on a strong Board that will not warp; you may do these Questions by the Compasses and one of the Lines (or the outmost Circle); as if 20 require 30, what will 50 require? In the second half Circle, the Extent of a pair of Compasses from 20 to 30, will extend from 50 to 75, the fourth Proportional or Answer.

The compound Interest of any Sum for Years, is found by the two Lines of Numbers: as that of  $l. 650$  for 5 Years at 5 per Cent. the fifth Power of 1.05 in the Amount of  $l. 1$  for five Years, which multiplied by  $l. 650$  gives  $l. 829 : 11 : 0$  the Answer: For

Turn 1 in the inner Circle, to 1.05 in the outer Circle, then against  $\left. \begin{array}{l} 1.05 \\ 1.1 \\ 1.16 \\ 1.22 \end{array} \right\}$  In the inner Circle stands  $\left. \begin{array}{l} 1.1 \\ 1.16 \\ 1.22 \\ 1.28 \end{array} \right\}$  In the outer Circle. = the Amount of  $1 l.$  for 5 Years.

And turning 1 in the inner Circle to 1.28 in the innermost Circle; against 650 (the Principal) in the inner Circle, you will find 829½ in the outer Circle, the Amount required.

Many other Questions might be proposed, and Uses shewn of this Instrument; but I leave it to the Reader to find out, of his own accord, to exercise his Parts.

*The End of Instrumental Arithmetic.*

## CHAP. X.

### ALGEBRAICAL ARITHMETIC.

I. **A**LGEBRA being a Word derived from the Arabic *Al-giabr*, makes it probable, that the *Arabs* were the most ancient Professors, and great Proficients in this Art. Although some have ascribed the first Invention to the *Chinese*, *Indians*, or *Persians*, some to the *Grecians*, and that it was first propagated by *Plato*; though it was kept a Secret, from the Principle, that

A a a

the



the divulging a Truth was prophaning of it. The first Treatise of this Art is said to be wrote by *Diophantus*, extant in *Latin* and *Greek*; but the first *European* Author, was, most probably, *Lucas de Burgo*, a Friar, who publish'd a Book of this Subject in *Italian*, printed at *Venice*, Anno 1494; yet *Scipio, Cardan*, and others about their Time, first shewed the Solution of Cubical Equations. The famous *Vieta* was the Inventor of *Specious Algebra* about 1590, which was improv'd by *Oughtred*; and the Methods of solving Quadratics, by changing the second Term, and by compleating of the Square, were, with many other excellent Improvements, introduced by the celebrated *Harriot*, who was contemporary with *Oughtred*, and flourished about the Year 1630. And so much may suffice for the Invention and Improvement of this great Art.

II. Some define *Algebra* to be, *The Art of Restitution and Comparison*. It is sometimes called, *The Analitick*, or *the Art of Enquiring into the fundamental Nature and Reason of Propositions, and of composing and solving Equations, and discovering Canons and Theorems accordingly*.

III. *Algebra* is either *Numeral* or *Literal*; the former, when a Question is solved by the Numbers themselves; and *Literal*, when done by Symbols, Letters, or Species, representing the Numbers in a Question.

What I have proposed for this first Section, I shall give the Reader in this Method.

1. Shew him the Characters and Signs used in *Algebra*.
2. Say what is necessary of the Symbols or Letters, representing Numbers in this Case.
3. Of the Notation, Numeration, or way of reading Compound *Quantity*.
4. Of the Definitions and Explanations of Terms peculiar to, and used in *Algebra*.

IV. The learned *Algebraists*, for the more commodious and expeditious Way of Procefs in solving Problems, have very wisely agreed upon certain Characters, whereby Quantities, or the Powers of them, are expressed and connected, and the Mind of the Artist is briefly and methodically explained, which would appear inconveniently confused, should Words at length be used: Thus,

Signs

| Signs or Characters. | Their Signification.                                       | Literal Examples ;                  | Numeral Examples ;                  |
|----------------------|------------------------------------------------------------|-------------------------------------|-------------------------------------|
| +                    | More.                                                      | as,<br>$b + a.$                     | or,<br>$3 + 2 = 5.$                 |
| -                    | Less.                                                      | $b - a.$                            | $3 - 2 = 1.$                        |
| =                    | Equal to                                                   | $b + a = c.$                        | $3 + 2 = 5.$                        |
| +                    | { Transpos'd to the Right.                                 | $a + b = c, \text{ is } a = c - b.$ | $2 + 3 = 5, \text{ is } 2 = 5 - 3.$ |
| x                    | Multiply'd by                                              | $b \times a = ba.$                  | $3 \times 2 = 6.$                   |
| +                    | Transposed to the Left.                                    | $b = c - a, \text{ is } a + b = c.$ | $3 = 5 - 2, \text{ is } 2 + 3 = 5.$ |
| ÷                    | Divided by                                                 | $b \div a = \frac{b}{a}$            | $3 \div 2 = \frac{3}{2} = 1.5.$     |
| ∞                    | Difference between                                         | $b \infty a$                        | $3 \infty 2 = 1.$                   |
| ⊥ (or ⊂)             | Lesser than                                                | $a \perp b.$                        | $2 \subset 3.$                      |
| ⊃ (or ⊇)             | Greater than                                               | $b \supset a.$                      | $3 \supset 2.$                      |
| ::                   | { So is (when in the middle of 4 Geometrical Proportions.) | $b : a :: c : \frac{ac}{b}$         | $3 : 2 :: 5 : \frac{10}{3}.$        |
| ∴                    | { Continual Geometrical Proportions.                       | $a . b . d . b . r . \div \div$     | $1 . 3 . 9 . 27 . 81 \div \div$     |
| √                    | The Square Root of                                         | $\sqrt{bb} = b.$                    | $\sqrt{9} = 3.$                     |
| $\sqrt[3]{}$         | The Cube Root of                                           | $\sqrt[3]{aaa} = a.$                | $\sqrt[3]{8} = 2.$                  |
| $\sqrt[4]{}$         | The Biquadrat Root of                                      | $\sqrt[4]{aaaa} = a.$               | $\sqrt[4]{16} = 2.$                 |
| $\sqrt[5]{}$         | { The Root of the 5th Power, or Surfsolid Root.            | $\sqrt[5]{bbbbbb} = b.$             | $\sqrt[5]{243} = 3. \text{ for}$    |
| aa (or $a^2$ )       | The Square of a                                            | $a^2 = aa.$                         | $2^2 = 4.$                          |
| aaa (or $a^3$ )      | The Cube of a                                              | $a^3 = aaa.$                        | $2^3 = 8.$                          |
| aaaa (or $a^4$ )     | The Biquadrat of a                                         | $a^4 = aaaa.$                       | $2^4 = 16.$                         |
| aaaaa (or $a^5$ )    | { The 5th Power, or Surfsolid of a                         | $a^5 = aaaaa.$                      | $2^5 = 32.$                         |

Other Characters less used are;  $a\sqrt{b}$  used in Surds, signifies  $a$  is multiplied in the Square Root of  $b$ .

(⊥) Right Angle. (⊠) Square. (<) Angle. (Δ) Triangle. ( $\frac{1}{\square}$ ) Equilateral. (⊗) Involution, obviating that such a Quantity is squared; and (ω) Evolution (or that the Root is extracted).

V. *The Species or Symbols* used in *Algebra*, are Letters, which are placed in an Operation to represent the Numbers given in any Question; and these are commonly Consonants, as *b, c, d, &c.* But the Letters put in an Operation by *Algebra* to stand for the unknown Numbers (or those sought for) are usually Vowels. And I shall observe this way in the following Work, but with those Limitations which I think are necessary.

1. I shall put such Consonants to stand for the known Numbers as are most expressive of the Things to which those Numbers relate; and the like for unknown, or if there is but one unknown Number sought in a Question, I shall always put *u* for it.

2. I shall not, as has been usual, put the Letters in a Product promiscuously, and without Order, but shall place them as they have Order or Priority in the Alphabet; whereby one and the same Quantity is easier seen in contracting manifold compound Quantities, and several other good Uses may be made thereof.

VI. *Of the Notation, Numeration, or way of reading Algebraical Quantities*, it may be observed from the foregoing Signs.

+ 1. Such as are connected with the Affirmative Sign +, as  $a + b + c + d = p$ , are to be read, *a* more *b*, more *c*, more *d*, equal to *p*.

Note, Which *p*, though it hath no Sign before, is (and the like is always to be observed) supposed to have +, as being always affirmative.

— 2. Such Quantities as are connected with the Negative Sign —, as,  $a - b - c$ , are to be read thus; *a* less *b*, less *c*. And as in the first Example, the Numbers signified by *b, c*, and *d*, are to be added together to make the Number represented by *p*: So in this second Example, either the Sum of the Numbers represented by *b* and *c*, are to be deducted from *a*; or else *b* and *c* taken from *a* singly, *b* from *a*, and then *c* from the Remainder.

∞ Or suppose  $b + c - d = a \text{ or } p$ ; that is, the Number represented by *d*, being deducted from the Sum of those represented by *b* and *c*; the Remainder is equal to the difference between the Numbers represented by *p*, and that unknown Number by *a*.

Note, That such Quantities as have no Numbers standing to the left hand, are supposed to have a Unit or 1; thus *b* is 1 *b*, *c* is 1 *c*, &c.

In  $c - b + d - p \text{ or } a$ : Here the Difference between *p* and *a*, is to be deducted from the Remainder, when *b* more *d* is taken from *c*; for 'tis read, *c* less *b* more *d*, less the difference between *p* and *a*; for



for  $a$  being unknown, it is so likewise whether the Excess lieth in  $a$  or  $p$ , therefore the  $\infty$  to represent the Difference is necessary.

3. Such Quantities as have the Sign  $\times$  between them, are commonly expressed without it, and the Letters put as in a Word; thus  $a \times b$ , is,  $a$  multiplied in  $b$ , or  $ab$ ;  $c \times b \times p$ , is  $c$  multiplied in  $b$ , and that Product in  $p$ , but is mostly put thus  $bcp$ ; and  $b \times$  in  $b$  is  $bb$ ,  $c \times c \times d \times d$  is  $ccdd$ , or  $c$  squared in  $d$  squared.

Or  $a \infty c + b \times p \times r$  is  $a \infty c + bpr$ , that is the Product of  $b, p$ , and  $r$  added to the difference between  $a$  and  $c$ .

4. So all Quantities having the Sign  $\div$  may be, and are most  $\div$  commonly expressed by way of Fraction, the Quantity to be divided being placed over the other thus,  $a - b \div c + p$  will stand  $\frac{a-b}{c+p}$ ; i. e.  $a$  less  $b$ , divided by  $c$  more  $p$ .

5. Surd Quantities are expressed with their proper Marks before them, as  $\sqrt{g-a}$  is the Square Root of the Remainder, when  $\sqrt{a}$  is deducted from  $g$ . So also  $\sqrt[3]{p+q-r}$  is the Cube Root of the Remainder, when  $r$  is deducted from the Sum of  $p$  and  $q$ . Lastly,  $\sqrt[4]{b-c+\frac{d}{q}}$ , which is the biquadrat Root of the Sum when  $c$  is taken from  $b$ , and that Remainder added to the Quote of  $d$ , divided by  $q$ .

*But some Examples admit of a double Entendre.*

As under the 2d of this 6th Head, wherein  $c - \overline{b+d}$ : If it were not for the Line over  $b$  and  $d$ , instead of deducting  $b+d$  from  $c$ ,  $b$  would only be shewed to be deducted from the Sum of  $c$  and  $d$ .

So also in the 4th above, if there were a Line over  $b \div c + p$ , it would stand thus  $a - \overline{b \div c + p}$ , which without the Line (supposing  $a=40$ ,  $b=20$ ,  $c=6$ , and  $p=4$ ) would be but 2; whereas with the Line 'tis = 38, for

|                                  |        |                                                                                                    |
|----------------------------------|--------|----------------------------------------------------------------------------------------------------|
| $a - b = 20$                     | is = 2 | But (with the Line) $a = 40$<br>Less $(b)$ }<br>Divided by $c + p$ } = 2<br>Which deducted is = 38 |
| And that divided by $c + p = 10$ |        |                                                                                                    |

so there is 36 difference. According to the 5th Example above, there is nothing more frequent in solving Quadratic Equations, than to have occasion in the Canon or Answer to use Lines to distinguish what Quantities are to have their Roots extracted, and which.

which are not, as  $u = \sqrt{bc + p} - d$ ; that is, ( $u$ ) is equal to the Remainder, when ( $d$ ) is deducted from the Square Root of the Sum of ( $p$ ), and ( $b$ ) multiplied in ( $c$ ), as if  $b$  be 16.

The Numbers to which Equation discover, from the Letters, the Value of ( $u$ ), and so you must unfold any Canon of Species.

But had not the Line been drawn over  $bc + p$ , it would have shewed that the Root is to be extracted of  $bc + p - d$ , which would give too much.

$$\begin{array}{r} c=3 \\ \hline \text{Product}=48 \\ \text{More } p=16 \\ \hline \text{Sum}=64 \\ \text{Square Root of which is } 8 \\ \text{Less } d=3, \text{ so } (u=5.) \end{array}$$

7. In the new Notation  $\frac{1}{a}$  is often expressed  $a^{-1}$ ;  $\frac{1}{a^2}$  is  $a^{-2}$ ;  $\frac{1}{a^3}$  is  $a^{-3}$ ;  $\frac{1}{a^4}$  is  $a^{-4}$ , &c. And is read 1 divided by  $a$ ; 1 divided by  $a$  squared; 1 by  $a$  cubed, &c.

Also  $a^7 + b^8$ , &c. is the 7th Power of  $a$ , plus, the 8th Power of  $b$ . And  $\frac{1}{a+b}$  is wrote  $\overline{a+b}^{-1}$ , &c.  $a^7 + b^7 \times \frac{1}{a+b}$  is  $a^7 + b^7 \times \overline{a+b}^{-1}$  that is, 1 divided by  $a$  more  $b$ ; and the 7th Power of  $a$  more the 7th Power of  $b$  multiplied in 1 divided by  $a$  more  $b$ .

Likewise  $\frac{1}{a^2} b$ ,  $\frac{1}{a^3} b^2$ , &c. is in the new Notation  $a^{-2} b$ ;  $a^{-3} b^2$ , &c. that is, 1 divided by the Square of  $a$  multiplied in  $b$ ; 1 divided by the Cube of  $a$  multiplied in  $b$  squared.

And  $\frac{a^3 + b^3}{a + b} = a^2 + b^2 \times \frac{1}{a + b} = a^2 + b^2 \times \overline{a + b}^{-1}$ ; i.e. the Sur-solid of  $a$ , more that of  $b$ , divided by  $a$  more  $b$ ; is expressed also  $a^2 + b^2 \times \frac{1}{a + b}$ . Or rather  $a^2 + b^2 \times \overline{a + b}^{-1}$ . And for Proof suppose  $a=10$ , and  $b=20$ ; then is  $a^2 + b^2 = 3300000$ ; and  $a + b$  being  $= 30$ ,  $3300000 \div 30 = 110000$ . And so likewise is  $3300000 \times 1 \div 30$ : or  $3300000 \times .3 \bar{r} = 109999.9 \bar{r}$  or  $110000$ : which plainly proves the Truth of the manner of new Notation.

VII. An Alphabetical Explanation and Definition of most Terms used in *Algebra*, which the Reader may have recourse to as he finds occasion; for having well understood what is above in this Section, he may pass this Head, and proceed to *Addition*, &c.

*Absolute*

*Absolute Quantity, or Absolute Number.*] Is such a known one as possesseth one Side of an Equation, as  $u + cu = b$ , here  $b$  is the Absolute Quantity, whose Value is known, and is free from any Dependence or Power of any other. Or it is 1 marked with the Sign  $+$ , contrary to Negative marked  $-$ .

*Affirmative Quantities.*] Such as have either no Sign before them, or are marked with  $+$ ; so  $a + b$  are both Affirmative Quantities, as by the last.

*Algorism.*] Is the Operations in the several Parts of *Algebra* practically handled.

*Algorithm.*] The Parts of single Arithmetick, as *Numeration, Addition, Subtraction, &c.*

*Arithmetic of Infinites.*] The same as infinite Series, or converging Series.

*Adfected Equations.*] When in a Quadratic Equation, (which see Sect. 10. in this Chap.) the unknown Quantity is in one Term, and its Square in another.

*Analogies.*] The same as Proportions, as the Analogy of these four Quantities,  $a : b :: c : d$ , &c. and because the Rectangle of the two Extremes is equal to that of the means; therefore  $ad = bc$ , the Analogy being converted to an Equation.

*Approximation.*] See *Converging Series*.

*Arithmetic of Negatives.*] See *Negative*, under *N*.

*Binomial Quantities or Roots.*] Those consisting of 2 Names, Terms, or Members, connected together by the Sign  $+$ , as  $u + d$ , &c.

*Coefficients.*] Those Numbers or known Quantities in a Quadratic, &c. Equation, which are (in the 2d Term commonly) multiplied in the Root of the first or unknown Quantity, as  $uu + 2bu$ , here  $2b$  is the Coefficient: And note, That where there is no Number nor Quantity multiplied in the Root in the 2d Term, then 1 or a Unit is the Coefficient, as  $uu + u$  is  $= uu + 1u$ . See *Compleating the Square*.

*Compound Quantities.*] Are such as are either the Product, Sum, or Difference of two or more single Quantities: Thus  $(up)$  is a Rectangle compounded of  $u$  and  $p$ ,  $u + d$ , or  $u - d$ , the Sum or Difference of those two Quantities  $u$  and  $d$ .

*Compleating a Square.*] This is to square half the Coefficient: As suppose two Members of a Square  $uu + bu$  be given; to find what Quantity must be put to compleat the Square, is to square half the Coefficient or  $\frac{b}{2}$ , which Square is  $\frac{bb}{4}$ , so that  $uu + bu + \frac{bb}{4}$  is a compleat Square: For Proof of which you may multiply

$u +$



$u + \frac{b}{2}$  (which is the Root of that Square) in itself. And if this were a Quadratic Equation, this Square of half the Coefficient must be added to both Parts of the Equation, as admit  $uu + bu = c$ , half the Coefficient being squared and added, will make it  $uu + bu + \frac{bb}{4} = c + \frac{bb}{4}$ .

*Composing an Equation,*] Is the way of proceeding (to resolve Problems) till we make some Quantities in the Question equal to some others; (see *Resolution of Equations*) as if I were asked, what Number that is which being multiplied by 7, and that Product divided by 3, the Quotient will make 1000, abating half the Number sought? Here, to compose this Equation, I put  $u$  for the unknown Number, or that sought; then 7 times that is  $7u$ ; that divided by 3, makes  $\frac{7u}{3}$ . Then putting  $b$  to represent 1000, this Equation is

composed from the Nature of the Question, viz.  $\frac{7u}{3} = b - \frac{u}{2}$ .

(See this Equation resolved under the Word *Resolution*.) And where the Quantities in the Proposition or Problem can be reduced naturally into Analogy, then that Analogy is reduced, or an Equation composed from it: See *Analogy*.

*Construction of an Equation,*] Is to demonstrate Geometrically the Truth of the Equation, and Canon, from Lines or Geometrical Figures laid down for that purpose, as simple and quadratic from the Sides of the Triangles and Arcs of Circles; and cubical from the Parabola and Circle, &c.

*Conversion of Equations,*] Is properly the converting of an Equation into that which is not so, i. e. into Analogy, as if  $ad = bc$ ; then it follows, that as the Quantity  $a$  is to  $c$  or  $b$ , so is the other of them to  $d$ , viz.  $a. b :: c. d$ . See *Analogy*, of which Example this is the Converse, and they ought both to be remember'd. But some call this following, and the like, a Conversion, (but I call it Reduction by Multiplication, of which more under *Reduction of Equations*.) viz.

$\frac{u}{r} + cd = p$ , this by multiplying each Member by the Denominator  $r$ , gives  $u + cdr = pr$ , &c. of any other.

*Converging Series.*] Lines converge, when two right ones proceed nearer and nearer to each other, till they meet in a Point where they

they make an Angle: So in Numbers, it is the orderly approaching nearer and nearer to the Truth (as to the Root of a surd Number or Quantity) by the common Rules. Examples hereof, see *Section* the 13th following: *Note*, Sometimes this Method of Process is called also *Arithmetic of Infinites, Infinite Series, and Approximation*.

*Contraction.*] This in *Algebra* is to leave out all repeated or superfluous Terms, or single Quantities; as  $4ab - 3c + 5c - 3ab$ , this contracted is  $ab + 2c$ ; for  $4ab$ , less  $3ab$ , leaves  $ab$ ; and  $5c$ , less  $3c$ , leaveth  $2c$ . So also  $7uu + 3pu - du - 3pu$  is  $7u - d$ ; for first,  $u$  being in each Member  $1u$ , may be left quite out, and then 'tis  $7u + 3p - d - 3p$ ; secondly,  $+3p$  and  $-3p$  destroy each other: So there rests but  $7u - d$  of the Total, and the like may be observed in *Contraction* of other Quantities.

*Calculus.*] A Term in Fluxions: It is either *Calculus Integralis*, which is the Method of finding a flowing Quantity of any Fluxion given; or else *Calculus Differentialis*; which is the way of finding a Fluxion of any flowing Quantity.

*Canon.*] As it relates to *Trigonometry*, is the Rule of Proportion by which an Angle or sides of a  $\Delta$  is found; the Tables of Artificial Numbers (Logarithms, Sines, and Tangents) are sometimes called *Canons*; but in *Algebra* it is a resolved Equation, being the Result of an Operation, and is the Value of some unknown Quantity in known Species, or Numbers, which being wrote down in Words, contains a plain general Rule for the Solution of any Question of like Nature; which is one of the great Excellencies of *Algebra*.

*Cosick Powers.*] Algebraick Powers of Number or Quantity; for the first, see *Chap. I. Sect. 1.* of this Treatise; whence it follows, that the Powers of any Quantity will stand thus,  $u$  = a Root;  $u^2$  = the Square or second Power;  $u^3$  = the Cube or third Power;  $u^4$  = the Biquadrat or fourth Power of  $u$ , &c. See *Extraction of Roots* in this Chapter, *Sect. 6.*

*Cubic Equations.*] Vide *Sect. 12.* of this Chapter.

*Depression.*] This Word is used in the Solution of Cubic &c Equations, where the Equation is depressed or reduced from one of the third Dimension to a Quadratic; that being one of the ways of resolving Cubical Equations: Or Depression is the reducing of an Equation by Division; as  $puu + duu = cu - du$  by dividing by  $p + d$ , that Equation is reduced to  $u = \frac{c-d}{p+d}$ , first expunging or casting out  $u$ , because found in each Member or Term of the Equation: See this Chapter, *Sect. 5.*

*Discrete Proportionals*] Are Numbers or Quantities, where there is the same Proportion between the third and fourth, as between the first and second; but not the same between the second and third, as in  $u. b :: b. k$ , there is the same Analogy between  $b$  and  $k$ , as between  $u$  and  $b$ , but not the like between  $b$  and  $b$ ; so that 'tis also called *Disjunct Proportion* in Number,  $3. 7 :: 12. 28$ .

*Dimensions*] In Equations, where the first Term is  $uuuu$ , that is one of 4 Dimensions;  $uuu$  is an Equation of 3 Dimensions;  $uu$  of 2, &c.  $u$  being unknown.

*Equation*] An Equation in *Algebra* is, when one or several known or unknown Quantities are made equal to other Quantities upon a due and regular Process of *Composing*, (see that Word,) as if  $u + c - d = rs + tu$ , that is,  $u$  more  $c$  less  $d$  is equal to  $r$  multiplied in  $s + tu$ , which Equation is resolved as under *Resolution*; see also Simple, Quadratic, Cubical and Lateral Equations, in this part.

*Evolution*] Is marked  $\omega$ , (or the Extraction of a Root) and tells us, that the Number or Quantity before which it stands, is the Square Root of some other Step referred to in an Algebraical Operation.

*Exponents*] Are those small Figures placed almost over any Quantity, to shew (or expound) what Power of the said Quantity is expressed. See *Cosic Powers* above. And

*Exponential Quantities*] Is a Term in Fluxions, these being such Quantities, whose Exponents are variable or flowing Quantities.

*Fluxions*] Are defined to be the Doctrine, or Algorithm of the infinitely small Increment or Decrement of variable and indeterminate Quantities: For the best Notion we have of generating a Line, is by the Motion of a Point; of a Surface by the Motion of a Line; and of a Solid by the Motion of a Superficies; Now as this Increase of a Figure by Local Motion, is termed a *flowing Quantity*; so the Velocity of that Increase of the flowing Quantity is called a *Fluxion*; and is distinguish'd from the Character or Symbol of the flowing Quantity by a Point over it, &c. which is a kind of Artificial *Algebra*, but is wrought by Rules peculiar to itself, being a late Invention and a great Improvement.

*Flowing Quantities*] See *Fluxions* above.

*Generating*] In Geometry Points Lines and Superficies generate as under the word *Fluxion*; or in Arithmetic, what is produced by Multiplication or Involution is a *generated Number or Quantity* in *Algebra*.

*Homogeneous*] Is of the same kind: So such surd Roots as have the same radical Sign are *Homogeneous*, as  $\sqrt{ab}$ ,  $\sqrt{ac}$  &c;  $\sqrt[3]{cd}$ ,  $\sqrt[3]{brke}$ ;  $\sqrt[4]{lp}$ ,  $\sqrt[4]{g}$ , &c. See this Word in the *Introduction*, Sect. 1.

*Homologous*.]



*Homologous.*] In a Proportion of four Quantities. If  $u. b :: c. d$ . Here the two Antecedents  $u$  and  $c$  are *Homologous*, as are the two Consequents  $b$  and  $d$ , &c. and  $u$  is *Homologous* to  $c$ , as  $b$  to  $d$ .

*Heterogeneous.*] (Of different kind;) So Surds are *Heterogeneous* when their radical Sines are not the same, as  $\sqrt[3]{bc}$ ;  $\sqrt[4]{tu}$ ,  $\sqrt[5]{rst}$ , &c.

*Infinite Series.*] See *Converging Series* above, and Chap. 10. following, Sect. 13.

*Involution.* Is marked  $\textcircled{Q}$ , and shews that the Quantity before which it stands is the Square of some other which is referred to; the contrary to *Evolution*.

*Ineffable Quantities.*] Surds, whose Roots can only be expressed by the proper radical Signs.

*Irrational Quantities.*] The same as *Ineffable*, last above.

*Lateral Equations.*] Simple Equations which have but one Root, or in figurate Numbers they are those of the first Order, as 1, 2, 3, 4, &c. See Sect. 14 of this Chapter.

*Like Signs.*] Those Quantities in *Algebra* which have the same Sign, as  $+b+c+d$ , or  $-b-c-d$ , &c.

*Like Quantities.*] Such as have the same Symbols or Species, as  $23bcd$ ,  $14bcd$ ,  $2bcd$ , &c. there being the same Number of each Letter.

*Maximis and Minimis*] Is a Term used in *Fluxions*. It is a Method whereby Problems are resolved, which require the greatest or least Quantity attainable in that Case; it is a stable Quantity naturally; and therefore to determine to a *Maximum* or *Minimum* in any flowing Quantity, is to make it a permanent Quantity.

*Members of an Equation.*] Those Quantities contained between every Sign, as  $ab+du-cp=r$ ; here  $ab$ ,  $du$ ,  $cp$ , and  $r$ , are the four Members or Terms of this Equation.

*Moments.*] A Term in *Fluxions* according to some; being such Parts of Quantity as are in a continual Flux, either decreasing or increasing: Or rather, Moments are the beginning (of the generating Principle) of Magnitude, and are themselves no Magnitude conceivable, because infinitely small.

*Multinomial.*] See *Polynomial*.

*Negative Signs.*] See under the last Article (or Head) among the Characters used in *Algebra* for  $(-)$ .

*Negative, or the Arithmetic of Negatives.*] Is that kind wherein Negative Numbers or Quantities are employed, which are known by this mark  $(-)$  before them.

The *Numeration* is no more than only adding the Words [less than nothing] to what is wrote down; as  $-a$ , or  $-7$  is read *a* less than nothing, or 7 less than nothing, and  $-.75$  *cc* is that Decimal less than nothing.

*Addition*] Shews the adding of Negative to Negative or Affirmative, as by the Rule of adding the Quantities of like Signs together, and taking the Difference for the Sum; as the Sum of  $-7 + -3$  is  $-10$ ;  $-7 + 3$  is  $-4$ ; and  $12 + -13 = -1$ .

*Subtraction*] Is done by changing the Sign of the Subtrahend, and then adding the Quantities; as from  $-7$  take  $-3$ , rests  $-4$ ; from  $-7$  take  $3$ , rests  $-10$ ; and if from 12 you take  $-13$ , there rests 25, &c.

*Multiplication*] Is nothing difficult, for you only observe, that if Quantities of like Signs be multiplied, the Sign of the Product is Affirmative or  $+$ ; but if the Factors have unlike Signs, the Sign of the Product is Negative or  $-$ : Thus  $-7$  by  $-3$  is 21, and so  $+7$  by  $+3$ ; but 7 by  $-3$  is  $-21$ , and  $-12$  by 13 is  $-156$ , &c. And the same Rule serveth for

*Division of Negatives*.] As to prove those in Multiplication,  $21 \div -7$  quoteth  $-3$ ;  $-21 \div -3$  quoteth 7; and  $-156 \div 13 = -12$ . All which you'll find farther illustrated, with variety of Examples, *Sect. 3. of Chap. 7.* also *Sections 2, 3, 4, and 5 of this Chap. X.* and the last of my Examples in *Converging Series* near the end of the Book.

*Nome*.] Whence Binomial, Trinomial, and Multi- (or Poly-) nomial, *i.e.* Roots of 2, 3, or more Names; see those Words.

*Numeral Algebra*.] When the Work is performed Algebraically with Figures or Numbers, and not Letters.

*Periodical Degrees, or Parodical*.] Of the Terms of a Cubical *cc* Equation, are when the Exponents of the Powers therein rise or fall gradually in an Arithmetical Ratio, as  $u^4 - 59u^3 + 1154u^2 - 8080u = -12000$ , &c. are regular Biquadratics, having all their Terms in a gradual Order.

*Permanent Quantities*.] In Fluxions such Quantities as are invariable without Motion, or at Rest, and consequently have no Fluxions to them.

*Powers of Quantities*.] See that of Numbers, *Chap. I. Sect. 1.* or Exponents here below; as  $u$  or  $c$ , &c. are Roots or first Powers,  $uu$  or  $cc$  are Squares or second Powers,  $uuu$  or  $ccc$  the Cubes or third Powers.

Power

*Power to raise the Powers of Numbers to that of any other.]* As suppose I raise the Root  $b$  to the same Power as  $cccc$  or  $c^4$ , this is to multiply  $b$  in itself  $ec$ , to bring it to the fourth Power or Biquadrat: Or suppose  $\sqrt[3]{82}$ , were to be multiplied by 2, here 2 must be raised up to the same Power or Cube with the 82, and it will stand either  $2\sqrt[3]{82}$ , or  $\sqrt[3]{8 \times 82} = \sqrt[3]{656}$ ; this will be of Use to observe, when you come to the Doctrine of surd Quantities, *Seet. 7.* of this Chapter.

*Polynomial Roots or Quantities.]* Those which consist of above three Names or Numbers; as  $u + s + t + n$ , &c.

*Positive Quantities,]* The same with Affirmative; which are those that are marked with or follow the Sign  $+$ , or else such as have no Sign before them.

*Quantity.]* In *Algebra*, either that sought for, or those given in a Question, or they are all those Symbols which represent any Number. See *Simple, Compound, Surd, and Variable Quantity.*

*Quadratic Equations]* Are such (or reduceable to such) as have the Square of the unknown Quantity in the first Term, and the Root (or that multiplied in some known Quantity) in the second Term, as  $uu + bu + c = rs$ . See two Sorts, *Seet. the 10th* of this Chapter.

*Rational Quantities.]* Either such whose Roots can be extracted accurately; or such as are already the Root of some Quantities, or which want no Extraction.

*Raise the Powers of Number or Quantity.]* See *Powers.*

*Radical Signs]* Are  $\sqrt{\quad}$  for the Square Root;  $\sqrt[3]{\quad}$  for the Cube;  $\sqrt[4]{\quad}$  for the Biquadrat Root, &c. see *Roots* and the *Powers* in the Article preceeding, of *Algebraical Characters*: And also

*Roots of Quantities.]* Are either (in *Algebra*) put actually down (if of Rational Quantities) or if of Surd Quantities the Root is expressed by putting the proper Radical Sign before the Quantity; thus in the first Case, the Square Root of  $bb$  is  $b$ , of  $cc$  is  $c$ , &c. the Cube Root of  $uuu$  is  $u$ , &c. and in the second Case, the Square Root of  $u + s$  is  $\sqrt{u + s}$ ; the Cube Root of  $u + s$  is  $\sqrt[3]{u + s}$ ; the Biquadrat Root of it is  $\sqrt[4]{u + s}$ , and the like of any other Surd Quantity. See *Binomial, Trinomial, Polynomial, and Residual. Note,* That the Surd  $c\sqrt{d}$ , &c, is  $c$  multiplied in the Square Root of  $d$ , &c.

*Resolution*



*Resolution of (or to solve) Equations.*] After an *Algebraist* has done with the composing an Equation, as under that Word, he resolves it by several Rules requisite thereto, as by *Multiplication*, *Division*, *Addition*, *Subtraction*, *Evolution*, &c. so the Equation composed under the word *Composing* as above, which is  $\frac{7u}{3} = b - \frac{u}{2}$ ; being resolved by multiplying by the Denominators of the Fraction, and adding  $3u$  to each Part of the Equation, dividing by 14 and 3 (or 17) gives the Solution of the Question, or the Canon  $u = \frac{6b}{17}$ .

Now  $b$  we find is = 1000 in the Question, so  $u$  the unknown Quantity is easily found by dividing 6000 by 17. And the Equation under that Word, viz.  $u + c - d = rs + tu$  being resolved, is  $u = \frac{d - c + rs}{t - 1}$ . Particular Rules for resolving Equations, see *Seet.*

the 9th of this Chapter; where it appears, that an Equation is resolved when the unknown Quantity alone possesseth one Side of the Equation as here  $u$  doth.

*Residual Roots or Quantities.*] Such as have a Negative Sign between them, as  $be - d$ , &c. or  $40 - 10 = 30$  the Residue.

*Simple Quantities.*] In a strict Sense are those single Letters which represent the several Numbers in a Problem, given or required: Or some take them in a larger Sense, for such Quantities as are not connected by  $+$  or  $-$  to any other, though they be multiplied in other; of the first sort are  $b$ , or  $c$ , or  $d$ , &c. of the second are  $4b$ ,  $bc$ ,  $drs$ , &c.

*Signs.*] See the *Table of Characters*, Article IV. of this Section; also *Affirmative*, *Negative*, and *Radical*.

*Simple Equations.*] Such as are free from the Involution or Powers of the unknown Number or Quantity.

*Simple Quadratic Equations.*] Such as have (or are reduceable to have) the Square of the unknown Number or Quantity in only one of the Members or Terms, as  $uu = pq$ , that is,  $u = \sqrt{pq}$ , or  $uu + nb + udc$ , which by the Seclusion of  $u$  in each, is  $u = \sqrt{b + dc}$ .

*Series.*] A Rank, Row, or Column of Numbers or Quantities orderly placed. See *Progression and Converging Series*.

*Species.*] In *Algebra*, are the Letters which represent the Numbers in a Question, they are also called *Symbols*; so specious Arithmetic is not Numeral, but Literal *Algebra*.

*Substitution.*]

*Substitution*] Is the putting one Symbol or Quantity into the Progress of Algebraical Operations, in the room of some other: As in the Solution of a Quadratic Equation, by changing the second Term, and as in Fluxions.

*Surd Quantities.*] Such as cannot have the Root expressed, and are therefore marked with their proper *Radical Sign*, which Term see above, and also *Roots of Quantities*.

*Terms*] Of an Equation, are commonly said to be the several Members thereof; (see that Word above) but if in a Quadratic, as  $uu + ub + ud = k + c$ , the Terms are called but three Terms, as  $uu$  the first;  $ub + ud$  (because the known Quantities are multiplied in the same power of the unknown) the second; and  $k + c$  the third Term.

*Transposition*] Is the changing the Places of the Members of an Equation, in order to make the unknown Quantities to possess one Side, and the known the other Side of the Equation; whereby the Knowledge is gained of what Value the Number sought in a Question, really is, by finding what it is equal to. See *Sett.* 9. of this Chapter, for Rules to perform the same.

*Trinomial Roots or Quantities*] Are such as have three Names or Names, as  $u + r + tu$ , &c.

*Vanish, or be Wanting.*] A Quantity is so, when by Reduction or Contraction it is expunged or thrown out of an Equation: Or as in a Cubic, &c. When the highest and lowest Powers of the unknown Quantity are only expressed, as  $x^3 + x$ : here  $x^2$  is wanting.

*Variable Quantities*] Are such as are supposed in continual Motion, whereby they generate Lines, Superficies, and Solids, the same as *Flowing Quantities*.

*Unciæ*] Are the Numbers prefixed to the Quantities, which are the Powers of some Binomial Roots; as in the Square of  $a + b = aa + 2ab + bb$ , the *Unciæ* is 1, 2, 1, for  $aa$  and  $bb$  are 1  $aa$  and 1  $bb$ ; so in the Cube of  $a + b$ , viz.  $a^3 + 3aab + 3abb + bbb$ , the *Unciæ* is 1, 3, 3, 1, &c. for according to Sir Isaac Newton's Rule, the *Unciæ* of any Powers are found (as suppose the third Power or Cube) by this Rule;  $1 \times \frac{3-0}{1} (3 \times \frac{3-1}{2} (3 \times \frac{3-2}{3} (1$ , which in Words is read thus: One multiplied in 3 the Power, less 0, divided by 1 is 3 = the second of the *Unciæ*; 3, the second, multiplied in 3 the Power, less 1, divided by 2, gives 3 = the third of the *Unciæ*; and 3, the third multiplied in the Power 3, less 2, divided by 3, gives 1 = the fourth of the *Unciæ*.

And

And by the same Rule, the *Unciæ* of the Surfolid is 1, 5, 10, 10, 5, 1, for every Quote and the first 1, are the *Unciæ* arising.

This  $(1 \times \frac{5-0}{1}) (5 \times \frac{5-1}{2}) (10 \times \frac{5-2}{3}) (10 \times \frac{5-3}{4}) (5 \times \frac{5-4}{5}) (1; \text{ see also Sect. 6.})$

*Vinculum*] Is a Compound Surd Quantity multiplied by a Fluxion; in which this is a Term.

*Unfold.*] To unfold an Algebraick Canon, expressed Literally, is to compare the known Quantities with the Numbers which they represent; and working with them, as the Canon directs, we discover the Value of the unknown Number, or that sought.

*Wanting.*] See *Vanish*.

Having in this Section laid a plain and copious Foundation; I shall be something briefer in the subsequent Rules, but no farther than is consistent with evident Demonstration.

## SECT. II. Addition of Algebra.

Note  $b = 70$

Example 1.

Literal  $9b = 630$  Numeral

$10b = 700$

$b = 70$

Sum  $20b = 1400$  Sum or Proof.

Note  $c = 35$

Example 2.

Literal  $-12c = -420$  Numeral

$-7c = -245$

$20c = 700$

Sum  $= c = 35$  Sum or Proof.

Compound Quantities.

Note,  $d=4$ ;  $g=16$ ;  $b=6$ .

$3ddb + bbg = 288 + 576$

$ddb - 2bbg = 96 - 1152$

$-5ddb + bbg = -480 + 576$

Sum  $-ddb$

$0 = -96 + 0$  Sum for Proof.

The General Rule for Addition.

That the Quantities of like Signs are to be added together, and where there are different Signs, as in the second and third Examples, you must subtract the Sum under one Sign from that under another, and put the Difference down marked with the same Sign as that Sum, wherein was the Excess: As I deduct  $19c$  from  $20c$  (in the second Example) and the Difference is  $c$  for the Sum required.

And in the third Example, (Col. 1.) the Excess lies in the Negative Sign, for taking  $4ddb$  from  $-5ddb$ , the Remainder is  $-ddb$  (or  $-1ddb$ .)

And



And in the second Series of Quantities in that third Example, the Signs making the Quantities to balance; I therefore put (o) down, all which is proved by the respective Numbers.

And whereas it may be thought strange, that a Sum Total should be  $ddb$ , or 96 less than nothing; it is no more than what falls out in many Cases: For if a Merchant draws on his Factor 480  $l$ . (as the Number is in *Example 3.*) and that Factor has but 288  $l$ . and 96  $l$ . = viz. 384  $l$ . to pay the Bills in his Hands; 'tis plain, when the Factor has paid what is drawn on him, the Merchant will have 96  $l$ . less than o in the Factor's Hands, &c. And this is also plain from the Reason given for adding Affirmative and Negative Indices of Logarithms, *Chap. IV.* foregoing; which they who have considered, the Addition and Substraction of *Algebra* will to them appear evident and reasonable.

And when Numbers are to be added that have different Quantities annexed, there is nothing to do but to put them down in one Line with their Signs between them; as to add  $5ab$  to  $3ad$ , is =  $5ab + 3ad$ ; and  $abc$  to  $rst$  is =  $-rst + abc$ , &c. And the Quantities in the foregoing Examples are truly added by the same way, being afterward contracted as under the word *Contraction* in the last Section foregoing.

## SECT. III. Substraction of Algebra.

The General Rule, &c. see after the Examples.

|                 |            |                  |
|-----------------|------------|------------------|
| Note, $b=70$    | Example 1. | Example 2.       |
| From $20b=1400$ |            | From $c=35$      |
| Literal =       | = Numeral  | Take $-19c=-665$ |
| Take $-11b=770$ |            |                  |
| Refts $9b=630$  |            | Refts $20c=700$  |

Note (as in Addition of Compound Quantities)  $d=4$ ;  $g=16$ , and  $b=6$ .

|                                   |         |            |
|-----------------------------------|---------|------------|
| From $-ddb$                       | = $-96$ | Example 3. |
| Take $-5ddb - 2gbb = -480 - 1152$ |         |            |
| Refts $4ddb + 2gbb = 384 + 1152$  |         |            |

And as a fourth Example; if from  $4ddb + gbb + bb$ , I take  $bb + gg - bb - d$ . The Remainder will stand thus:

$$4ddb + gbb + bb; -bb - gg + bb + d,$$

C c c

For

*For Proof of which.* By the last Section it appears that  $4ddb = 384$   
 From this Sum 5860 taking the  $gbb = 576$   
 Sum of the four Quantities in the  $bb = 4900$   
 Subtrahend (added according to Sum 5860  
 the Rules of Addition,) which is *viz.*  $+bb = 420$  } 676 }  
 $+gg = 256$  }  $= 636$   
 $-bb = 36$  }  $-40$  }  
 by the Margin  $-d = 4$  } 5224.

And so it will be by this Rule.

*The General Rule for Subtraction, i. e.*

*Change the Sign of the Quantities to be subtracted, and then proceed as in Addition.*

So in the first Example,  $20b - 11b = 9b$ , for  $1400 - 770 = 630$ ; which is  $9b$ , or 9 times 70.

And in the second Example,  $c + 19c = 20c$ , or  $35 + 665 = 700$ ; which is 20 times 35.

In the third Example; I change the Sign of  $2gbb$ , (or 1152), and add the same to (0), makes  $+2gbb = +1152$ ; and  $-5ddb$ , or  $-480$  having the Sign changed and added to  $-ddb$ , and  $-96$  gives  $4ddb$ , or  $384 =$  the Remainder as you see. And

In the fourth Example; If the four Signs of the Subtrahend be changed and made as in the Remainder above, that Remainder will prove true and agreeable to the Deduction foregoing, altho' it be done differently and according to the General Rule for Subtraction of Algebraical Integers, as by the Margin appears, where  $5860 - 636 = 5224$ .

*Note,* These Examples prove the Truth of those in Addition.

$$\begin{array}{r} 4ddb = 384 \\ gbb = 576 \\ bb = 4900 \\ \hline \text{Sum} = 5860 \end{array}$$

$$\begin{array}{r} \text{And } -bb = -420 \\ -gg = -256 \\ +bb = +36 \\ +d = +4 \\ \hline \text{Refts} = 5224 \end{array}$$

*The Reason of the General Rule for Subtraction.*

1<sup>st</sup>, Either the Sign of the Subtrahend is  $+$  or  $-$ , if  $+$  as to take 2 from 6, or  $\left. \begin{array}{l} +6 \\ +2 \end{array} \right\}$  to take 2 from 6 is (speaking Algebraically) to say  $6 - 2$  (or 6 less 2) whereby the Sign of the 2 is of Course changed.

2<sup>dly</sup>, And

# SECT. IV. Multiplication of Algebra.

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2dly, And if the Sign of 2 (*cc*) as given to be subtracted were (—), then by what is said of Subtraction of Indices of Logarithms, as well as from the Reason of the Thing itself; if I take (0) from 6, there will remain 6; but if — 2 (2 less than 0) from 6, 'tis the same as to add 2 to 6, so there must remain 8; for by how much the less any Number is that is deducted, by so much the greater is the Remainder; and by how much soever the Number subtracted is less than 0, by so much does it make the Remainder greater than the Sum subtracted from.

## SECT. IV. Multiplication in Algebra.

Note,  $u=25$ ;  $b=12$ ;  $c=20$ ;  $d=4$ ;  $g=5$ ;  $h=10$ .

A General Rule, see after the Examples.

Prop. 1. To multiply simple Quantities by Simple.

|         |                  |                 |         |
|---------|------------------|-----------------|---------|
|         | Multiply $u=25$  | Multiply $d=4$  | Cafe 2. |
| Cafe 1. | by $b=12$        | by $g=5$        |         |
|         | Product $bu=300$ | Product $dg=20$ |         |

|         |                   |                   |         |
|---------|-------------------|-------------------|---------|
|         | Multiply $c=-20$  | Multiply $u=25$   |         |
| Cafe 3. | by $b=10$         | by $c=-20$        | Cafe 4. |
|         | Product $cb=-200$ | Product $cu=-500$ |         |

Prop. 2. To multiply compound Quantities by Simple.

|         |                                   |         |
|---------|-----------------------------------|---------|
| Cafe 1. | Multiply $2bu+d=500+4$            | Cafe 1. |
|         | by $c=20$                         |         |
|         | Product $=2cbu+cd=10000+80=10080$ |         |

|         |                                  |         |
|---------|----------------------------------|---------|
| Cafe 2. | Multiply $cd-b=-80-10$           | Cafe 2. |
|         | by $-c=-20$                      |         |
|         | Product $+ccd+cb=+1600+200=1800$ |         |

|         |                                     |         |
|---------|-------------------------------------|---------|
| Cafe 3. | Multiply $2bu+d=500+4$              | Cafe 3. |
|         | by $-c=-20$                         |         |
|         | Product $-2cbu-cd=-10000-80=-10080$ |         |

Ccc 2

Prop.



*Prop. 3.* To multiply compound Quantities by Compound.

$$\begin{array}{r} \text{Case 1. Multiply } bb+g=100+5=105 \\ \text{by } gg+b=25+10=35 \end{array}$$

$$\begin{array}{r} ggbb+ggg \\ +bbb+gb \\ \hline \end{array} \begin{array}{r} 525 \\ 315 \\ \hline \end{array}$$

$$\text{Product } ggbb+ggg+bbb+gb=3675 \quad \text{Product.}$$

$$\begin{array}{r} \text{Case 2. Multiply } -bb-g=-100-5=-105 \\ \text{by } -gg-b=-25-10=-35 \end{array}$$

$$\begin{array}{r} ggbb+ggg \\ +bbb+gb \\ \hline \end{array} \begin{array}{r} 525 \\ 315 \\ \hline \end{array}$$

$$\text{Product } ggbb+ggg+bbb+gb=3675 \quad \text{Product.}$$

$$\begin{array}{r} \text{Case 3. Multiply } bb+g=100+5=105 \\ \text{by } -gg-b=-25-10=-35 \end{array}$$

$$\begin{array}{r} -ggbb-ggg \\ -bbb-gb \\ \hline \end{array} \begin{array}{r} -525 \\ -315 \\ \hline \end{array}$$

$$\text{Product } = -ggbb-ggg-bbb-gb=-3675 \quad \text{Product.}$$

To multiply *Cosick Powers*, see at the end of *Division of Algebra*.

*The General Rule for Multiplication.*

*If the Signs of both the Factors are like, then the Sign of the Product is Affirmative: But if the Signs of the Factors are different, the Sign of the Product is Negative.*

This Rule will appear to be observed in the nine Cases of *Multiplication* as above, which I have demonstrated by Numbers, which agree to the Quantities of the Factors and Products: For Instance, the Product of the first Case of *Prop. 3.* is as in the Margin, where the Value of the Product is equal to the Rectangle of those Numbers 105 and 35 (the Values of the Factors) being multiplied together.

| Quantities. | Values.  |
|-------------|----------|
| $ggbb$      | $= 2500$ |
| $ggg$       | $= 125$  |
| $bbb$       | $= 1000$ |
| $gb$        | $= 50$   |
| Sum         | $= 3675$ |

*The*

*The Reason of the General Rule for Multiplication.*

1<sup>st</sup>, Multiplication being nothing but the Work of many Additions; it follows that  $-ab$  being added to  $-ab$ , is  $-2ab$ , which is the same as multiplying  $-ab$  by 2; and so  $-gb - 2gb - 3gb$  added, is  $-6gb$ , which is the same as  $-gb \times 6$ ; which shews, that the Negative Quantity  $-gb$  by the Affirmative 6, must give the Negative  $-6gb$ , and the like Reason holds for any other Negative by a positive Quantity.

2<sup>dly</sup>, And as in Logic, two Negatives make an Affirmative; so in Algebra, one Negative Number or Quantity multiplied by another Negative, produceth an Affirmative: For the Sign  $-$  being directly contrary to  $+$ , must make a Product of a direct contrary Nature when the Multiplier is  $-$ , then (as above) when it is  $+$ ; as subtracting a Negative, is adding really so much as we seem to subtract, as  $-2$  from 4 is 6.

## SECT. V. Division of Algebra.

I shall make use of the same Letters and their Values as in the last Section, and the Examples shall be such as to prove both themselves and those in the last Section.

Prop. 1. To divide a simple Quantity by a Simple.

Case 1. Divide  $bu = 300$  by  $-b = -12$   $-b) bu (-u = \text{Quote} = 25$ ; for  $-12) 300 (-25 = -u$

$$\begin{array}{r} bu \\ -12 \overline{) 300} \\ \underline{60} \\ 0 \end{array}$$

Case 2 Divide  $dg = 40$  by  $g = 5$   $g) dg (d \text{ the Quote; for } 5) 40 (8 = d$

$$\begin{array}{r} dg \\ 5 \overline{) 40} \\ \underline{40} \\ 0 \end{array}$$

Case 3. Divide  $-cb = -200$  by  $b = 20$   $b) -cb (-c \text{ the Quote for } 10;) -200 (-20 = -c$

$$\begin{array}{r} -cb \\ 20 \overline{) -200} \\ \underline{200} \\ 0 \end{array}$$

Prop.

Prop. 2. To divide a Compound by a simple Quantity.

Case 1. Divide  $2cbu + cd = 10000 + 80$  by  $c = 20$  |  $c) 2cbu + cd (2bu + d (20) 10080 (504$  | Quote.  
 $= 2bu + d$   
 $\frac{+cd}{cd}$   
 $\frac{cd}{0}$  refts.

Case 2. Divide  $ccd + cb = 1600 + 200$  by  $-c = -20$  |  $-c) ccd + cb (-cd - b (-20) 1800 (-90$  | Quote.  
 $= -cd - b$   
 $\frac{ccd}{+cb}$   
 $\frac{cb}{cb}$   
 $\frac{0}{0}$

Case 3. Divide  $-2cbu - cd = -10000 - 80$  by  $-c = -20$  |  $-c) -2cbu - cd (2bu + d (-20) -10080 (504$  | Quote.  
 $= 2cb + d$   
 $\frac{-2cbu}{-cd}$   
 $\frac{-cd}{0}$

Prop. 3. To divide compound Quantities by Compound.

Case 1. Divide  $ggbb + ggg + bbb + gb = 3675$  by  $gg + b = 35$  |  $gg + b) ggbb + ggg + bbb + gb (bb + g (35) 3675 (105 = bb + g$  | Quote.  
 $\frac{ggbb + bbb}{ggg - bbb + bbb + gb}$  Deduct  
 $\frac{ggg - bbb + bbb + gb}{rest of the (Dividend.)}$   
 $\frac{ggg + gb}{ggg + gb}$  Deduct.  
 $\frac{0}{0}$  refts.

Case 2. Divide  $ggbb + ggg + bbb + gb = 3675$  by  $-bb - g = -105$  |  $-bb - g) ggbb + ggg + bbb + gb (-gg - b (-105) 3675 (-35 = Quote.$   
 $\frac{ggbb + ggg}{+bbb + gb}$   
 $\frac{+bbb + gb}{+bbb + gb}$   
 $\frac{0}{0}$  refts 0

Case



Case 3. Divide  $-ggbb - ggg - bbb - gb$   
 by  $-gg - b$   

$$\begin{array}{r} -ggbb - ggg - bbb - gb \\ -ggbb - bbb \\ \hline -ggg + bbb - bbb - gb = \text{the rest of} \\ \text{(the Dividend.)} \end{array}$$
  
 Or  $-ggg - gb$  if Contracted.  

$$\begin{array}{r} -ggg - gb \\ \hline 0 \text{ refts.} \end{array}$$

$-35) -3675(105$   
 $\underline{175}$   
 $\bullet$

## General Rule as to the Signs.

It is easily seen by the 9 Cases above, that if you divide Quantities by those having like Signs with those divided, then the Signs of those put in the *Quote* are + or *Affirmative*; but if the Signs of the *Dividend* and *Divisor* are unlike, then the Sign of those Quantities put in the *Quote* are *Negative*.

## General Rule as to the Quantities.

Proceed in the Method of common *Division*, putting those Quantities in the *Quote* which are not in the *Divisor*, as in the last Case; I ask how often  $-gg$  in  $-ggbb$ ? the Answer therefore is  $bb$ , which I put in the *Quote*, and multiplying the *Divisor* by  $bb$ , is  $-ggbb - bbb$  (in the last Case above), so the *Remainer* is  $-ggg + bbb$  (because the Sign must be changed of  $bbb$ , to deduct it from  $-ggg$ ); then because here is  $+bbb - bbb$ , they destroy one another, (as is shewn under the Word *Contraction*, in Sect. 1. of this Chap.) so there refts but  $-ggg - gb$ : and I find I can have  $-gg$  of the *Divisor* in  $-ggg$  of the *Dividend*  $+g$ , by which multiplying the *Divisor*  $-gg - b$ , produceth  $-ggg - gb$ , which deducted from the last compound Quantity contracted, leaves 0, or nothing for a *Remainer*.

But tho' this is pretty enough, and serves to shew the Coherence of *Division* in *Algebra*, with that in other kinds of *Arithmetic*; yet it is far from falling in use, so much as where the *Divisor* in Species will not divide any part of the *Dividend*; or especially where it wont divide it all, but some part; and the rest as a Fraction of the *Divisor* being put under the remaining part of the *Dividend*.

The first Kind of more common Cases in Division are these.

To divide  $gb$  by  $r$ , quoteth  $\frac{gb}{r}$ ; so  $pr + st - m$  by  $c$ , is  $\frac{pr + st - m}{c}$

and







A TABLE shewing the several Ways of expressing the Powers,  
both of Numbers and Species.

|                    | Power.    | Power.                                       | Power.                                       | Power.                                       | Power.                                       | Power.                                       | Power.                                       | Power.                                       | Power.                                       | Power.                                          |
|--------------------|-----------|----------------------------------------------|----------------------------------------------|----------------------------------------------|----------------------------------------------|----------------------------------------------|----------------------------------------------|----------------------------------------------|----------------------------------------------|-------------------------------------------------|
| Exponents.         | 1st       | 2d                                           | 3d                                           | 4th                                          | 5th                                          | 6th                                          | 7th                                          | 8th                                          | 9th                                          | 10th                                            |
|                    | Root or — | Square or —                                  | Cube or —                                    | Biquadrat or —                               | Surfolid or —                                | Squared Cube or —                            | Second Surfolid or —                         | Squared Biquadrat or —                       | Cubed Cube or —                              | Squared Surfolid or —                           |
|                    | 1         | 1                                            | 1                                            | 1                                            | 1                                            | 1                                            | 1                                            | 1                                            | 1                                            | 1                                               |
|                    | 2         | 4                                            | 8                                            | 16                                           | 32                                           | 64                                           | 128                                          | 256                                          | 512                                          | 1024                                            |
|                    | 3         | 9                                            | 27                                           | 81                                           | 243                                          | 729                                          | 2187                                         | 6561                                         | 19683                                        | 59049                                           |
|                    | 4         | 16                                           | 64                                           | 256                                          | 1024                                         | 4096                                         | 16384                                        | 65536                                        | 262144                                       | 1048576                                         |
|                    | 5         | 25                                           | 125                                          | 625                                          | 3125                                         | 15625                                        | 78125                                        | 390625                                       | 1953125                                      | 9765625                                         |
|                    | 6         | 36                                           | 216                                          | 1296                                         | 7776                                         | 46656                                        | 279936                                       | 1679616                                      | 10077696                                     | 60466176                                        |
|                    | 7         | 49                                           | 343                                          | 2401                                         | 16807                                        | 117649                                       | 823543                                       | 5764801                                      | 40353607                                     | 282475249                                       |
|                    | 8         | 64                                           | 512                                          | 4096                                         | 32768                                        | 262144                                       | 2097152                                      | 16777216                                     | 134217728                                    | 1073741824                                      |
|                    | 9         | 81                                           | 729                                          | 6561                                         | 59049                                        | 531441                                       | 4782969                                      | 43046721                                     | 387420489                                    | 3486784401                                      |
| Or more brief.     | 9         | 9 <sup>2</sup>                               | 9 <sup>3</sup>                               | 9 <sup>4</sup>                               | 9 <sup>5</sup>                               | 9 <sup>6</sup>                               | 9 <sup>7</sup>                               | 9 <sup>8</sup>                               | 9 <sup>9</sup>                               | 9 <sup>10</sup>                                 |
| In Species thus :  | u         | uu                                           | uuu                                          | uuuu                                         | uuuuu                                        | uuuuuu                                       | uuuuuuu                                      | uuuuuuuu                                     | uuuuuuuuu                                    | uuuuuuuuu                                       |
| Or rather.         | u         | u <sup>2</sup>                               | u <sup>3</sup>                               | u <sup>4</sup>                               | u <sup>5</sup>                               | u <sup>6</sup>                               | u <sup>7</sup>                               | u <sup>8</sup>                               | u <sup>9</sup>                               | u <sup>10</sup>                                 |
| If compound thus : | u b       | u u b b                                      | u u u b b b                                  | &c.                                          | ...                                          | ...                                          | ...                                          | ...                                          | ...                                          | ...                                             |
| Or better thus :   | u b       | u <sup>2</sup> b <sup>2</sup>                | u <sup>3</sup> b <sup>3</sup>                | u <sup>4</sup> b <sup>4</sup>                | u <sup>5</sup> b <sup>5</sup>                | u <sup>6</sup> b <sup>6</sup>                | u <sup>7</sup> b <sup>7</sup>                | u <sup>8</sup> b <sup>8</sup>                | u <sup>9</sup> b <sup>9</sup>                | u <sup>10</sup> b <sup>10</sup>                 |
|                    | b c d     | b <sup>2</sup> c <sup>2</sup> d <sup>2</sup> | b <sup>3</sup> c <sup>3</sup> d <sup>3</sup> | b <sup>4</sup> c <sup>4</sup> d <sup>4</sup> | b <sup>5</sup> c <sup>5</sup> d <sup>5</sup> | b <sup>6</sup> c <sup>6</sup> d <sup>6</sup> | b <sup>7</sup> c <sup>7</sup> d <sup>7</sup> | b <sup>8</sup> c <sup>8</sup> d <sup>8</sup> | b <sup>9</sup> c <sup>9</sup> d <sup>9</sup> | b <sup>10</sup> c <sup>10</sup> d <sup>10</sup> |

And the Notation of Powers with Fractional Exponents is thus :

| Roots.                           | Squares.           | Cubes.             | Biquadrat.         | Surfolid.          | Squared Cube.      | Second Surfolid.   | Squared Biquadrat. | Cubed Cube.        | Squared Surfolid.         |
|----------------------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|---------------------------|
| $a^{\frac{1}{2}}$                | $a$                | $a^{1\frac{1}{2}}$ | $a^2$              | $a^{2\frac{1}{2}}$ | $a^3$              | $a^{3\frac{1}{2}}$ | $a^4$              | $a^{4\frac{1}{2}}$ | $a^5$                     |
| $a^{\frac{1}{3}}$                | $a^{\frac{2}{3}}$  | $a$                | $a^{1\frac{1}{3}}$ | $a^{1\frac{2}{3}}$ | $a^2$              | $a^{2\frac{1}{3}}$ | $a^{2\frac{2}{3}}$ | $a^3$              | $a^{3\frac{1}{3}}$        |
| $a^{\frac{1}{4}}$                | $a^{\frac{1}{2}}$  | $a^{\frac{3}{4}}$  | $a$                | $a^{1\frac{1}{4}}$ | $a^{1\frac{1}{2}}$ | $a^{1\frac{3}{4}}$ | $a^2$              | $a^{2\frac{1}{4}}$ | $a^{2\frac{1}{2}}, \&c.$  |
| Or with Literal Exponents, thus: |                    |                    |                    |                    |                    |                    |                    |                    |                           |
| $\frac{m}{n}$                    | $\frac{2m}{n}$     | $\frac{3m}{n}$     | $\frac{4m}{n}$     | $\frac{5m}{n}$     | $\frac{6m}{n}$     | $\frac{7m}{n}$     | $\frac{8m}{n}$     | $\frac{9m}{n}$     | $\frac{10m}{n}$           |
| $a^{\frac{m}{n}}$                | $a^{\frac{2m}{n}}$ | $a^{\frac{3m}{n}}$ | $a^{\frac{4m}{n}}$ | $a^{\frac{5m}{n}}$ | $a^{\frac{6m}{n}}$ | $a^{\frac{7m}{n}}$ | $a^{\frac{8m}{n}}$ | $a^{\frac{9m}{n}}$ | $a^{\frac{10m}{n}}, \&c.$ |

Some

*Some Observations upon this Table.*

1. Any two Exponents added, give that of the Product of the respective Powers: As where the Root is 3, and the Exponents 4 and 5. I say, the Sum of 4 and 5 = 9, which is equal to the Exponent of the Rectangle of the respective Powers  $243 \times 81 = 19683$ .

2. Double any Exponent, gives the Exponent of the Square of the respective Power, as in the 4th Power of 5, which is 625. I say, double the Exponent is 8, which is the Exponent of the Rectangle of  $625 \times 625$ , viz. of 390625.

3. I observe that 3, 4, *ec* times any Exponent, gives the Exponent of the 3d, 4th, &c. Powers: As three times the Exponent 2 is 6, so the Cube of the Powers 9, 16, *ec* under the Exponent 2, gives 729, 4096, *ec* the respective Powers under the Exponent 6.

4. If an 8th Power be to be divided by a 2d Power, the Exponent 2 taken from the Exponent 8, leaves the Exponent 6; so any of the Numbers under the Exponent 8, (as suppose 256,) divided by any of those respectively under the Exponent 2, (as for Instance 4,) the Quote will be respectively the Powers under the Exponent 6, (in this Example 64.)

5. The Cube *ec* Root of any Power, is the Power respectively of the Exponent of that Power given, divided by 3 *ec*; so the Cube Root of 262144 (its Exponent being 9, a third of which is 3,) is 64, found under the Exponent 3, and against the Power given. So likewise the Surfsolid Root of 1024, (or any of the Numbers under the Exponent 10,) is 4, or the respective Numbers under the Exponent 2, the Quote of 10 divided by 5; for  $4^5 = 1024$  the Proof. So much for raising the Powers of Quantities compounded by *Multiplication*; I proceed to shew,

III. How to raise the Powers of Quantities compounded by Addition or Subtraction, i. e. from a Binomial or Residual Root, and to extract Roots.

$u + b$  = the Root or 1st Power.

$$u + b$$

$uu + 2bu + bb$  = the Square or 2d Power of  $u + b$

$$\begin{array}{r} u + b \\ \hline uu + 2bu + bb \\ \hline \end{array}$$

$u^3 + 3buu + 3bbu + bbb$  = the Cube or 3d Power of  $u + b$ .

$$\begin{array}{r} u + b \\ \hline u^3 + 3buu + 3bbu + bbb \\ \hline \end{array}$$

$u^4 + 4bu^3 + 6b^2u^2 + 4ub^3 + b^4$  = the Biquadrat or 4th Power of  $u + b$ .

$$\begin{array}{r} u + b \\ \hline u^4 + 4bu^3 + 6b^2u^2 + 4ub^3 + b^4 \\ \hline \end{array}$$

$u^5 + 5bu^4 + 10b^2u^3 + 10u^2b^3 + 5ub^4 + b^5$  = the Surfolid or 5th Power of  $u + b$ .

$$\begin{array}{r} u + b \\ \hline u^5 + 5bu^4 + 10b^2u^3 + 10u^2b^3 + 5ub^4 + b^5 \\ \hline \end{array}$$

$u^6 + 6bu^5 + 15b^2u^4 + 20u^3b^3 + 15u^2b^4 + 6ub^5 + b^6$  = the Squared Cube (of  $u + b$ ).

$$\begin{array}{r} u + b \\ \hline u^6 + 6bu^5 + 15b^2u^4 + 20u^3b^3 + 15u^2b^4 + 6ub^5 + b^6 \\ \hline \end{array}$$

$u^7 + 7bu^6 + 21b^2u^5 + 35u^4b^3 + 35u^3b^4 + 21u^2b^5 + 7ub^6 + b^7$  = the 2d (Surfolid of  $u + b$ ).

$u^8 + 8bu^7 + 28b^2u^6 + 56u^5b^3 + 70u^4b^4 + 56u^3b^5 + 28u^2b^6 + 8ub^7 + b^8$  (= the 8th Power.

$u^9 + 9bu^8 + 36b^2u^7 + 84u^6b^3 + 126u^5b^4 + 126u^4b^5 + 84u^3b^6 + 36u^2b^7 + 9ub^8 + b^9$  = the 9th Power.

$u^{10} + 10bu^9 + 45b^2u^8 + 120u^7b^3 + 210u^6b^4 + 252u^5b^5 + 210u^4b^6 + 120u^3b^7 + 45u^2b^8 + 10ub^9 + b^{10}$  = the 10 Power of  $u + b$ .

Observations



*Observations from this last Table of Powers.*

1. That the Sum of the Square of the Parts of any Number or Quantity more the double Rectangle is = the Square of the whole, as per Euclid.

2. That the foregoing Example of Powers may serve as a Table for any one to know any Power of any other Binomial Roots by, though of other Letters, *mutatis mutandis*; the Unciæ and Powers being the same as in this.

3. That the 10th Power is easily produced from the 9th or any subsequent Power from the preceding, by first finding the Unciæ as per the Rules foregoing, or as under the 5th Head following, and considering the Course of the Powers that precede that which you would find: By which Method I have produced the 8th, 9th, and 10th Powers without any Multiplication.

4. And that the Reader may not want a proper Way of expressing in Words any Power, I shall give an Example how the 10th Power is properly read; which is thus:

The squared Sur-solid of  $u$ , more 10 times the Rectangle of  $b$  in the cubed Cube of  $u$ ; more 45 times the Square of  $b$  in the squared Biquadrat of  $u$ ; more 120 times the 2d Sur-solid of  $u$  in the Cube of  $b$ ; more 210 times the squared Cube of  $u$  in the Biquadrat of  $b$ ; more 252 times the Sur-solid of  $u$  in the Sur-solid of  $b$ ; more 210 times the Biquadrat of  $u$  in the squared Cube of  $b$ ; more 120 times the Cube of  $u$  in the 2d Sur-solid of  $b$ ; more 45 times the Square of  $u$  in the squared Biquadrat of  $b$ ; more 10 times  $u$  in the cubed Cube of  $b$ ; more the squared Sur-solid of  $b$ .

5. A very brief Way of finding the Unciæ of any Power by that of the next preceding Power is this; (which I have not seen given before.)

*A second Way of finding the Unciæ.*

Add the Unciæ of the 1st and 2d Members (for Example of the 9th Power to find the Unciæ of the 10th) together they make 10, which is that of the 2d Member of the 10th Power, 36 in the 3d, and 9 in the 2d Members of the 9th Power is the 3d Member in the 10th or 45; 36 and 84 in the 3d and 4th Members is 120 belonging to the 4th Member of the 10th Power, &c. of the rest.

And the Powers themselves, and their proper Exponents are found by the like Method, observing, That in which of the 2 Powers soever added,

added, is the greatest Exponent, that Exponent must be put in the Sum.

Thus to make the 10th Power by these Rules from the 9th. The Unciæ of the 1st and 2d Members is  $10 =$  the 2d in the 10th; and the Sum of the Powers is  $bu^9$ ; (because 9 is found the highest Exponent of  $u$  and  $b$  is only the 1st Power.) So  $10bu^9$  is the 2d Member of the 10th Power. The 3d Member of the 10th is the 2d and 3d Members of the 9th, &c. done as in that last mentioned.

And this will be made plain by a Numeral Example of finding the Unciæ to the 15th Power of a Binomial, by an easy Addition of only two Numbers; which Unciæ, with those of all the intermediate, are found much speedier than that of the 15th Power alone can be by the Method before-mentioned; which requires Subtraction, Multiplication, and Division, and those pretty tedious ones before those of the 15th Power can be exhibited by this Method.

*A new Table of the Unciæ to the 15th Power of a Binomial Root,  
made by Addition only.*

[illegible]

*This Method, to me, is intirely New, and my own Contrivance:  
But see a Third Way a little farther.*

6. It is observable, that the Unciæ encrease till you come to the middle, and then decrease; there being the same Unciæ to the right hand of the highest, as towards the left; so that when you

you have found half the Unciæ, you have all: Observing farther, That there are two highest Unciæ in the middle of the odd Powers, as the 3d, 5th, 7th, &c. Powers.

And that the Exponents of several Members after you come to the highest, do change, as in the 10th Power in the former Table above; the Member toward the left hand of the highest is  $210u^6b^4$ , but toward the right of the highest, the Power is  $210u^4b^6$ , which is the same, only the Exponent's changed: So the 2d Member towards the left hand is  $120u^7b^3$ ; but 'tis  $120u^3b^7$ . The 3d =  $45u^8b^2$ ; but 'tis  $45u^2b^8$ . The 4th =  $10u^9b$ ; but 'tis  $10ub^9$ , &c. } toward the right.

7. You may see that four of the Unciæ, viz. the two first and two last are always known in all Powers, being one; and the Exponent of the Power. And by these Rules the Unciæ, the Powers, and Exponents of any Binomial Root may be found with much less Trouble than the common Way, or any I have seen before.

8. But the Unciæ of any Term, or Member of any of the Powers in the Table above, may be found without any of the previous Unciæ of Powers or their Terms, by easy Operations in Multiplication and Division, as by these 6 Examples.

*A third Way of finding the Unciæ.*

| Term or Power Member. | Factors.       | Products.           | Quote or Unciæ required. |
|-----------------------|----------------|---------------------|--------------------------|
| 4 of the 4 {          | 1,2,3 =        | 6 the Divisor       | } = 4                    |
|                       | 4,3,2 =        | 24 the Dividend     |                          |
| 4 of the 5 {          | 1,2,3 =        | 6 the Divisor       | } = 10                   |
|                       | 5,4,3 =        | 60 the Dividend     |                          |
| 5 of the 10 {         | 1,2,3,4 =      | 24 the Divisor      | } = 210                  |
|                       | 10,9,8,7 =     | 5040 the Dividend   |                          |
| 7 of the 10 {         | 1,2,3,4,5,6 =  | 720 the Divisor     | } = 210                  |
|                       | 10,9,8,7,6,5 = | 151200 the Dividend |                          |
| 5 of the 15 {         | 1,2,3,4 =      | 24 the Divisor      | } = 1365                 |
|                       | 15,14,13,12 =  | 32760 the Dividend  |                          |
| 4 of the 16 {         | 1,2,3 =        | 6 the Divisor       | } = 560                  |
|                       | 16,15,14 =     | 3360 the Dividend   |                          |

Here it may easily be seen that I tell the Figures from one to within one of the Term given, and multiply them together for a Divisor.

And



And telling as many from the Power given backward; I multiply them one in another for a Dividend; whence the *Uncia* of any Term of any Power ariseth by Division.

For suppose  $p$  = the Exponent of any Power, and  $t$  = the Term, and  $u$  = the *Uncia* sought, the Rule will be

$$\begin{array}{l} p \times p - 1 \times p - 2 \times p - 3, \text{ \&c.} \\ t - 1 \times t - 2 \times t - 3 \times t - 4, \text{ \&c.} \end{array} = u \quad \left. \begin{array}{l} \text{? (in this Example the } \textit{Uncia} \text{ of} \\ \text{the 5th Term or Member of} \\ \text{the 10th Power} = 210.) \end{array} \right\}$$

9. Any Number may be squared by the Canon exhibited in the Square, or 2d Power of the above Table of the Powers raised from a Binomial Root. For Example; What is the Square of 1234?

The Analytical Square or Canon directing us how to work, is  $uu + 2ub + bb$ . Now suppose 1200 the 2 first Figures =  $a$

And 34 the 2 next the right hand =  $b$

It follows, That  $u^2 = 1200^2 = 1440000$

$$2ub \text{ or } u \times b \times 2 \text{ is } = 81600 = 1200 \times 34 \times 2$$

$$bb \text{ (or } 34 \times 34) \text{ is } = 1156$$

$$\text{Sum} = 1522756 = uu + 2ub + bb = 1234^2.$$

10. And by the Cube or 3d Power we have a Rule how to cube any Number. Example: The Canon =  $u^3 + 3u^2b + 3ub^2 + b^3$ . What is the Cube of 2232?

I say 2000 =  $u$

$$200 = b$$

$$8000000000 = uu u \text{ or } 2000^3$$

$$2400000000 = 3uub = 2000^2 \times 200 \times 3$$

$$240000000 = 3ubb = 2000 \times 200^2 \times 3$$

$$8000000 = bbb = 200^3.$$

$$10648000000 = \text{Sum or } 2200^3 \text{ or } uu u \text{ in the 2d Operation; where}$$

$$2200 = u$$

$$\text{and } 30 = b$$

$$435600000 = 3uub, = 2200^2 \times 30 \times 3$$

$$5940000 = 3ubb = 2200 \times 900 \times 3$$

$$27000 = bbb = 30^3.$$

$$11089567000 = \text{Sum, or } 2230^3. \text{ So in the 3d Operation } 2230 = u$$

$$\text{and } 2 = b$$

$$29837400 = 3uub = 2230^2 \times 2 \times 3$$

$$26760 = 3ubb = 2230 \times 4 \times 3$$

$$8 = bbb = 2^3.$$

$$11119431168 = 2232^3 = uu u + 3uub + 3ubb + bbb.$$

This

This proves that the third Power in the foregoing Table is the true Cube of  $u+b$ , as is here demonstrated by Numbers working with the Species exactly according to the Canon, without having regard to the way of placing Numbers in common *Multiplication*.

II. But that which renders the Powers of a Binomial Root more admirably useful is, that they shew plainly the Reason of all those mysterious Rules given in extracting the Root of any Power, as the Square, Cube, Biquadrat, &c. Roots in the former Part of this Book mentioned: For Example,

In Extracting the Square Root of 1522756.

Canon for the Square Root.

$$=aa+2ab+bb.$$

$$1522756(1000=a$$

$$1000000=aa \text{ deduct.}$$

$\div$  this Rest by  $2a=2000$ ) 522756 refts, ( $200=b$ =the Quote.

$$400000=2ab.$$

$$40000=bb.$$

$$440000=2ab+bb, \text{ deduct from the (last Rest.}$$

$\div$  by  $2a=2400$ ) 82756 remains, and now  $1000+200$   
 $(=1200=a30=b$ , the Quote.

$$72000=2ab.$$

$$900=bb.$$

$$72900=2ab+bb, \text{ deduct from the last (Remainer.}$$

$\div$  by  $2a=2460$ ) 9856=rest; and now  $1230$ =the 3  
 $(Roots=a4=b$ =the Quote or last

$$9840=2ab.$$

$$16=bb.$$

$$9856=2ab+bb, \text{ deduct from the (last Rest.}$$

0 refts.

E e e

And

And the Reason of Extracting the Cube Root, will appear in the following Example, proceeding according to this Canon  
 $aaa + 3aab + 3abb + b^3$ .

Extract the

Cube Root of 11119431168 (2000 =  $a$ .)

8000000000 =  $aaa$ , deduct.

12006000) 3119431168 resteth, which divide by  $3a + 3aa$ ,  
 (200 =  $b$ . (gives 200 =  $b$ .)

6000 =  $3a$ , the Treble Root.

12000000 =  $3aa$ , = the Treble Square of the  
 (Root  $a$ .)

12006000 =  $3a + 3aa$ , the Divisor.

8000000 =  $bbb$  or  $200^3$ .

240000000 =  $3abb$  =  $6000 \times 40000$ .

2400000000 =  $3aab$  =  $4000000 \times 200 \times 3$ .

2648000000 =  $bbb + 3abb + 3aab$  = the subtrahend  
 (to deduct from the last resolvend.

14526600) 471431168 resteth the second resolvend ( $30 = b$ , =  
 (the Quote.

6600 = Treble Root 2200 =  $3a$ .

14520000 =  $3aa$  =  $2200^3 \times 3$ .

14526600 =  $3a + 3aa$  = the Divisor for 2d Re-  
 (solvend, gives  $30 = b$ .)

27000 =  $bbb$  =  $30^3$ .

5940000 =  $3abb$  =  $6600 \times 900$ .

435600000 =  $3aab$  =  $4840000 \times 30 \times 3$ .

441567000 =  $bbb + 3abb + 3aab$  = the subtrahend.

14925390) 29864168 = third resolvend rests.  $2 = b$ , the Quote.

6690 =  $3a$ , the Treble Root =  $2230 \times 3$ .

14918700 =  $3aa$  =  $4972900 \times 3$ .

14925390 =  $3a + 3aa$ , the Divisor for the third  
 (Resolvend.

8 =  $bbb$  =  $2^3$ .

26760 =  $3abb$  =  $6690 \times 4$ .

29837400 =  $3aab$  =  $14918700 \times 2$ .

29864168 =  $bbb + 3abb + 3aab$ , the Sum or sub-  
 (trahend, which taken from the

(0) 3d Resolvend there Rests (0)

So the Root is found 2000 + 200 + 30 + 2.

And



SECT. VI. To Extract Roots of Numbers, &c. 395

And the Proceſs according to the Canon of the third Power of  $a+b$ , is repeated three times, viz.

$$\begin{array}{l|l} \text{1st, } 2000 = a, \text{ and } 200 = b & \text{or } 2000 \\ \text{2dly, } 2200 = a, \text{ and } 30 = b & + 200 \\ \text{3dly, } 2230 = a, \text{ and } 2 = b & + 30 \\ & + 2 \end{array} \left. \vphantom{\begin{array}{l} 2000 \\ 200 \\ 30 \\ 2 \end{array}} \right\} = 2232, \text{ the Root required.}$$

These Examples, according to the 2d and 3d Powers of the Binomial Roots in the Table above, fully ſhew the Reason of the Method uſed in the Extraction of the Square and Cube Roots. And by the obſerving of the Canon given by the Powers, the Root of any other, even the ſquared Surſolid may be extracted: As I did that of the Biquadrat in *Sett.* 6. of *Chap.* I. Article the 2d; having never ſeen it done but only proceeding purely as the 4th Power directs.

IV. The Roots of Algebraic Quantities are either thoſe of Rationals or Surds. 1. The Roots of Rational Quantities, whether thoſe Quantities be compounded by Multiplication of Single Binomial, or Reſidual Roots are thus expreſs'd.

1. How Single Roots of Powers are expreſs'd in Species and Numbers.

|              | Powers.     |          | Their Roots. |       |
|--------------|-------------|----------|--------------|-------|
|              | in Species. | in Numb. | Species.     | Numb. |
| Square       | $uu =$      | 4        | $u =$        | 2     |
| Cube         | $bbb =$     | 27       | $b =$        | 3     |
| Biquadrat    | $c^4 =$     | 256      | $c =$        | 4     |
| Surſolid     | $d^5 =$     | 3125     | $d =$        | 5     |
| Squared Cube | $b^6 =$     | 46656    | $b =$        | 6     |
| 2d Surſolid  | $k^7 =$     | 823543   | $k =$        | 7     |

2. How Binomials and Reſidual Roots are extracted.

| Binomial Powers. Their Roots. |                  |            |                  | Reſidual Powers. Their Roots. |                  |            |                  |
|-------------------------------|------------------|------------|------------------|-------------------------------|------------------|------------|------------------|
| Species.                      | N <sup>o</sup> . | Spec.      | N <sup>o</sup> . | Species.                      | N <sup>o</sup> . | Spec.      | N <sup>o</sup> . |
| $uu + 2ub + bb =$             | 25.              | $u + b =$  | 5                | $dd - 2du + uu =$             | 9.               | $d - u =$  | 3                |
| $uu + 2u + 1 =$               | 9.               | $u + 1 =$  | 3                | $bb - 2b + 1 =$               | 4.               | $b - 1 =$  | 2                |
| $16bb + 8bk + kk =$           | 961.             | $4b + k =$ | 31               | $16bb - 8bk + kk =$           | 289.             | $4b - k =$ | 17               |
| $bbuu + 2buu + uu =$          | 64.              | $bu + u =$ | 8                | $bbuu - 2buu + uu =$          | 16.              | $bu - u =$ | 4                |

E e e 2

2dly, Surd

3dly, Surd Roots, or Roots of Surd Quantities, are expressed thus by their own Radical Signs.

## Simple Surd Quantities.

|                    | Powers.            | Roots.                 |      | Powers.             | Roots. |
|--------------------|--------------------|------------------------|------|---------------------|--------|
| Square Root of $u$ | $= \sqrt{u}$       | or in N <sup>o</sup> . | 171  | $= \sqrt{171}$      | 171    |
| Cube Root          | $b = \sqrt[3]{b}$  | _____                  | 315  | $= \sqrt[3]{315}$   | 315    |
| Biquadrat          | $c = \sqrt[4]{c}$  | _____                  | 517  | $= \sqrt[4]{517}$   | 517    |
| Surfolid           | $d = \sqrt[5]{d}$  | _____                  | 735  | $= \sqrt[5]{735}$   | 735    |
| Squared Cube       | $g = \sqrt[6]{g}$  | _____                  | 379  | $= \sqrt[6]{379}$   | 379    |
| 2d Surfolid        | $h = \sqrt[7]{h}$  | _____                  | 1231 | $= \sqrt[7]{1231}$  | 1231   |
| Squared Biquad.    | $k = \sqrt[8]{k}$  | _____                  | 339  | $= \sqrt[8]{339}$   | 339    |
| Cubed Cube         | $l = \sqrt[9]{l}$  | _____                  | 1129 | $= \sqrt[9]{1129}$  | 1129   |
| Squared Surfol.    | $m = \sqrt[10]{m}$ | _____                  | 1057 | $= \sqrt[10]{1057}$ | 1057   |

## Compound Surd Quantities.

| Powers.                                   | Roots.                                    |
|-------------------------------------------|-------------------------------------------|
| $u + b$ _____                             | $\sqrt{u + b}$                            |
| $u + b + c$ _____                         | $\sqrt[3]{u + b + c}$                     |
| $u + b + c + d$ _____                     | $\sqrt[4]{u + b + c + d}$                 |
| $u + b + c + d - g$ _____                 | $\sqrt[5]{u + b + c + d - g}$             |
| $u + b + c - d + g + b$ _____             | $\sqrt[6]{u + b + c - d + g + b}$         |
| $u + b + c - d + g - b - k$ _____         | $\sqrt[7]{u + b + c - d + g - b - k}$     |
| $u + b - c + d + g + b + k + l$ _____     | $\sqrt[8]{u + b - c + d + g + b + k + l}$ |
| $u + b + c + d - g - b + k + l + m$ _____ | $\sqrt[9]{u + b + c + d - g + k + l + m}$ |
| $17r + gb - bc + m$ _____                 | $\sqrt[10]{17r + gb - bc + m}$            |

It must be observed, That the Sign connecting the Residual Root is always the same with that of the double Rectangle, & c. Negative as under the 2d above.

Having fully shewn the Nature of raising the Powers of Quantities, and extracting the Roots, and illustrated the same by Numbers,

bers, it naturally leads me to the *Arithmetic of Surd Quantities*; which I shall also demonstrate by Numbers.

SECT. VII. *The Algorithm of Surd Quantities.*

In regard there is often Occasion for Working the Roots of Species or Numbers, as by Addition, Subtraction, Multiplication, &c. of them; and that it would be tedious and less accurate to find near the real Roots of Numbers, and impossible to be done of all Algebraic Quantities; therefore the Business of this Section is to give Rules for such Operations in Powers without regarding the Roots farther than by the Radical Signs prefixed: So that the compleat Powers of Quantities, or Numbers may hereby be added, subtracted, &c. as well, (and much sooner) than if the real Roots were known, and the Work done by them. See Article 7. Sect. I. of this Chapter.

I. *Reduction of Surd Quantities.*

Reduction is here taught first, because Quantities are required to be reduced to a common Radical Sign before those that have different Signs can be multiplied, divided, &c.

*Example 1.* Reduce  $\sqrt{u}$  and  $\sqrt[4]{c}$  to Surds having a common Radical Sign; or Surds Heterogeneous to Homogeneous. See the Operation.

|                  |               |                                                    |           |
|------------------|---------------|----------------------------------------------------|-----------|
| 4,               | 4             | The common Radical Sign.                           | (ponents. |
| 2,               | 1             | The lowest Terms of the given Radical Signs or Ex- |           |
| $\sqrt{u}$ ,     | $\sqrt[4]{c}$ | The Surds given to be reduced.                     |           |
| $\sqrt[4]{uu}$ , | $\sqrt[4]{c}$ | The Answer or the Surds given in a common Radical  | (Sign.    |

By the Operation it is plain, that first I take half the Radical Sign 2 (given) and put it over the other given, or 4; and half the given Radical Sign 4, and place it over the other given or 2, which are the lowest Terms of the given Radical Sign.

2dly, I multiply the Radical Signs given by those standing over respectively, and they produce 4 = the common Radical Sign required.



3dly, I raise the Surd Roots given to the Powers of the lowest Terms of the given Radical Signs which stand over the respective Root; as  $u$  raised to the 2d Power is  $uu$ ,  $c$  in the 1st Power is  $=c$ , and placing the common Radical Sign before this  $uu$  and the  $c$ , I have the Answer  $\sqrt[4]{uu}$  and  $\sqrt[4]{c}$ .

Example 2d shall be in Numbers, as to reduce  $\sqrt[3]{9}$  and  $\sqrt[4]{16}$  to a common Radical Sign.

See the Work by the same Rule as above; and in Multiplication and Addition it is shewn how these prove each other, or how the Totals agree, and consequently prove the Truth of this Method of Reduction.

|                  |                |                            |
|------------------|----------------|----------------------------|
| 4,               | 4              | = Common Radical Sign.     |
| 2,               | 1              | = Least Terms of given     |
| —                | —              | (Radical Signs, placed     |
| —                | —              | interchangeably.           |
| $\sqrt[3]{9}$ ,  | $\sqrt[4]{16}$ | = Surds given.             |
| $\sqrt[3]{81}$ , | $\sqrt[4]{16}$ | = Surds required, or those |
| —                | —              | (given reduced.            |

## II. Multiplication of Simple Surds.

In this Article I shall shew,

1. How to multiply a Surd by a Rational Number or Quantity.
2. To multiply a Surd in others of the same kind, or the same Radical Sign.
3. To multiply Surds together having different Radical Signs.

### 1. To multiply a Surd in a Rational Number or Quantity.

This is expressed two Ways, for Example, to multiply  $\sqrt[4]{c}$  by  $b$ , the Answer is very often expressed thus,  $b\sqrt[4]{c}$  or  $b$  in the Biquadrat Root of  $c$ . But actually to multiply them is done by this

Rule.] Raise  $b$  up to the Power of the Surd given, as here  $b$  in the fourth Power is  $bbbb$  and putting the same Radical Sign before, is  $\sqrt[4]{bbbb}$ , which being multiplied by  $\sqrt[4]{c}$ , gives  $\sqrt[4]{bbbbc}$ .

Or in Numbers, suppose  $c=16$ , and  $b=8$ , then  $b^4=4096$ , and  $\sqrt[4]{16}$  multiplied in  $\sqrt[4]{4096}=\sqrt[4]{65536}$  the Product, and that Root being extracted is  $=16$ , the Product in a Rational Number.

To

To prove both which Ways of working, by raising the Power of the Rational Number, to be genuine,

I say,  $c$  being  $= 16$ , its  $\sqrt[4]{c} = 2$ , which multiplied by  $b = 8$ , gives 16 as before.

2. To multiply a Surd Number by a Surd, which are Homogeneous, or when the Radical Signs are the same.

*Rule.*] Multiply the one by the other, and the Product is the Answer, prefixing one of the Radical Signs given.

*Example.* Multiply  $\sqrt[3]{dr}$  by  $\sqrt[3]{u}$ , and the Product is  $= \sqrt[3]{dru}$ , and putting  $u = 8$ ;  $r = 9$ ; and  $d = 3$ . It is thus done in Numbers,  $\sqrt[3]{27}$  by  $\sqrt[3]{8} = \sqrt[3]{216} = 6$  the Product.

*Proved thus:* By the real Roots without Surds. I say  $\sqrt[3]{27} = 3$ , and  $\sqrt[3]{8} = 2$ , and 2 times 3 is 6, as before in the Numerical Way last above, which is the same as the Literal.

3. To multiply Heterogeneous or Surds, having different Radical Signs.

*Rule.*] Reduce them to a common Radical Sign, and then work as in the last.

*Example.* Multiply  $\sqrt[4]{c}$  by  $\sqrt[4]{u}$ . In a common Radical Sign, the Surds are  $\sqrt[4]{uu}$  and  $\sqrt[4]{c}$ ; whose Product is  $\sqrt[4]{cuu}$ .

Or in Numbers:  $c = 16$ ; and  $u = 9$ ; or  $\sqrt[4]{16}$  to be multiplied by  $\sqrt[4]{9}$ , which in a common Radical Sign are  $\sqrt[4]{16}$  and  $\sqrt[4]{81}$ , whose Product is  $\sqrt[4]{1296} = 6$ .

*Rule proved thus:* By the real Roots, for  $\sqrt[4]{16} = 2$ ; and  $\sqrt[4]{81} = 3$ ; and  $2 \times 3 = 6$ .

### III. Division of Surd Quantities.

There is nothing of Difficulty in this Rule; for if the Quantities of the Dividend can be divided by those of the Divisor, and the Radical Signs are the same, work as is taught in *Division of Algebra*, or of whole Numbers; as  $\sqrt{u b c} \div \sqrt{b c}$  the Quote  $= \sqrt{u}$ , &c.

Or

Or in Numbers;  $\sqrt{144} + \sqrt{9} = \sqrt{16} = 4$ , or  $= 12 \div 3$ . For Proof,  
 2. If the Radical Signs are not the same, they may be reduced to a common Radical Sign, and then divide the Power of the Dividend by the Divisor; or put it over that of the Divisor, as if the  $\sqrt[4]{cuu}$  be divided by  $\sqrt[4]{u}$ . In a common Radical Sign they are  $\sqrt[4]{cuu}$  for the Dividend, and  $\sqrt[4]{uu}$  for the Divisor; so the Answer is  $\sqrt[4]{\frac{cuu}{uu}} = \sqrt[4]{c}$ . And this proves the last Example in the last Rule, and is farther proved by dividing the Numeral Rectangle there by one Factor, for the Quotient will be the other Factor.

## IV. Addition of Surd Quantities.

This Rule is placed after *Multiplication*, because that is often used in this Rule of *Addition*. I shall shew

1. How to add when there are the same Quantities and Radical Signs.

2. When there are different Quantities of Species, but the same Radical Signs, which Quantities are supposed commensurable.

3. How to add Quantities, having the same Species, but different Radical Signs, or when both the Radical Signs are different, and the Quantities incommensurable.

1. To add  $\sqrt[4]{u}$ ,  $\sqrt[4]{u}$ , and  $\sqrt[4]{u}$ .

*Rule.]* Multiply any one of the Surds by the whole Number of them, as here  $\sqrt[4]{u} \times$  by  $3 = \sqrt[4]{81u}$ , (observing the first Rule of *Multiplication* last above,) which is the Answer.

Or in Numbers: Add  $\sqrt[4]{16}$ ,  $\sqrt[4]{16}$ , and  $\sqrt[4]{16}$ . Here 3 the Number of Roots raised to the fourth Power is 81; and  $\sqrt[4]{16} \times \sqrt[4]{81} = \sqrt[4]{1296} = 6$  proved thus;  $\sqrt[4]{16} = 2$ , and 3 times 2 is 6, or  $2 + 2 + 2$ , which proves the Rule.

2. When two unequal Surds are commensurable, and have the same Radical Signs, and when divided by their greatest common Measure, are reduced to such Quantities as are the compleat Powers of some Quantities or Numbers; they may be added thus, suppose  $\sqrt[3]{40}$  to be added to  $\sqrt[3]{135}$ .

2d Rule.]



2d Rule.] Divide the Surds by their common Measure  $\sqrt[3]{5}$ , and they are reduced to  $\sqrt[3]{8}$  and  $\sqrt[3]{27}$ , the Cube Roots of which are 2 and 3; the Sum of which is 5, which Sum of the Roots multiplied is  $5\sqrt[3]{5} = \sqrt[3]{625}$  the Answer.

And for Proof; Extract the Cube Roots of the Surds given in the common way of Extraction; and you'll find the Sum  $= \sqrt[3]{625} = 8.55$ .

*How to add Quantities having the same Surds, but different Radical Signs; or when the Radical Signs are either different or the same, and the Surds are incommensurable.*

Rule.] There is no Occasion for farther Operation, than to put the Radical Sign  $+$  between the given Surd Roots: As

To add  $\sqrt{cc}$  to the  $\sqrt[3]{cc}$  and  $\sqrt[4]{cc}$ ; the Sum is  $\sqrt{cc} + \sqrt[3]{cc} + \sqrt[4]{cc}$ .

Or the Sum of  $\sqrt{5}$ , the  $\sqrt[3]{6}$  and  $\sqrt[4]{13}$ , is  $= \sqrt{5} + \sqrt[3]{6} + \sqrt[4]{13}$ .

Or  $\sqrt{3} + \sqrt{7} + \sqrt{12}$ , &c. is the Sum of such Surds as  $\sqrt{3}$ ,  $\sqrt{7}$  and  $\sqrt{12}$ , &c.

There is a Rule which some pretend helps us to the Sum of these kind of Surds, when the Radical Sign is  $\sqrt{\phantom{x}}$ . The Rule is this; To the square Root of the Sum of the Squares of the Surds given, add the double Rectangle of these Surds, and the Sum is the Answer.

Example. To  $\sqrt{12}$ , add  $\sqrt{8}$ ; according to the Rule, the Sum of the Squares of the Surds is  $= 20$ , its Root  $\sqrt{20}$ , and the double Rectangle of the Surds is  $2\sqrt{96} = 8 \times 12 \times 4 = \sqrt{384}$ , by the first Example of Multiplication of Surds. So by this Rule, the Sum of  $\sqrt{12} + \sqrt{8}$  is reduced to  $\sqrt{20} + \sqrt{384}$ . I must confess I do not see the Use of the Rule, nor of the Alteration of the Surds, since to say the Answer is  $\sqrt{12} + \sqrt{8}$  is more true and intelligible, than to call the Sum  $\sqrt{20} + \sqrt{384}$ .

#### V. Subtraction of Simple Surds.

1. When the Surd Roots are commensurable or reduceable to Rational Quantities, as in the second Example in Addition; work

F f f

as

as is there directed, but when you come to the Roots, instead of taking their Sum, take their Difference, and multiply as in the same Example. To know the Difference between  $\sqrt[3]{40}$  and  $\sqrt[3]{135}$ : These  $\div \sqrt[3]{5}$  reduces them to  $\sqrt[3]{8}$  and  $\sqrt[3]{27}$ , whose Roots are 2 and 3, and the Difference of those Roots is 1, which multiplied by the common Measure  $\sqrt[3]{5}$ , gives  $\sqrt[3]{5}$  for the Remainder or Difference.

But when the Surds are incommensurable, as in the last Example in *Addition*, you have nothing to do but place the Sign (—) between the Surds given; so the Difference of  $\sqrt{b}$  above  $\sqrt{g}$ , is  $\sqrt{b} - \sqrt{g}$ .

#### VI. Addition and Subtraction of Compound Surds.

1<sup>st</sup>, For commensurable Surds, as those under the second Rule of *Addition* above, take this Example.

$$\begin{array}{l} \text{To } \sqrt{45} + \sqrt{294} \\ \text{Add } \sqrt{125} - \sqrt{96} \end{array} \quad \text{Sum} = \sqrt{320} + \sqrt{54}.$$

Here the Sum of  $\sqrt{45}$  and  $\sqrt{125} = \sqrt{320}$ ; and of  $\sqrt{294}$  and  $-\sqrt{96} = \sqrt{54}$  by the said second Rule.

2<sup>dly</sup>, For mixt Numbers, as Surds and Rational Numbers; see this Example.

$$\begin{array}{l|l} \text{To } 331 + \sqrt[4]{768} - \sqrt[4]{80} & \text{Or from } 331 + \sqrt[4]{768} + \sqrt[4]{80} \\ \text{Add } 153 + \sqrt[4]{243} + \sqrt[4]{1280} & \text{Take } 153 + \sqrt[4]{243} - \sqrt[4]{1280} \\ \hline \text{Sum} = 484 + \sqrt[4]{7203} + \sqrt[4]{80} & \text{Refts} = 178 + \sqrt[4]{3} + \sqrt[4]{1296} \end{array}$$

I need give no farther Examples, because the *Addition* and *Subtraction* of these Surds are done by the self-same Rules as those called *Simple-Surds*, as by the Rule under the second Example, and in *Subtraction* of those Surds.

The second Example last foregoing is done thus: To add  $\sqrt[4]{768}$  to  $\sqrt[4]{243}$ ; I divide them by the common Measure  $\sqrt[4]{3}$ , which reduceth them to  $\sqrt[4]{256}$  and  $\sqrt[4]{81}$ , whose Roots are 4 and 3, whose Sum

Sum is 7, so  $7\sqrt[4]{3}$  is the Answer, or  $7^4 \times \sqrt[4]{3} = \sqrt[4]{7203}$ , and so of the Rest.

*The Demonstration of the Rule for this last, as under Article IV.*

## Rule 2.

This I shall do by this plain Instance of Numbers adapted on purpose, as in the Margin; where  $\sqrt{64} + \sqrt{64}$ , are given to have their Sum found.

$$\begin{array}{l} \sqrt{4} \sqrt{64} + \sqrt{64} = \text{the supposed Surds given.} \\ \text{Quotes } \sqrt{16} = 4 \\ \text{Rt. } = + \sqrt{16} = 4 \quad \text{Roots:} \\ \hline 8 = \text{Sum of the Roots.} \\ \text{and } 8\sqrt{4} = \sqrt{256} = 16 \text{ the Answer.} \\ = \sqrt{64} + \sqrt{64} = 8 + 8. \end{array}$$

1. These divided by  $\sqrt{4}$ , are reduced to the Root  $\sqrt{16} + \sqrt{16}$ , whose real Roots are  $4 + 4 = 8$ . And  $\sqrt{4} \times 8$ , as by the first Rule of *Multiplication* above, gives the  $\sqrt{256}$ , whose Square Root being 16, is the Sum of the Roots of the two Surds given.

## VI. Multiplication of Compound Surds.

*Three Examples or Varieties done in Species two Ways.*

*The three Examples in Species proved by Numbers.*

Example 1.

To multiply  $b \sqrt{ac}$  } See the Operation, {  $4 \sqrt{64}$  } Multiply as  
in  $ac + \sqrt{b}$  } also in Numbers {  $2 + \sqrt{16}$  } the Species.

$$\underline{abc\sqrt{ac} + b\sqrt{abc} = \text{the Prod. or in N}^\circ. = 8\sqrt{64} + 4\sqrt{64}\sqrt{6} = 192}$$

Or reduced in- }  $= \sqrt{aaabbbccc} + \sqrt{abbbc}$  For  $4 \times 8 (= \sqrt{64}) = 32$   
to all Surds. } and that  $32 \times 2 + 4 (= \sqrt{16}) = 6 = 192$

Example 2.

To multiply  $b \sqrt{ac}$   
in  $ac + \sqrt{b}$

Example 2d proved thus:

$4\sqrt{64}$  } Multiply as the  
Multiply by  $2\sqrt{16}$  } Species are.

$$\underline{abc\sqrt{abc} = \text{the Prod. or in Numb.} = 8\sqrt{64}\sqrt{16} = 256}$$

Or reduced }  $= \sqrt{aaabbbccc}$  For  $4 \times 8 (= \sqrt{64}) = 32$   
to Surds. } And  $32 \times 2 \times 4 (= 2\sqrt{16}) = 8 = 256$



Example 3.

To multiply  $b + \sqrt{ac}$  Or in N<sup>o</sup>. thus:  $4 + \sqrt{64}$   
 in  $ac + \sqrt{b}$  x in  $2 + \sqrt{16}$  } By the 3d Examp.

$$abc + ac\sqrt{ac} + b\sqrt{b} + \sqrt{abc} \left\{ \begin{array}{l} = 8 + 2\sqrt{64} + 4\sqrt{16} + \sqrt{64}\sqrt{16} = 72 \\ \text{And so 'tis in reality for } 4 + \sqrt{64} = 12 \end{array} \right.$$

Or more }  
 in Surds }  $= abc + \sqrt{aaaccc} + \sqrt{bbb} + \sqrt{abc}$

And  $2 + \sqrt{16} = 6$ And  $12 \times 6 = 72$ 

1. Note, That in multiplying  $b$ , multiplied in the Square Root of  $ac$ , by  $ac$  more the Square Root of  $b$ ; I say  $b\sqrt{ac}$  by  $ac = abc\sqrt{ac}$ , and stands so in the Example 1. More  $b$  (in the Multiplicand)  $\sqrt{abc}$ , &c.

2. Note, In like manner by the Numbers, Example 1. I say 2. times  $4\sqrt{64}$  is  $8\sqrt{64}$ . 2dly  $4\sqrt{64}$  by  $\sqrt{16}$ , is  $= 4\sqrt{64}\sqrt{16}$ ; so the Product is  $= 8\sqrt{64} + 4\sqrt{64}\sqrt{16}$ . Now the Square Root of 64 is 8, and of 16, 4; so that this Product is  $8 \times 8 = 4 \times 8 \times 4 = 192$ : And that this is right multiplied, will appear by the Numbers given, or  $4 \times$  in 8, (which 8 is the Square Root of 64,) gives 32 for the Value of the Multiplicand.

And 2, more 4, (which 4 is the Square Root of 16,) makes 6, which is the Value of the Multiplier; so 6 times 32 is  $= 192$  as before, which is a full Proof of the Rule by the first Example, and the rest are performed and proved much after the same manner, which it would be needless to insist farther upon.

Example 4. To multiply  $\sqrt[3]{u} - cd$  by  $\sqrt[3]{u} + cd$ :  
 See the Operation in the Margin.

In Numbers demonstrated thus;  $u=8, c=1, d=2$ .

Then  $\sqrt[3]{8} - 2$ by  $\sqrt[3]{8} + 2$  $\sqrt[3]{64} - \sqrt[3]{64}$  $+ \sqrt[3]{64} - 4$  $\sqrt[3]{64} - 4 =$  the Sum or Product contracted.Now the Cube Root of  $64 = 4$ , and  $4 - 4 = 0$ 

So the  $\sqrt[3]{8} =$  the Multiplicand  $= 2$ , and  $2 - 2 = 0 \times 2$  in the Multiplier  
 (= 0 the Proof.

5. Hence

5. Hence it appears, that if you multiply two Surd Numbers together, whose Radical Signs are  $\sqrt{\quad}$ , the Square Root of the Product is the Answer in a Rational Number. But if the Root cannot be extracted without a Remainder, divide the Product by the greatest Square Number, that will do it without Remainder; so the Root of that Divisor in the Quote is the Answer, as in the first Case,  $\sqrt{16}$  in  $\sqrt{9} = \sqrt{144} = 12$  the Answer. And in the second Case,  $\sqrt{72}$  in  $\sqrt{8} = \sqrt{576}$ , which  $\div 144$  quoteth 4, and the  $\sqrt{144} = 12$ ; so that  $12 \sqrt{4}$  is the Answer.

### VII. Division of Compound Surds.

This will appear very easy to those who understand common Division in Species, as before taught, and Multiplication of Surds.

*Example,* Divide  $\sqrt{bu} + \sqrt{du}$  by  $\sqrt{u}$ ; see the marginal Work.

*Example 2.* To divide  $abc + ac\sqrt{ac} + b\sqrt{b} + \sqrt{abc}$  by  $b + \sqrt{ac}$ , or in Numbers,  $8 + 2\sqrt{64} + 4\sqrt{16} + \sqrt{64}\sqrt{16}$  by  $2 + \sqrt{16}$ .

Done Literally thus:

$$\begin{array}{r} b + \sqrt{ac} \overline{) abc + ac\sqrt{ac} + b\sqrt{b} + \sqrt{abc}} \quad (ac + \sqrt{b} = \text{the Quote.} \\ abc + ac\sqrt{ac} \end{array}$$

$$\begin{array}{r} b\sqrt{b} + \sqrt{abc} \\ b\sqrt{b} + \sqrt{abc} \\ \hline \end{array}$$

rests = 0

Done Numerally thus:

$$\begin{array}{r} 2 + \sqrt{16} \overline{) 8 + 2\sqrt{64} + 4\sqrt{16} + \sqrt{64}\sqrt{16}} \quad (4\sqrt{64} = \text{Quote.} \\ 8 + 4\sqrt{16} \end{array}$$

$$\begin{array}{r} 2\sqrt{64} - 4\sqrt{16} + 4\sqrt{16} \\ \hline \end{array}$$

Or Contracted, and the } =  $2\sqrt{64} + \sqrt{64}\sqrt{16}$   
rest brought down.

$$\begin{array}{r} 2\sqrt{64} + \sqrt{64}\sqrt{16} \\ \hline \end{array}$$

0 remains.

There

There is nothing of Difficulty in these Operations, observing to change the Signs of what you subtract: As in the Numeral Example, I am to take  $4\sqrt{16}$  from  $2\sqrt{64}$ , therefore I say that  $2\sqrt{64} - 4\sqrt{16}$  is the Remainder, to which if I bring down  $4\sqrt{16}$ , the  $+$  and  $-$  destroy the  $4\sqrt{16}$ : So that bringing down to the said Remainder  $2\sqrt{64}$ , the rest of the Dividend, which is  $\sqrt{64}\sqrt{16}$ , I find that  $2 + \sqrt{64}$  is contained in  $2\sqrt{64}$ ;  $\sqrt{64}$  times, which I put in the Quote, and multiply and deduct, so the Remainder is (0).

And the Truth is thus proved, 1<sup>st</sup>, In that it agrees with the Literal Example. 2<sup>dly</sup>, As it is the Product of Example the Third foregoing, divided by the Multiplier. And 3<sup>dly</sup>, and most plainly, thus:

*The Value of the several Members of the Numeral Dividend are,*

$$8 \text{ ————— } = 8$$

$$2\sqrt{64}, \text{ or } 2 \times 8 \text{ ————— } = 16$$

$$4\sqrt{16}, \text{ or } 4 \times 4 \text{ ————— } = 16$$

$$\sqrt{64}\sqrt{16}, \text{ or } 8 \times 4 \text{ ————— } = 32$$

Value of the Divisor  $2 + \sqrt{16} = 2 + 4 = 6$   $72 = \text{Sum}$  ( $12 = \text{Quote}$   
 $= 4 + 8 = 4 + \sqrt{64}$   
 for Proof.

#### SECT. VIII. Algebraical Fractions.

Although the Business of Fractions in Species are often used in the Solution of Analytical Questions: Yet since the Reader of this Part is supposed to have perfectly learnt *Reduction of Vulgar Fractions*, Chap. II. Sect. 1. and there being so very little Difference between the Method of working Fractions in Numbers and Species, therefore I shall not need to enlarge much on this Section.

1. *Note*, The fifth Case is done thus: Multiply the Quantity given by the proposed Denominator, and place the Product over the Denominator for the Answer.

2. And the sixth Case is only placing the Integral Quantity given as a Numerator, and a Unit for a Denominator.

3. And that the Quantities omitted, which are found in both the Terms, brings a Fraction into its lowest Terms.

The



*The Cases of Reduction of Algebraical Fractions.*

| No. of Cases Given. | To reduce                                                                     | Examples, two to each Case.                                                                                              | The Answers.                                                                                         | Done by Rules C <sup>b</sup> . II. Sect 1. and Cases. |
|---------------------|-------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------|-------------------------------------------------------|
|                     |                                                                               |                                                                                                                          |                                                                                                      |                                                       |
| 1                   | Mixt Quantities to Fractions.                                                 | $\frac{ab + \frac{b}{c}}{u + \frac{b+c}{d}}$                                                                             | $\frac{bcu + b}{c} \div \frac{du + b + c}{d}$                                                        | 1                                                     |
| 2                   | An improper Fraction to a whole or mixt Quantity.                             | $\frac{\frac{bcu + b}{c}}{\frac{du + b + c}{d}}$                                                                         | $bu + \frac{b}{c} \div u + \frac{b+c}{d}$                                                            | 2                                                     |
| 3                   | A Fraction to its lowest Terms.                                               | $\frac{ubbd}{ubc} \div \frac{14udc + 21dpu}{7dru}$                                                                       | $\frac{bd}{c} \div \frac{2c + 3p}{r}$                                                                | 4                                                     |
| 4                   | Fractions of different to common Denominators.                                | $\frac{u}{c}, \frac{m}{r}, \frac{s}{t} \div \frac{u+c}{m}, \frac{b+d}{gg}$                                               | $\frac{urt}{crt}, \frac{cmt}{crt}, \frac{crs}{crt} \div \frac{gg u + cgg}{ggm}, \frac{bm + dm}{ggm}$ | 6                                                     |
| 5                   | A whole Quantity to a Fraction of the same Value, whose Denominator is given. | $\frac{b+c}{u+m} \text{ to that of Denominator } r \div \frac{u+m}{u+m} \text{ to that whose Denominator } = r+s$        | $\frac{br + cr}{r} \div \frac{ru + mr + su + ms}{r+s}$                                               |                                                       |
| 6                   | An Integral Quantity to the Form of a Fraction.                               | $\frac{bu}{cu - pr + bb}$                                                                                                | $\frac{bu}{1} \div \frac{cu - pr + bb}{1}$                                                           |                                                       |
| 7                   | A Compound to a simple Fraction.                                              | $\frac{\frac{u}{b} \text{ of } \frac{c}{d} \text{ of } \frac{r}{m}}{\frac{3ub + cd}{m} \text{ of } \frac{r - 5st}{b+n}}$ | $\frac{cru}{b d m} \div \frac{3bru + cbr - 15bstu - 5cdst}{bm + mn}$                                 | 3                                                     |

II. Addition, Subtraction, Multiplication, and Division of  
Algebraical Fractions; two Examples to each Rule.

|                | Propositions or Examples.                                             | Solutions or Answers.                    | Rules.                                                                                                                                                         |
|----------------|-----------------------------------------------------------------------|------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Addition       | 1. To $\frac{am}{u}$ add $\frac{2bb}{m}$                              | Sum = $\frac{amm + 2bbu}{mu}$            | Reduce the Fractions to a common Denominator and add the Numerators, as in Vulgar Fractions.<br>Note, $-gr$ and $+gr$ destroy each other.                      |
|                | 2. To $\frac{u-r}{g}$ add $\frac{r}{g+u}$                             | $\frac{gu - gr + uu - ru + gr}{gg + gu}$ |                                                                                                                                                                |
| Subtraction    | 1. From $\frac{amm + 2bbu}{mu}$ Take $\frac{2bb}{m}$                  | Refts = $\frac{am}{u}$                   | Reduce to a common Denominator and Work as in Vulgar, reducing the Remainders to their lowest Terms.                                                           |
|                | 2. From $\frac{gu - gr + uu - ru + gr}{gg + gu}$ Take $\frac{r}{g+u}$ | Refts $\frac{u-r}{g}$                    |                                                                                                                                                                |
| Multiplication | 1. $\frac{2bu}{c}$ by $\frac{b}{n}$                                   | Product $\frac{2bbu}{cn}$                | Multiply the Numerators give $2bbu$ , and the Denominator $= cn$ as in vulgar Fractions reducing $u + \frac{b}{r}$ into a Fraction, as by the first Reduction. |
|                | 2. $u + \frac{b}{r}$ by $\frac{3mp}{s}$                               | Prod. = $\frac{3mpru + 3bmp}{rs}$        |                                                                                                                                                                |
| Division       | 1. Divide $\frac{2bbu}{cn}$ by $\frac{b}{n}$                          | Quote = $\frac{2bu}{c}$                  | Work as in Vulgar, and reducing the Quotes to lowest Terms, shall be as you see.                                                                               |
|                | 2. $\frac{3mpru + 3bmp}{rs}$ by $\frac{3mp}{s}$                       | Quote = $u + \frac{b}{r}$                |                                                                                                                                                                |

These Examples above are very easy to such as have sufficiently acquainted themselves with *Vulgar Fractions*, and the two Examples in *Subtraction* prove those in *Addition*, as those in *Division* do

do those in *Multiplication*; there is no need to illustrate these by Numbers, because that is done by comparing these with the like Examples in *Vulgar Fractions*. So that I proceed to

SECT. IX. *Rules to resolve Simple Equations.*

The Business of this is, (when unknown Quantities are intermixt with known,) to bring the unknown Quantity to possess solely one Side of the Equation (commonly that towards the left hand) now these complex Quantities being so either by the unknown Quantity, being added to, subtracted from, multiplied in, or divided by known, or by most or all of these together; therefore Equations are resolved, or the Value of the unknown Quantity is discovered by *Addition*, *Subtraction*, *Multiplication*, *Division*, and *Evolution*, &c. and that in a contrary Way to that by which the known and unknown Quantities are connected and placed together, i. e. those of *Subtraction* solved by *Addition*; *Addition* by *Subtraction*; *Multiplication* by *Division*; and those divided and standing Fraction-Ways are solved by *Multiplication*, as by these Examples.

Simple Equations Resolved.

1. By Addition. Note  $\begin{cases} b=20 \\ c=12 \\ d=8 \end{cases}$  And that  $u$  is put for the unknown Quantity required.

| Literally                     |  | Numerally                  |  |
|-------------------------------|--|----------------------------|--|
| $u - d = b - c$               |  | $u - 8 = 20 - 12$          |  |
| Add $d$ $d$ Add               |  | Add 8      8 Add           |  |
| Sum $u = b - c + d$ the Answ. |  | Sum $u = 20 - 12 + 8 = 16$ |  |

2. By Subtraction.

|                                 |  |                         |  |
|---------------------------------|--|-------------------------|--|
| $u + d = b + c$                 |  | $u + 8 = 20 + 12$       |  |
| Deduct $d$ , $d$ Deduct         |  | Deduct 8,      8 Deduct |  |
| Rests $u = b + c - d$ the Answ. |  | $u = 20 + 12 - 8 = 24$  |  |

3. By Multiplication.

|                                   |  |                                                     |  |
|-----------------------------------|--|-----------------------------------------------------|--|
| $\frac{u}{4} = \frac{dc}{b}$      |  | $\frac{u}{4} = \frac{96}{20}$                       |  |
| $u = \frac{4dc}{b}$ = the Answer. |  | Answer $u = \frac{96 \times 4}{20} = 19\frac{1}{5}$ |  |

G g g

4. By



## 4. By Division.

$$du + cu = 5b$$

$$u = \frac{5b}{d+c} \text{ the Answer.}$$

$$8u + 12u = 20 \times 5$$

$$u = \frac{100}{8+12} = 5$$

## 5. By Addition, and Subtraction, or Transposition

$$u + c - d = b$$

$$u = b - c + d \text{ the Answer.}$$

$$u + 12 - 8 = 20$$

$$u = 20 - 12 + 8, = 16$$

## 6. By Addition, Subtraction, and Multiplication.

$$\frac{u-d}{2} + c = b-d$$

$$u-d + 2c = 2b-2d$$

$$u = 2b - d - 2c \text{ the Answer.}$$

$$\frac{u-8}{2} + 12 = 20-8$$

$$u-8 + 24 = 40-16$$

$$u = 40 - 16 + 8 - 24$$

$$\text{Or } u = 8$$

## 7. By Addition, Subtraction, Multiplication, and Division.

$$\frac{cu + du}{b} + d - b = 5c$$

$$cu + du + bd - bb = 3bc$$

$$cu + du = 3bc - bd + bb$$

$$u = \frac{3bc - bd + bb}{c+d} \text{ the Answer.}$$

$$\frac{12u + 8u}{20} + 8 - 20 = 3 \times 12 \text{ or } 36$$

$$12u + 8u + 20 \times 8 - 20 \times 20 = 36 \times 20$$

$$12u + 8u = 720 - 160 + 400$$

$$u = \frac{720 - 160 + 400}{12+8} \text{ (or } u = 48)$$

Eight General Rules for resolving Simple Equations, applied to the foregoing Examples.

1. When one or more known Negative Quantities are connected with an unknown Affirmative, they are separated, and the unknown made to possess solely one Side of the Equation (which is the same as to say that such Equations are resolved) by Addition only. So in the Example of resolving by Addition, I add the Quantity connected with  $u$ , as  $d$  to both Parts of the Equation, and the Sum is very plainly  $u = b - c + d$ .

The Reason of which is plain, and shews the Foundation of Transposition: (which Word see in the Alphabetical Account near the beginning of

of this Section) For adding the same Quantity or Number to both Parts of the Equation does preserve the Equality; but this Advantage is gained thereby, that  $+d$  added to  $-d$ , destroys the Quantity  $-d$ , so that it vanishes and leaves  $u$  in Possession of one Side of the Equation, and that  $+d$  being added to the other part of the Equation, makes  $b - c + d$ , to which  $u$  is therefore found equal. And the Reason appears still more plain in the Numerical Operation; for if  $u - 8 = 8$ , (or  $20 - 12$ ,) it must follow, that  $u = 8 + 8$  or 16, and this is no more than what is taught in *Subtraction of intire natural Numbers*.

2. When one or more known affirmative Quantities are connected with an unknown, that Equation is resolved by subtracting such known Quantities from each part of the Equation. Thus in the Example of the Resolving by Subtraction,  $d$  taken from  $+d$  (which is connected with  $u$ ) leaves nothing but  $u$  on that Side of the Equation; and  $d$  deducted from the other part of the Equation, there rests  $b + c - d$ , to which therefore  $u$  is equal, and the Equation is resolved.

The Reason of this is evident from what was said of adding in the first Rule above; for if equal Numbers or Quantities be either added to or taken from equal, the Sums or Remainders will be equal. And the Reason plainly appears also in the Numerical Work; for if  $u + 8 = 32$ , it follows that  $u = 32$ , abating only the 8.

3. When in an Equation, there are several known Quantities on the same Side with the unknown, some Affirmative, and some Negative; it follows from the above Rules, that you may transpose such known Quantities to the other Side of the Equation, changing their Signs, as in the fifth Example: And this you may do both ways to the Right, or thence towards the left hand of the  $=$ , and that with either known or unknown Quantities, keeping close to the Rules above, your own Reason, and the Nature of the Question: I mean, you will not think it reasonable to make the unknown Quantity a Negative, by transplacing it, when the Reason and Nature of the Question does dictate, that what is on the other Side of the Equation, is so far from being equal to  $-u$ , that it ought to be at least equal to  $u$ , or to some absolute Number or Quantity which  $-u$  is not.

4. When in an Equation, the unknown Quantity is divided by some Number or known Quantity, possessing one Side of the Equation; then every Member of such Equation must be multiplied by the Denominator of the Fraction, whereof the unknown Quantity is the Numerator, as in the third Example; I multi-

ply  $\frac{u}{4}$  by 4, produceth  $u$ ; and  $\frac{dc}{b}$  by 4, produceth  $\frac{4dc}{b}$ ; and so the Equation is resolved, the unknown Quantity being cleared and found  $= \frac{4dc}{b}$ , as the Numeral Work farther demonstrates; for equal Quantities multiplied by equal, produce equal, as well as adding and subtracting.

*The Reason is very evident*: For if a fourth Part of  $u$  be  $= \frac{dc}{b}$ , it must follow, that the whole  $u$  must be equal to four times the  $\frac{dc}{b}$ , and the like holds for any other Divisor of the unknown Quantity, or Denominator as abovesaid.

5. Hence it appears, that in multiplying by the Denominator of such a Fraction, 1<sup>st</sup>, *That every Member of both Parts of an Equation must be multiplied.* And 2<sup>dly</sup>, *That the Numerator of that Fraction is multiplied by putting it down as a whole Number, as  $\frac{1}{2} \times$  by 2, is  $= 1$ ;  $\frac{1}{4}$  by 4, is  $= 3$ ;  $\frac{1}{6}$  by 6, is  $= 5$ , &c.*

6. *When Quantities known and unknown stand multiplied together in an Equation possessing one Side thereof, such Equations are solved by Division; in which Case the Quantities so multiplied in the unknown, are to be your Divisor; as in the fourth Example or Division above. To separate  $u$  (in the two Members  $du + cu$ ) from the known Quantity  $d$  and  $c$ ; I divide both Parts of the Equation by  $d + c$ , and the*

Quotient is  $u = \frac{5b}{d+c}$ ; for equal Quantities divided by one and the same Quantity, must exhibit equal Quotients; and at the time you free one Side of the Equation of complex Quantities, and shew the known, which the unknown Quantity is equal to. Thus in the Numeral Example, the Matter is plain, for where  $8u + 12u = 100$ ;  $20u$  is  $= 100$ ; therefore  $u$  only must be equal to  $\frac{100}{20}$ , which is the same as in the Process, according to the Rule above; for  $8u + 12u \div 8 + 12 = u$ ; and  $20 \times 5$ , or  $100 \div 8 + 12 = 5$ .

*But Note*, That when you have two Members, one mixt with a known Quantity, the other intirely unknown; in this Case, the known  $+$  or  $-$  is your Divisor, as if  $8u - u = 49$ , then  $u = \frac{49}{8-1}$ , and if  $6u - u = 25$ , then  $u = \frac{25}{6-1} = 5$ .

7. When



7. When (as in the seventh Example) 'tis found that unknown Quantities are so mixt with known, as to have Products, Quotients, Sums, and Remainders; you have nothing to do but to work as by these foregoing Rules, as in  $\frac{cu+du}{b} + d - b = 3c$ : For if the whole is multiplied by  $b$ , it gives the next Step  $cu + du + bd - bb = 3bc$ . And if  $bd - bb$  be transposed, (as being all known Quantities,) the third Step stands thus;  $cu + du = 3bc - bd + bb$ . And if the third Step be divided by  $c + d$ , (because those are the known Quantities that are multiplied in the unknown,) the fourth Step or Quote, which is the Answer, is  $u = \frac{3bc - bd + bb}{c + d}$ ; each of which Steps appears still more plain in the Literal Work, which shews in this last Equation  $u = 48$ .

8. There is one other Thing to be observed, but that is rather in composing or forming the Equation, than in the Resolution, *i. e.* If  $u. c :: b. d$ . what is the Value of  $u$ ?

Here the second Step is found, as being the Rectangle of the two Extreams, which is equal to that of the means. And dividing by  $d$ , gives the Canon. See the Words *Analogy* in the beginning of this Chapter, and

*Equation*. And if you would reduce an Equation into an Analogy, it is as  $u$  to  $c$  or  $b$ ; so is the other to  $d$  in  $\left\{ \begin{array}{l} u. b :: c. d \\ u. c :: b. d, \&c. \end{array} \right.$  the two Examples above, or

$$u. c :: b. d$$

$$ud = bc$$

$$u = \frac{cb}{d} \text{ the Answer.}$$

#### SECT. X. Resolution of Quadratic Equations.

What simple and adfected Quadratics are, is shewed in the foregoing Definition of Terms near the beginning of this Chapter; where under the Words *Simple Quadratics*, I have said enough relating to that kind. As for adfected Quadratic Equations, (which Word see,) there are, by some Algebraists, reckoned three Sorts, but I make but two really distinct; to one of which, all other adfected Equations are reducible. I shall give Examples,

I. Of the two distinct Kinds of Square adfected Equations.

2dly, Of those reducible thereto, and how performed.

II. Shew how these Equations are Resolved by completing the Square.

III. How Resolved by Substitution; how done four different Ways, two of which (being the best) are new.

1st, The

1<sup>st</sup>, The two distinct Forms of Quadratic Equations are these only.

In Species.

In Numbers.

$$1^{\text{st}}, uu + du = bc \quad \text{---} \quad \text{---} \quad \text{---} \quad uu + 6u = 135$$

$$2^{\text{dly}}, uu - du = \frac{bc}{5} \quad \text{---} \quad \text{---} \quad \text{---} \quad uu - 6u = 27$$

2<sup>dly</sup>, Equations reducible to one of the two Forms above, are these and such like.

| Equations given.                                  | Reduced to these Quadratics.                                |
|---------------------------------------------------|-------------------------------------------------------------|
| 1 <sup>st</sup> , $du - uu = m$                   | $uu - du = -m$ , as the 2 <sup>d</sup> Form.                |
| 2 <sup>dly</sup> , $u + b = \frac{c-d+b}{2cu}$    | $uu + bu = \frac{c-d+b}{2c}$ , as the 1 <sup>st</sup> Form. |
| 3 <sup>dly</sup> , $u = \frac{c+db}{3bu} + b$     | $uu - bu = \frac{c+db}{3b}$ , as the 2 <sup>d</sup> Form.   |
| 4 <sup>thly</sup> , $u = \frac{cu+ru}{3buu} - bb$ | $uu + bbu = \frac{c+u}{3b}$ , the 1 <sup>st</sup> Form &c.  |

*Notes upon these Reductions.*

1<sup>st</sup>, That the first is performed by changing the Signs of all the Members, (as is directed in Rule the Third foregoing to *Simple Equations*) which makes  $-m = -du + uu$ , which is the same as to say  $uu - du = -m$ , as in the second Form; though some make this a distinct Sort, but with no more Reason than the three subsequent; for the Square is compleated, the Root extracted, and the Canon produced exactly as in one of the two Kinds abovementioned.

2<sup>dly</sup>, The second Equation above is reduced, as taught in the 4<sup>th</sup> and 6<sup>th</sup> Rules to the Examples of *Simple Equations*, multiplying by  $2cu$ , and dividing by  $2c$  each part of the Equation.

3<sup>dly</sup>, The third Equation above is reduced by the second, fourth, and sixth Rules to the Examples of *Simple Equations*, transposing  $b$ , multiplying by  $3bu$ , and dividing by  $3b$ .

4<sup>thly</sup>, The fourth is reduced by multiplying by  $3buu$ , expunging  $u$  in each Member, and dividing the rest by  $3b$ .

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II. *The Steps of resolving Quadratic Equations by completing the Square.*

| Equations proposed.       | Squares completed.<br>1 <sup>st</sup> ,                | Roots extracted.<br>2 <sup>dly</sup> ,                 | Canons exhibited.<br>3 <sup>dly</sup> ,                |
|---------------------------|--------------------------------------------------------|--------------------------------------------------------|--------------------------------------------------------|
| $uu + du = bc$            | $uu + du + \frac{dd}{4} = bc + \frac{dd}{4}$           | $u + \frac{d}{2} = \sqrt{\frac{bc + dd}{4}}$           | $u = \sqrt{bc + \frac{dd}{4}} - \frac{d}{2}$           |
| $uu - du = \frac{bc}{5}$  | $uu - du + \frac{dd}{4} = \frac{bc}{5} + \frac{dd}{4}$ | $u - \frac{d}{2} = \sqrt{\frac{bc}{5} + \frac{dd}{4}}$ | $u = \sqrt{\frac{bc}{5} + \frac{dd}{4}} + \frac{d}{2}$ |
| $uu + 2cu = 3b$           | $uu + 2cu + cc = 3b + cc$                              | $u + c = \sqrt{3b + cc}$                               | $u = \sqrt{3b + cc} - c$                               |
| $uu + u = 2b$             | $uu + u + \frac{1}{4} = 2b + \frac{1}{4}$              | $u + \frac{1}{2} = \sqrt{2b + \frac{1}{4}}$            | $u = \sqrt{2b + \frac{1}{4}} - \frac{1}{2}$            |
| $uu + 6u = 135$           | $uu + 6u + 9 = 135 + 9$                                | $u + 3 = \sqrt{135 + 9}$                               | $u = \sqrt{135 + 9} - 3$                               |
| $uu - 6u = \frac{135}{5}$ | $uu - 6u + 9 = \frac{135}{5} + 9$                      | $u - 3 = \sqrt{\frac{135}{5} + 9}$                     | $u = \sqrt{\frac{135}{5} + 9} + 3$                     |
| $uu + u = 90$             | $uu + u + \frac{1}{4} = 90 + \frac{1}{4}$              | $u + \frac{1}{2} = \sqrt{90 + \frac{1}{4}}$            | $u = \sqrt{90 + \frac{1}{4}} - \frac{1}{2}$            |

*Rules to be observed in this Method (which is the best) of the Resolution of Quadratic Equations, applied to the Examples above.*

1<sup>st</sup>, *Whatsoever known Quantities are multiplied in the Root, or unknown, they are called the Co-efficients; as d, 2c, and 1 in the second Terms of the four first Examples: In the fourth of which there being no Co-efficient, therefore 1 must be taken for it, because a Unit is supposed to stand before every Quantity, as u is = 1 u, c = 1 c, &c.*

2<sup>dly</sup>, *To compleat any Square having only the two first Members given, is done by adding the Square of half the Co-efficient:*  
Thus in the first and second Examples, half the Co-efficients  $\frac{d}{2}$ ,  
(the



(the Square of which is  $\frac{dd}{4}$ ) added to the two other Members standing towards the left hand, the compleat Squares are  $uu + du + \frac{dd}{4}$  and  $uu - du + \frac{dd}{4}$ . And in the third and fourth Examples  $2c$  and  $1$  are the Co-efficients, the half of which are  $c$  and  $\frac{1}{2}$ ; the Squares of which Halves are  $cc$  and  $\frac{1}{4}$ , the former of which compleats the Square of the two Members towards the left hand of the third Example, and the later (or  $\frac{1}{4}$ ) added, doth compleat the Square of the fourth Example: See at the beginning of this Chapter.

3dly, And because this Addition of the Square of half the Co-efficients is made to one part of the Equations, where the unknown Quantities are; therefore it must be added also to the other part of the Equations, *That so the Equality may be continued, (as is said in the first and second Rules to the Examples of Simple Equations,)* and thus the second Column is made in all the seven Examples above.

4thly, The third Column from the left hand, are the Roots of those in the second.

1st, For in all compleat Squares, the Square Roots are exactly those of the first and third Member.

2dly, The Sign connecting the Quantities of the Root, is always the same as that before the second Term of the compleat Square, or of the Equation given: Thus the Roots of

Squares.

Roots.

$uu + du + \frac{dd}{4}$  in the second Column, is  $u + \frac{d}{2}$  in the third Column.

$uu - du + \frac{dd}{4}$  is  $u - \frac{d}{2}$ .

$uu + 2cu + cc$  is  $u + c$ .

$uu + u + \frac{1}{4}$  is  $u + \frac{1}{2}$ , &c.

And for a Proof, any of these Roots being multiplied in itself, produces the respective Square above.

3dly, The

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3dly, The Root of the 1st Part being extracted, in order to discover the Value of the single unknown Quantity; therefore the Roots of the Quantities known, standing on the other part of the Equation, (as is said under the 3d of adding the Square of half the Coefficient) must have their Roots extracted; which is done by prefixing the Radical Sign, because they are generally Surds.

4thly, To reduce the 3d Column to the 4th, or to the Canons or Answers, you have nothing to do but to transpose the Root of the 3d Member of the compleat Square, as  $\frac{d}{2}$ ,  $c$ , and  $\frac{1}{2}$  in the four first Examples changing the Sign as taught.

*A farther Demonstration or Proof of the Method for the Solution of Quadratics, by the Numeral Operations above.*

In the Numeral Work these Numbers are equal to the Letters in the Specious, viz.  $d=6$ ,  $b=45$ ,  $c=3$ , and  $u$  is found equal to 9.

Here (proceeding by the Rules for Solution above) in the Equation  $uu+6u=135$ ,  $u$  in the 4th Column is found  $=\sqrt{135+9}-3$ .

Now  $135+9$  is  $=144$ , the Square Root of which is 12, whence take 3, and the rest is  $=9$ , so  $u=9$ .

And  $u$  being found  $=9$  by the Canon, grounded upon the Method of Process above said; if supposing  $u=9$ , the 2 Parts of the Equations proposed prove equal, it fully proves the Truth of the Method for Solution.

So  $uu+6u=81+54$  in the 5th Equation given, is  $=135$  the other part of that Equation given.

$uu-6u=81-54$  in the 1st part of the 6th Equation is  $=27$  the 2d part of that Equation.

And  $uu+u=81+9$  in the 1st part of the 7th Equation is  $=90$  in the 2d part of that Equation, &c.

Thus I have given the most plain and demonstrative Rules and Examples for the understanding and proving the Method of solving Square Equations by the way of compleating the Square. I proceed to shew,

## III. The Method of Solving Quadratic Equations by Substitution.

| Steps towards the Answer. |                                                    | How each Step is produced.                                                                     | Process towards an Answer. |                                                                        | Rules for each Step.                                                                           |
|---------------------------|----------------------------------------------------|------------------------------------------------------------------------------------------------|----------------------------|------------------------------------------------------------------------|------------------------------------------------------------------------------------------------|
| No. of Steps.             |                                                    |                                                                                                | No. of Steps.              |                                                                        |                                                                                                |
| 1                         | $uu + cu = 18d$ —                                  | = Equation given.                                                                              | 1                          | $uu - cu = 9d$ —                                                       | the Equation given                                                                             |
| 2                         | for $u$ is put $a - \frac{c}{2}$ in this 1st Case. |                                                                                                | 2                          | for $u$ is put $a + \frac{c}{2}$ in this 2d Case $u = a + \frac{c}{2}$ |                                                                                                |
| 3                         | $aa - ac + \frac{cc}{4}$ —                         | $\begin{cases} = \square a - \frac{c}{2} = uu \\ \text{or } a - \frac{c}{2} \odot \end{cases}$ | 3                          | $aa + ac + \frac{cc}{4}$ —                                             | $\begin{cases} = \square a + \frac{c}{2} = uu \\ \text{or } a + \frac{c}{2} \odot \end{cases}$ |
| 4                         | $ac - \frac{cc}{2}$ —                              | $= a - \frac{c}{2} \times c - cu$                                                              | 4                          | $-ac - \frac{cc}{2}$ —                                                 | $= a + \frac{c}{2} \times -c$                                                                  |
| 5                         | $aa - ac + \frac{cc}{4} + ac - \frac{cc}{2} = 18d$ | = 3d Step + 4th.                                                                               | 5                          | $aa + ac + \frac{cc}{4} - ac - \frac{cc}{2} = 9d$                      | = 3d + 4th Steps.                                                                              |
| 6                         | $aa - \frac{cc}{4} = 18d$ —                        | = 5th contracted.                                                                              | 6                          | $aa - \frac{cc}{4} = 9d$ —                                             | = 5th contracted.                                                                              |
| 7                         | $a = \sqrt{18d + \frac{cc}{4}}$ —                  | $= 6th\ w,$ and $\frac{cc}{4}$ +                                                               | 7                          | $a = \sqrt{9d + \frac{cc}{4}}$ —                                       | = 6th $w$ and $\frac{cc}{4}$ +                                                                 |
| 8                         | $u + \frac{c}{2} = \sqrt{18d + \frac{cc}{4}}$      | $u + \frac{c}{2}$ put for $a$                                                                  | 8                          | $u - \frac{c}{2} = \sqrt{9d + \frac{cc}{4}}$ —                         | $\begin{cases} u - \frac{c}{2} \text{ put for } a \\ \text{in the 7th Step.} \end{cases}$      |
| 9                         | $u = \sqrt{18d + \frac{cc}{4}} - \frac{c}{2}$      | $\frac{c}{2}$ in the 8 +                                                                       | 9                          | $u = \sqrt{9d + \frac{cc}{4}} + \frac{c}{2}$ —                         | $= -\frac{c}{2} +$<br>in the 8th Step.                                                         |

A 2d Example see towards the right hand.



SECT. X. *Quadratic Equations by Inspection.*

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I have here inserted the Operations at large, to make them more plain and easy; which the 3d and 6th Columns from the right hand do so fully contribute to, that I have little to add, only,

1. To put the Reader in mind, that the Signs connecting the substituted Quantities are always contrary to that which connecteth the 2 Members of the Square given, as is plain by the 2 Examples above.

2. That any Letter as well as  $a$  (if it be not in the Equation given)  $+$  or less half the Co-efficient, as above, may be substituted.

3. In the 8th Step I omit  $a$ , and substitute in place thereof  $u + \frac{c}{2}$  in the 1st Example, because if  $a - \frac{c}{2} = u$ , then by transposing  $-\frac{c}{2}$ ,  $a$  is  $= u + \frac{c}{2}$ , and in the 2d Example  $a = u - \frac{c}{2}$ .

*A New and Better Way.*

But tho' this is the only Way I know extant of solving these Equations by Substitution; yet I am sure it is a better, to make the Vowel and half the Co-efficient in the Quantities given to be your Substitute: and then it will fall right at the last, remembering that half the Co-efficient is to be added or deducted, and 'twill save some Steps of working.

For Example, putting  $u - \frac{d}{2}$  for  $u$ , in the Equation  $uu + du = bc$ .

Here it only needs to be noted, that when the Sign connecting the 2 first Terms of the Equation given is  $+$ , I make the half Co-efficient added to the Surd Root of the Answer  $-$ ; and the contrary.

But in truth this way of solving Equations by Substitution, is at the best not so brief and easy a Method as that by compleating the Square; for if that be well considered, there arises from it,

$$uu - du + \frac{dd}{4} = \square u - \frac{d}{2}$$

$$du - \frac{dd}{2} = u - \frac{d}{2} \times d.$$

$$uu - \frac{dd}{4} = bc; \left\{ \begin{array}{l} \text{the Sum of the 2} \\ \text{last contracted.} \end{array} \right.$$

$$u = \sqrt{bc + \frac{dd}{4}} - \frac{d}{2} = \text{the Canon.}$$

IV. *These Examples and Rules for solving most Quadratic Equations by Inspection only.*

Examples. Note,  $c=3$ , and  $u$  found  $=9$ .

1. Equations }  $uu + 10u = 57c$   
proposed. }

Canon —  $u = \sqrt{57c + 25} - 5$

Demonstrations of the Truth of the Solutions or Canons.

$\sqrt{57c + 25} - 5 = 9$ : so  $u = 9$  as per Canon 1.

and  $57c = 171$ : so is  $uu + 10u = 171$  as per Equation 1.

H h h 2

2. Equations

|                                            |                                                                                                                                                             |
|--------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 2. Equations<br>proposed. } $uu - 6u = 9c$ | $\sqrt{9c + 9} + 3 = 9 : \text{so } u = 9 \text{ as}$<br><i>per Canon 2.</i><br>$9c = 27 : \text{so is } uu - 6u = 27 \text{ as}$<br><i>per Equation 2.</i> |
| Canon — $u = \sqrt{9c + 9} + 3$            |                                                                                                                                                             |

*The Rules for Solving a Quadratic Equation by Inspection.*

1. What I mean here, by *Inspection*, is the putting down only the Canon or Answer without any previous Operation: To do which, take the Root of the Square of the unknown Quantity, which here is  $u$ .

2. Make the Sign of Equation, and to the right hand of that, place the Root of the unknown Quantity or Quantities given, with the Sign  $\sqrt{\phantom{x}}$  before, as in Equation the 1st,  $u = \sqrt{57c}$ .

3. Square half the Co-efficient, and add to the last mentioned Quantity with the Sign  $+$  always before it, and it will be  $u = \sqrt{57c + 25}$ .

4. To the last add half the Co-efficient, when the Sign is  $-$ , and subtract it when the Sign before the Quantity where the Co-efficient is, is  $+$ ; as in this Example I put  $-5$ , because  $10u$  is  $+$   $10u$ : so is the Canon  $u = \sqrt{57c + 25} - 5$ .

And thus you may give the Canon for any Quadratic Equation when reduced into either of the two Forms above.

Here is the proper place for giving the Geometrical Construction of Simple and Quadratic Equations; which I omit for these four Reasons: 1. Because it is done by so many other Authors. 2. For that this Treatise is designed to be as easy as possible, in order to instruct and encourage the Tyro, rather than to make a shew of Learning. 3. It cannot be expected, where so many things are contained in one small Volume. And, 4. It is sufficiently proved and demonstrated above, by comparing the Parts of the Canon and those of the Equation given, as grounded on the Solution, especially in the Numeral Operations.

SECT. XI. *Containing for Exercise, some Questions resolved by Simple and Quadratic Equations, and by various Positions.*

I. **B**Y Simple Equations. I had the following proposed to me, which the Querist, my Friend, had a real Occasion to know, as falling in his Business.

*Problem-1.*] 365 Pounds is received for the Use of a publick Building, and the Receiver was to have 4d per Pound for what he paid in: what was the Poundage, and what paid in?

*Steps*

| Steps. | Done Literally thus.                                | The Column<br>of Rules to<br>both Ways | Numerally thus.                          | Steps. |
|--------|-----------------------------------------------------|----------------------------------------|------------------------------------------|--------|
| 1      | $u = \text{the Poundage, and } b \text{ is } = 365$ |                                        | $u = \text{the Poundage.}$               | 1      |
| 2      | $b - u = \text{the Sum paid in.}$                   |                                        | $365 - u = \text{the Sum paid in.}$      | 2      |
| 3      | $60 u = b - u$                                      |                                        | $60 u = 365 - u$                         | 3      |
| 4      | $61 u = b$                                          | $u \text{ in } 3d +$                   | $61 u = 365$                             | 4      |
| 5      | $u = \frac{b}{61} \text{ the Canon.}$               | $\text{the } 4th \div$<br>$61$         | $u = \frac{365}{61} = \text{the Canon.}$ | 5      |
| <hr/>  |                                                     |                                        |                                          |        |
|        | So $\frac{b}{61} = \text{the Poundage.}$            |                                        | $365 \div 61 = 1.5:19:8$                 |        |
|        | And $b - \frac{b}{61} = \text{the Sum paid in.}$    |                                        | and $365 - \frac{365}{61} = 359:0:4$     |        |

Problem 2.] *The whole Length of a May-Pole is 16 Yards, the major part of which is broke by the Wind, tho' not quite off, but so that the Top of the Pole extends from the Fracture to the Ground at 8 Yards distant from the Appearance of the Pole above the Surface of the Earth in which it was placed: how much is broken off, and how much is standing?*

| Steps                                              | I put $16 = k$ and $8 = n$                           |                                  |                                                          | Steps |
|----------------------------------------------------|------------------------------------------------------|----------------------------------|----------------------------------------------------------|-------|
| 1                                                  | $u = \text{the Perpendicular or}$<br>(Part standing. | Rules, viz.                      | $u = \text{the part standing.}$                          | 1     |
| 2                                                  | $k - u = \text{Hypothenufe.}$                        |                                  | $16 - u = \text{the Hypothenufe}$                        | 2     |
| 3                                                  | $nn + uu = kk - 2ku + uu$                            | $n \odot + 1st \odot + 2d \odot$ | $8^2 + u^2 = 16^2 - 2 \times 16u + u^2$                  | 3     |
| 4                                                  | $2ku = kk - nn$                                      | $2ku +, nn +$                    | $2 \times 16u = 16^2 - 8^2$                              | 4     |
| 5                                                  | $u = \frac{kk - nn}{2k} = \text{the Canon.}$         | $4th \div 2k$                    | $u = \frac{16^2 - 8^2}{2 \times 16} = \text{the Canon.}$ | 5     |
| <hr/>                                              |                                                      |                                  |                                                          |       |
| Demonstration of both.                             |                                                      |                                  |                                                          |       |
| $k^2 = 256 - n^2 = 64 = 192 \div 2k = 32 = u = b.$ |                                                      |                                  | $16^2 = 256 - 8^2 = 64 = 192$                            |       |
|                                                    |                                                      |                                  | $\div 32 = 6 = \text{the part}$                          |       |
| Vid. Euclid's Elem. Lib. 1. Prop. 47.              |                                                      |                                  | (standing.)                                              |       |

Problem 3.] *Numbers in Geometrical Progression being given, to find the Rule for the Aggregate of any Series. This is shewn in Progression, Chap. II. pag. 82.*

Problem 4.] *Supposing the Pendulum of a Clock which vibrateth Seconds (or 60 times in a Minute) be 39.2 Inches in length; how long must a Pendulum be to, vibrate as often in a Minute as it bath Inches in length? This also was a Question proposed for me to answer, which I did thus:*

Steps.



I put  $k=60$  Seconds: and  $n=39.2$  Inches.

| Steps. |                                                                                                             | Rules for the Work.                                                                                                                    |                                                    | Steps. |
|--------|-------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------|--------|
| 1      | $n =$ the No of Vibrat. requir'd.<br><i>The Analogy.</i><br>$\square$ Vib. Le. Pen. $\square$ Vib. Le. Pen. |                                                                                                                                        | $u =$ No of Vibrat.                                | 1      |
| 2      | $kk. \quad n. \quad uu. \quad \frac{kkn}{uu}$                                                               | $\left\{ \begin{array}{l} \text{Or } uu. kk :: n. \frac{kkn}{uu} \\ \text{in a direct Pro-} \\ \text{(portion.)} \end{array} \right\}$ | $60^2.39.2 : uu.39.2$<br>$\times 60^2 \div uu$     | 2      |
| 3      | $u = \frac{kkn}{uu}$                                                                                        | $\left\{ \begin{array}{l} uu \cdot uw = \frac{kkn}{uu} \\ \text{or } \sqrt{uu} = \&c. \end{array} \right\}$                            | $u = \frac{60^2 \times 39.2}{uu}$                  | 3      |
| 4      | $uuu = kkn$                                                                                                 | 3d Reduced.                                                                                                                            | $uuu = 60^2 \times 39.2$                           | 4      |
| 5      | $u = \sqrt[3]{kkn}$                                                                                         | $= \sqrt[3]{\text{of the 4th}}$                                                                                                        | $u = \sqrt[3]{60 \times 39.2}$<br>(viz. of 141120) | 5      |

$$kk = 3600, \times n (=39.2) = 141120; \text{ and } \sqrt[3]{141120} = 52 \text{ near.}$$

$$\sqrt[3]{141120} = 52 \text{ near.}$$

And Inches for the Length of such a Pendulum.

Problem 5.] What Age is that Person when  $\frac{2}{3}$  of the Time from his Birth is equal to  $\frac{1}{3}$  of what he wants of 60?

| Steps. |                                   | Rules for the Steps.         |                                                                     | Steps. |
|--------|-----------------------------------|------------------------------|---------------------------------------------------------------------|--------|
| 1      | $u =$ the Person's supposed Age.  |                              | $u =$ his Age.                                                      | 1      |
| 2      | $k - u$ the time he wants of 60.  | or $60 - u$                  | $60 - u$                                                            | 2      |
| 3      | $\frac{u}{n} = \frac{k - u}{t}$   | $u \div n, \&c k - u \div t$ | $u \div \frac{2}{3} = 60 - u \div \frac{1}{3}$                      | 3      |
| 4      | $u = \frac{kn - nu}{t}$           | the 3d $\times$ by $n$       | $u = 60 \times \frac{2}{3} - u \times \frac{2}{3} \div \frac{1}{3}$ | 4      |
| 5      | $tu = kn - nu$                    | the 4th $\times$ by $t$      | $\frac{7u}{8} = 60 \times \frac{2}{3} - u \times \frac{2}{3}$       | 5      |
| 6      | $tu + nu = kn$                    | $nu$ in 5, +                 | $\frac{7u}{8} + \frac{2u}{3} = 60 \times \frac{2}{3}$               | 6      |
| 7      | $u = \frac{kn}{t+n} =$ the Canon. | 6th $\div t + n$             | $u = 60 \times \frac{2}{3} \div \frac{1}{3} + \frac{2}{3}$          | 7      |

So that  $\frac{kn}{t+n} =$  the Age; and

$k - \frac{kn}{t+n} =$  it wants of 60.

And  $\frac{kn}{t+n} \div n = k - \frac{kn}{t+n} \div t$  for

Proof.

$u = 25 \frac{1}{3}$  Years, the Age, and

$60 - 25 \frac{1}{3} = 34 \frac{2}{3}$  Years, the want of 60.

And  $25 \frac{1}{3} \div \frac{2}{3} = 34 \frac{2}{3}$  Years  $\div \frac{1}{3}$  for a Proof.

Problem 6.] Two Persons, A and B, have each a certain Number of Pounds, so that if B give A 3 of his, then they will have each a like Number: But if A give B three of his Pounds, then B will have 5 times as many as A: What Number of Pounds had each at first?

I put  $n=3$  and  $k=5$ .

|                                         |                                   |                                                                        |                                             |   |
|-----------------------------------------|-----------------------------------|------------------------------------------------------------------------|---------------------------------------------|---|
| 1                                       | $u=B$ hath at first.              | $u$ supposed                                                           | $u=B$ hath at first.                        | 1 |
| 2                                       | $u-2n=A$ hath at first.           | 1st $\frac{2n}{2}$                                                     | $u-3 \times 2$                              | 2 |
| 3                                       | $u-n=B$ will have.                | <i>ibid.</i> as 3d.                                                    | $u-3$                                       | 3 |
| 4                                       | $u-n=A$ will have.                | $=u-2n-n$                                                              | $u-3$                                       | 4 |
| 5                                       | $u-3n=A$ will after have.         |                                                                        | $u-9$                                       | 5 |
| 6                                       | $u+n=B$ will then have.           |                                                                        | $u+3$                                       | 6 |
| 7                                       | $ku-3kn=u+n$                      | 5th $\times k+6th$                                                     | $5u-3 \times 5 \times 3 = u+3$              | 7 |
| 8                                       | $ku-u=n+3kn$                      | $u+8 \times 3ku+$                                                      | $5u-u=3+3 \times 5 \times 3$                | 8 |
| 9                                       | $u=\frac{n+3kn}{k-1}$ the Answer. | 8th $\div k-1$                                                         | $u=\frac{3+3 \times 5 \times 3}{4}$ Answer. | 9 |
| $\frac{u+3kn}{k-1}=4n$ , which $B$ had. |                                   | $\frac{3+3 \times 5 \times 3}{4}=12$ , w <sup>ch</sup> $B$ at 1st had. |                                             |   |
| And $A$ had $2n$ : for $2n-n$           |                                   | And $A$ had 6: for $6-3=3$ ; and                                       |                                             |   |
| $=n$ : and $n+4n=5n$ .                  |                                   | $3+12=5 \times 3=15$ .                                                 |                                             |   |

Problem 7.] A Hare, 50 of her Steps or Paces before a Greyhound, takes 4 Steps to the Dog's three; but then the Dog steps as much at twice as the Hare does at thrice: How many Steps must the Dog make before he catch the Hare? This old Algebraic Question I answer thus:

I put  $k=50$ ,  $n=3$ ,  $s=2$ , and  $b=4$ .

|   |                                                                        |                                                      |   |
|---|------------------------------------------------------------------------|------------------------------------------------------|---|
| 1 | $u$ = the supposed Answer (or Dog's Steps.                             | $u$                                                  | 1 |
| 2 | $n.b::u.$ $\frac{bu}{n}$ all the Hare's (actual Steps.                 | $3.4::u.$ $\frac{4u}{3}$                             | 2 |
| 3 | $n.s::\frac{bu}{n}+k.$ $\frac{bsu}{nn}+\frac{ks}{n}$ (the Dog's Steps. | $3.2::\frac{4u}{3}+50.$ $\frac{8u}{9}+\frac{100}{3}$ | 3 |
| 4 | $u=\frac{bsu}{nn}+\frac{ks}{n}$ the 4th $\times nn$ , &c.              | $u=\frac{8u}{9}+\frac{100}{3}$                       | 4 |
| 5 | $nnu-bsu=kns$                                                          | $9u-8u=\frac{900}{3}$ . Ergo                         | 5 |
| 6 | $u=\frac{kns}{nn-bs}$ the Canon for (Answer.                           | $u=\frac{900}{3}=300$ Steps, the (Answer.            | 6 |

*Problem 8.]* A Cistern that hath four Vents of different Dimensions containeth 72 Barrels of Liquid: If one Vent or Cock be opened, the Liquid will be spent in 6 Hours; if a second be opened alone, it will run out in 8 Hours; if the third Cock be turned alone, the Liquid will be exhausted in 9 Hours; and the fourth Cock or Vent is so small, that if it alone be opened, it will be twelve Hours before the Cistern is empty: Now the Question is, in what Time the Cistern will be void of Liquid, if all the four Vents be opened at the same Moment of Time. This Question (which I have not as I know of, seen before answered by Algebra) I solve thus; tho' there is somewhat a shorter way: but I design it plain to the Learner, and if 72 or  $k$  be omitted and supposed 1, it will come to the same Canon, as you may make trial at your leisure, for  $k$  is thrown out even in this Example.

For 72 I put  $k$ ,  $6 = n$ ,  $8 = s$ ,  $9 = t$ ,  $12 = r$ .

|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p><math>u = \text{the unknown Time or sup-}</math><br/><math>(\text{posed Answer.})</math></p> <p>1 <math>n. k :: u. \frac{ku}{n}</math></p> <p>2 <math>s. k :: u. \frac{ku}{s}</math></p> <p>3 <math>t. k :: u. \frac{ku}{t}</math></p> <p>4 <math>r. k :: u. \frac{ku}{r}</math></p> <p>5 So that <math>\frac{ku}{n} + \frac{ku}{s} + \frac{ku}{t} + \frac{ku}{r} = k</math></p> <p>6 <math>\frac{u}{n} + \frac{u}{s} + \frac{u}{t} + \frac{u}{r} = 1</math></p> <p>7 <math>u + \frac{nu}{s} + \frac{nu}{t} + \frac{nu}{r} = n</math></p> <p>8 <math>su + nu + \frac{nsu}{t} + \frac{nsu}{r} = ns</math></p> <p>9 <math>stu + ntu + nsu + \frac{nstu}{r} = nst</math></p> <p>10 <math>rstu + nrtu + nrsu + nstu = nrst</math></p> <p>11 <math>u = \frac{nrst}{rst + nrt + nrs + nst} = \text{Ans.}</math></p> <p>And <math>\frac{nrst}{rst + nrt + nrs + nst} = \frac{5184}{2520} \text{ Hours.}</math></p> | <p><math>u</math></p> <p>6. <math>72 :: u. \frac{72u}{6}</math></p> <p>8. <math>72 :: u. \frac{72u}{8}</math></p> <p>9. <math>72 :: u. \frac{72u}{9}</math></p> <p>12. <math>72 :: u. \frac{72u}{12}</math></p> <p><math>\frac{72u}{6} + \frac{72u}{8} + \frac{72u}{9} + \frac{72u}{12} = 72</math></p> <p>5th Contract. <math>\frac{u}{6} + \frac{u}{8} + \frac{u}{9} + \frac{u}{12} = 1</math><br/>or <math>\div</math> by <math>k</math></p> <p>the 6th <math>\times n</math> <math>u + \frac{6u}{8} + \frac{6u}{9} + \frac{6u}{12} = 6</math></p> <p>the 7th <math>\times s</math> <math>8u + 6u + \frac{6u \times 8}{9} + \frac{6u \times 8}{12} = 6 \times 8</math></p> <p>the 8th <math>\times t</math> <math>8u \times 9 + 6u \times 9 + 6u \times 8 + \frac{6u \times 8 \times 9}{12} = 6 \times 8 \times 9</math></p> <p>the 9th <math>\times r</math> <math>8u \times 9 \times 12 + 6u \times 9 \times 12 + 6u \times 8 \times 12 + 6u \times 8 \times 9 = 6 \times 8 \times 9 \times 12</math></p> <p>the 10th <math>\div</math> all <math>u = \frac{8 \times 9 \times 12 + 6 \times 9 \times 12 + 6 \times 8 \times 12 + 6 \times 8 \times 9}{2520}</math><br/>that are <math>\times u</math></p> <p>Numer. = 5184; Denom. = 2520 = 2:3:25<br/>which is the accurate Answer.</p> |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|



*Problem 9.]* How shall I find out a Canon for discovering the Discount of Money? See the Operation.

$d$  = Days to come.  $c$  = 100*l*.

$p$  = Principal.  $r$  = Rate of Int.

$y$  = a Year, or 365 Days.

$u$  = the Discount fought.

$$y.r :: d.\frac{dr}{y}$$

$$\text{And } c + \frac{dr}{y} :: p.u$$

$$cu + \frac{dr u}{y} = \frac{dpr}{y}$$

or  $cuy + dru = dpr$ , and consequently  $u = \frac{dpr}{cy + dr}$  the Canon

for Discount.

*Problem 10.]* To find a Canon for discovering the present Worth of Money due at the end of any Days. Symbols as above.

$$y.r :: d.\frac{dr}{y}$$

$$c + \frac{dr}{y} :: p.u$$

$$cu + \frac{dr u}{y} = cp$$

$cuy + dru = cpy$ ; and consequently  $u = \frac{cpy}{cy + dr}$  the Canon

for present Worth.

These Canons apply'd under Sect. 8. Chap. 3. find the Discount of Money much sooner than by the common Way of the Rules of Proportion.

II. To resolve Questions by Quadratic Equations, which they form.

*Problem 1.]* There are three Numbers in Geometrical Proportion continued; the Mean = 42, the Difference between the Extremes = 112: What are the two Extremes?

I put  $k=112$ ;  $n=42$ .

1  $u$  = the lesser Extreme.

2  $u + k$  the greater.

3  $uu + ku = nn$

$$4 \quad u = \sqrt{nn + \frac{kk}{4}} - \frac{k}{2} \text{ Answ.}$$

$u$  = the lesser Extreme.

$u + 112$  = the greater.

$uu + 112u = 42^2$

$$u = \sqrt{42^2 + 56^2} - 56$$

$$\sqrt{nn + \frac{kk}{4}} - \frac{k}{2} = 14 = \text{the lesser.}$$

$$\sqrt{42^2 + 56^2} - 56 = 14 \text{ the lesser.}$$

And  $14 + k$  = the greater Extreme.

And  $14 + 112 = 126$  the greater.

And for Proof  $14 \times 126 = 42^2 = 1764$ .

*Problem 2.]* To divide a Line into extreme and mean Proportionals: or (which is one Use thereof) to find the side of the Dodecahedron

hedron by having that of the Octahedron given, whereof this is the greater proportional part.

A Line is thus divided, when the Square of the greater Part is = the Rectangle under the whole Line and the lesser Part.

I put  $k =$  to the Side of the Octahedron ( $=ab$ )  $a \text{ --- } | \text{ --- } b$

|                                                     |                                                                                                                                                       |                                                      |   |
|-----------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------|---|
| 1 $u =$ the greater part (or the                    | $\left\{ \begin{array}{l} \text{the 1st } \odot; \\ \& k \times \text{ in 2d} \\ \text{the 3d } uw \\ \text{and } \frac{k}{2} + \end{array} \right\}$ | $u = ac$ of the Line $ab$ .                          | 1 |
| [side of the Dodecahedron]                          |                                                                                                                                                       |                                                      |   |
| 2 $k - a =$ the lesser part ( $ascb$ )              |                                                                                                                                                       | $120 - u = cb$ of the Line $ab$ .                    | 2 |
| 3 $uu = kk - ku$                                    |                                                                                                                                                       | $uu = 120^2 - 120u$                                  | 3 |
| 4 $uu + ku = kk$                                    |                                                                                                                                                       | $uu - 120u = 120^2$                                  | 4 |
| 5 $u = \sqrt{kk + \frac{kk}{4} - \frac{k}{2}} = ac$ |                                                                                                                                                       | $u = \sqrt{120^2 + \frac{120^2}{4} - \frac{120}{2}}$ | 5 |
| (Answ.)                                             |                                                                                                                                                       |                                                      |   |

So that  $u = ac$  of the Line  
 $ab =$  the Side required.

And  $\sqrt{120^2 + \frac{120^2}{4} - \frac{120}{2}} = 74.16$   
(near.)

**Problem 3.]** A Merchant sold a Ship for  $l. 2400$ , and gained after the Rate of what the Ship cost by laying out  $10000$  Pounds: What did the Vessel cost him?

I put  $s = 2400$ , and  $t = 10000$ .

|                                                |                                                                                                                                                                                                       |                                         |   |
|------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------|---|
| 1 $u =$ supposed Cost.                         | $\left\{ \begin{array}{l} \text{or the 1st} \\ \text{4th Pro-} \\ \text{portional,} \\ \text{4th } \times u \text{ and} \\ \text{2 } tu + \\ \text{5th Step} \\ \text{uw, \&c.} \end{array} \right\}$ | $u$                                     | 1 |
| 2 $s - u =$ the Gain.                          |                                                                                                                                                                                                       | $2400 - u$                              | 2 |
| 3 $u \cdot s - u :: t$                         |                                                                                                                                                                                                       | $u \cdot 2400 - u :: 10000$             | 3 |
| 4 $u = \frac{st - tu}{u}$                      |                                                                                                                                                                                                       | $u = \frac{2400 - u \times 10000}{u}$   | 4 |
| 5 $uu + tu = st$                               |                                                                                                                                                                                                       | $uu + 10000u = 24000000$                | 5 |
| 6 $u = \sqrt{st + \frac{tt}{4} - \frac{t}{2}}$ |                                                                                                                                                                                                       | $u = \sqrt{24000000 + 25000000 - 5000}$ | 6 |
| $= 2000$                                       |                                                                                                                                                                                                       | $(= 2000)$                              |   |

And for Proof  $2000. 400 :: 10000. 2000$

**Problem 4.]**

# SECT. XI. Questions by Quadratic Equations.

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Problem 4.] Admit *bin* (Fig. 9. Plate C.) to be the Curve of an Hyperbola, and that there are given

*i* (= *io*) the Latus transversum = 7.4

*l* (= *ib*) the Latus rectum = 1.76

*o* (= *zb*) the greatest Ordinat = 4.68

What is the Length of the Abscissa, (*iz*) for which I put *u* because unknown?

|   |                                                         |                                         |                                                                               |  |  |  |   |
|---|---------------------------------------------------------|-----------------------------------------|-------------------------------------------------------------------------------|--|--|--|---|
| 1 | $oo = lu + \frac{luu}{t} = \text{the Pro-}$             |                                         |                                                                               |  |  |  |   |
|   | (perty of the Curve.)                                   |                                         |                                                                               |  |  |  |   |
| 2 | $ltu + luu = oot$                                       | ist $\times$ by <i>t</i>                | $4.68^2 = 1.76 \times u + \frac{1.76 \times uu}{7.4}$                         |  |  |  | 1 |
| 3 | $uu + tu = \frac{oot}{l}$                               | 2d $\div$ by <i>l</i>                   | $uu + 7.4 \times u = \frac{4.68^2 \times 7.4}{1.76}$                          |  |  |  | 2 |
| 4 | $u = \sqrt{\frac{oot}{l} + \frac{tt}{4}} - \frac{t}{2}$ | the 3d <i>uw</i> ,<br>& $\frac{t}{2} +$ | $u = \sqrt{\frac{4.68^2 \times 7.4}{1.76} + \frac{7.4^2}{4}} - \frac{7.4}{2}$ |  |  |  | 3 |
|   | $= 6.7$                                                 |                                         | $= 6.7$                                                                       |  |  |  | 4 |

Problem 5.] What Number is that, whose Square multiplied by 10, being added to its Biquadrate, the Sum makes 30251?

I put  $k = 30251$ , and  $10 = n$

|   |                                                    |                                                                                            |                                                                |  |  |  |   |
|---|----------------------------------------------------|--------------------------------------------------------------------------------------------|----------------------------------------------------------------|--|--|--|---|
| 1 | $u = \text{the Answer unknown.}$                   |                                                                                            |                                                                |  |  |  |   |
| 2 | $u^4 + nuu = k$                                    | $\left\{ \begin{array}{l} 2d + \frac{n^2}{2} \end{array} \right\}$                         | $uuuu + 10uu = 30251$                                          |  |  |  | 1 |
| 3 | $uuuu + nuu + \frac{nn}{4} = k + \frac{nn}{4}$     | or $\left\{ \begin{array}{l} \frac{nn}{4} \end{array} \right\}$                            | $u^4 + 10uu + \frac{10^2}{4} = 30251 + \frac{10^2}{4}$         |  |  |  | 2 |
| 4 | $uu + \frac{n}{2} = \sqrt{k + \frac{nn}{4}}$       | 3d <i>uw</i>                                                                               | $uu + \frac{10}{2} = \sqrt{30251 + \frac{10^2}{4}}$            |  |  |  | 3 |
| 5 | $u = \sqrt{\sqrt{k + \frac{nn}{4}} - \frac{n}{2}}$ | 4th Step<br><i>uw</i> and<br>$\left\{ \begin{array}{l} \frac{n}{2} + \end{array} \right\}$ | $u = \sqrt{\sqrt{30251 + \frac{10^2}{4}} - \frac{10}{2}} = 13$ |  |  |  | 4 |

And for Proof  $13^2 = 169 \times 10 = 1690 + 13^4 (= 28561) = 30251$ .

The last Canon for the Answer = *u*, is thus to be read: That if half *n* be deducted from the Square Root of  $k + \frac{nn}{4}$ , the Square Root of the Remainder is = *u* = 13.



**Problem 6.]** A Farmer left his Son and a Nephew £. 400 to be so divided between them; that their Parts being alternately divided by each other, the Sum of the Quotients will be 30; but the greater part he devised to his Son: What did he leave to each?

Some make this Question, which I propose as above, to be solved by various Positions, others by the Doctrine of Surds: But I am sure it is most naturally reducible to the 2d of my 2 Cases of Quadratics before mentioned. And because I have not seen it done before, I shall insert the Solution at large as it is most easily understood, and that by the numeral Way; but shall waver the literal Method on purpose that the Reader may try his own Proficiency therein.

|    |                                                  |                                                           |
|----|--------------------------------------------------|-----------------------------------------------------------|
| 1  | $u = \text{the Son's Share.}$                    |                                                           |
| 2  | $400 - u = \text{the Nephew's Share.}$           | 400 — the 1st Step.                                       |
| 3  | $\frac{u}{400 - u} + \frac{400 - u}{u} = 30$     | 1st $\div$ 2d, and 2d $\div$ by 1st                       |
| 4  | $u + \frac{160000 - 800u + uu}{400 - u} = 12000$ | the 3d $\times$ by $400 - u$                              |
| 5  | $uu + 160000 - 800u + uu = 12000(400 - u)$       | the 4th $\times$ by $u$                                   |
| 6  | $160000 = 12800u - 32uu$                         | { the 5th contracted,<br>and $2uu - 800u +$               |
| 7  | $5000 = 400u - uu$ ; and Ergo                    | the 6th $\div$ by 32                                      |
| 8  | $uu - 400u = -5000$ an Equation of (the 2d sort. | { 5000 (in the 7th +<br>and $400u - uu +$                 |
| 9  | $uu - 400u + 40000 = 35000$                      | $\frac{400}{2}$ @ and added to each part of the Equation. |
| 10 | $u - 200 = \sqrt{35000}$                         | the 9th $uu$                                              |
| 11 | $u = \sqrt{35000} + 200$                         | — 200 in the 10th +                                       |

And

And  $\sqrt{35000} + 200 = \text{Son's Share} = 1387.083$ ; and  $400 - 387.083 = 12.917$

And for Proof  $387.083 \div 12.917 = 29.966$ ; and  $12.917 \div 387.083 = .034$ ; and  $.034 \times 29.966 = 30$ ; which answers in every part the Requisites.

*Problem 7.]* What Number is that by which if we divide 20, and to that Quote add 4 times that unknown Number the Sum is 24?

I put  $b = 4$ ,  $t = 20$ ,  $c = 24$ .

$$\frac{t}{a} + ab = c$$

$$t + aab = ac$$

$$aab - ac = -t$$

$$aa - \frac{ac}{b} = -\frac{t}{b}$$

$$aa - \frac{ac}{b} + \frac{cc}{4bb} = -\frac{t}{b} + \frac{cc}{4bb}$$

$$a - \frac{c}{2b} = \sqrt{-\frac{t}{b} + \frac{cc}{4bb}}$$

$$a = \sqrt{-\frac{t}{b} + \frac{cc}{4bb}} + \frac{c}{2b} = \text{true}$$

(Answer  $= 5 = 5 = a$ )

I have inserted this last Question that the Learner may see how to manage when the 2d Term (or the known Number multiplied in the Root) is a Fraction; as in the 4th Step foregoing.

### III. The Resolution of Questions which require various Positions.

Having in Sect. 9, and 10. given Rules for solving Simple and Quadratic Equations; there was no need in Sect. 11. that I should repeat any thing farther than what I have done in the middle Column of Rules: But in this Head of various Positions having said nothing thereon before, I shall make it as plain as I can.

1. Then

1. Then by various Positions we mean Questions which have more Numbers than 1 sought for, and are discovered by assuming a Vowel for each Number sought.

2. And we proceed in the Work first with one unknown Number, not regarding the rest but as known: And when we have found the first unknown Quantity equal to some others, we omit that Quantity, and take instead thereof what it is equal to. Examples, and Rules thereupon, will make the matter plain.

*Problem 1.]* There is a Composition of 20 Integers, whose total Value is 82; but there are 2 different prized Ingredients in the 20, 1 of 4, the other of  $4\frac{1}{4}$ : How much of each Simple is there in the said Composition, and what the Value of each Ingredient in the whole of it, to make up the 82?

| I put $q=20$ , $4=k$ , $4\frac{1}{4}=n$ , and $82=t$ . |                                         |  | Rules for the Operations.  |  |                                         |        |
|--------------------------------------------------------|-----------------------------------------|--|----------------------------|--|-----------------------------------------|--------|
| Steps.                                                 |                                         |  |                            |  |                                         | Steps. |
| 1                                                      | $u$ = the Quantity at 4                 |  |                            |  | $u$                                     | 1      |
| 2                                                      | $a$ = the Quantity at $4\frac{1}{4}$    |  |                            |  | $a$                                     | 2      |
| 3                                                      | $ku + an = t$                           |  | { the 1st and 2d }         |  | $4u + 4\frac{1}{4}a = 82$               | 3      |
| 4                                                      | $ku = t - an$                           |  | { $\times k$ and $nan +$ } |  | $4u = 82 - 4\frac{1}{4}a$               | 4      |
| 5                                                      | $u = \frac{t - an}{k}$                  |  | 4th $\div$ by $k$          |  | $u = \frac{82 - 4\frac{1}{4}a}{4}$      | 5      |
| 6                                                      | $\frac{t - an}{k} + a = q$              |  | { 2d part of 5th }         |  | $\frac{82 - 4\frac{1}{4}a}{4} + a = 20$ | 6      |
| 7                                                      | $t - an + ak = kq$                      |  | the 6th $\times$ by $k$    |  | $82 - 4\frac{1}{4}a + 4a = 80$          | 7      |
| 8                                                      | $t - kq = an - ak$                      |  | $-an + ak +$ in 7th        |  | $82 - 80 = 4\frac{1}{4}a - 4a$          | 8      |
| 9                                                      | $a = \frac{t - kq}{n - k}$ = the Canon. |  | the 8th $\div$ $n - k$     |  | $\frac{82 - 80}{4\frac{1}{4} - 4} = a$  | 9      |

Hence appears the Canon to find the Quantity at  $4\frac{1}{4}$ , viz.  $\frac{82 - 80}{4\frac{1}{4} - 4}$

$$= 2 \div \frac{1}{4} = 8. \text{ and } 20 - 8 = 12 = a.$$

For Proof of which, 12 at 4 = 48; and 8 at  $4\frac{1}{4}$  = 34: Now  $48 + 34 = 82$  = the total Value.

*Problem 2.]* A Person dying, left by Will to 2 Sons each a Sum of Money; so that if to 5 times what he left the elder you add 3 times what



SECT. XI. Questions by various Positions.

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what he gave to the younger, the Sum will be 1675: And if from 7 times the Share of the elder Brother you deduct 8 times the Share of the younger, the Remainder will be 210: What Sum did the Father leave to each?

I put  $s = 1675$ ,  $d = 210$ ,  $k = 5$ ,  $n = 3$ ,  $m = 7$ , and  $t = 8$ .

|                                                    |                                                                                                                                                                          |   |
|----------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---|
| 1 $u =$ the Legacy<br>(of the Elder.               | $u$                                                                                                                                                                      | 1 |
| 2 $a =$ that of the<br>Younger.                    | $a$                                                                                                                                                                      | 2 |
| 3 $ku + an = s$ .                                  | $5u + 3a = 1675$                                                                                                                                                         | 3 |
| 4 $u = \frac{s - an}{k}$ .                         | $3d \div k; \text{ and } an + u = \frac{1675 - 3a}{5}$                                                                                                                   | 4 |
| 5 $\frac{ms - amn}{k} - at = d$                    | $\left\{ \begin{array}{l} \text{2d part, 4th} \times \\ \text{by } m - at = d \end{array} \right\} \frac{1675 \times 7 - 3a \times 7}{5} - 8a = 210$                     | 5 |
| 6 $ms - amn - akt = dk$                            | $5\text{th} \times \text{by } k$                                                                                                                                         | 6 |
| 7 $ms - dk = amn + akt$                            | $\left\{ \begin{array}{l} amn - akt + \\ \text{and } dk + \end{array} \right\} \frac{1675 \times 7 - 210 \times 5}{3 \times 7 + 8 \times 5} = 3a \times 7 + 8a \times 5$ | 7 |
| 8 $\frac{ms - dk}{mn + kt} = a$<br>$= 175$ . Ergo. | $7\text{th} \div mn + kt$                                                                                                                                                | 8 |
| 9 $\frac{s - 175n}{k} = u$<br>$= 230$ .            | $4\text{th compared with 8th.}$<br>$\frac{1675 - 175 \times 3}{5} = u = 230$                                                                                             | 9 |

And for Proof  $230 \times 5 + 175 \times 3 = 1675$ . And  $230 \times 7 - 175 \times 8 = 210$ , as per the Question.

Problem 3.] A Surveyor having measured three Fields of Enclosure, hath lost the Contents: But remembers, that if 847 Acres be added to the 1st, the Sum will be equal to that of the other two Fields; if 847 Acres be added to the 2d Enclosure, the Sum will be double

double to the Content of the 1st and 3d; and if 847 Acres be added to the 3d Field, it will make treble the 1st and 2d: How many Acres is in each Field?

I put  $a = 1st$ ,  $e = 2d$ ,  $u = 3d$ , and  $k = 847$ .

|                                                |                                                                                                                                                                       |                                                         |
|------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------|
| 1 $a + k = e + u$ ; so that $a = e + u - k$    | $k +$ in 1st.                                                                                                                                                         | $a + 847 = e + u$ or $a = e + u - 847$                  |
| 2 $e + k = 2a + 2u$ $e = 2a + 2u - k$          | $k +$ in 2d.                                                                                                                                                          | $e + 847 = 2a + 2u$ $e = 2a + 2u - 847$                 |
| 3 $u + k = 3a + 3e$ $u = 3a + 3e - k$          | $k +$ in the 3d.                                                                                                                                                      | $u + 847 = 3a + 3e$ $u = 3a + 3e - 847$                 |
| 4 $e = 2e + 2u - 2k + 2u - k$                  | $\left\{ \begin{array}{l} \text{the 1st} \times \text{in } 2 + 2u \\ - k \text{ in 2d.} \end{array} \right\}$                                                         | $e = 2e + 2u - 847 \times 2, + 2u - 847$                |
| 5 $e = 3k = 4u$                                | the 4th contracted.                                                                                                                                                   | $e = 847 \times 3, - 4u$                                |
| 6 $u = 9k - 12u + 3u - 3k + (9k - 12u - k)$    | $\left\{ \begin{array}{l} \text{the 5th} \times 3 + 3u \\ - k +, \text{ the 5th} \\ \times 3 - k \end{array} \right\}$                                                | $u = 847 \times 9, - 12u + 3u - 847 \times 3$           |
| 7 $u = 14k - 21u$                              | 6th Step contracted.                                                                                                                                                  | $u = 847 \times 14 - 21u$                               |
| 8 $u = \frac{7k}{11}$                          | $\left\{ \begin{array}{l} 21u + \text{in the 7th} \\ \text{and } \div 22, \&c. \end{array} \right\}$                                                                  | $u = \frac{847 \times 7}{11} = 539 = 3d \text{ Field.}$ |
| 9 $e = 3k - \frac{28k}{11}$ or $\frac{5k}{11}$ | $\left\{ \begin{array}{l} = 3k - 4u \text{ as in} \\ \text{the 5th, and } u \text{ in the} \\ \text{8th is } = \frac{7k}{11} \times 4 \\ = 9th. \end{array} \right\}$ | $e = \frac{847 \times 5}{11} = 385 = 2d \text{ Field.}$ |
| 10 $a = 3k - \frac{21k}{11} - k$               | $\left\{ \begin{array}{l} a = \text{as per 1st} \\ \text{compared with} \\ \text{Steps 8, and 5.} \end{array} \right\}$                                               | $a = 847 \times 3 - \frac{847 \times 21}{11} - 847$     |
| 11 $a = 2k - \frac{21k}{11} = \frac{k}{11}$    | $\left\{ \begin{array}{l} \text{the 10th Step} \\ \text{contracted.} \end{array} \right\}$                                                                            | $a = \frac{847}{11} = 77 = \text{the 1st Field.}$       |

|                                           |                                                    |
|-------------------------------------------|----------------------------------------------------|
| Now $\frac{k}{11} = a$ the 1st Field = 77 | Acres. And for Proof, $77 + 847 = 924 = 385 + 539$ |
| $\frac{5k}{11} = e$ the 2d Field = 385    | $= \text{the 2d and 3d.}$                          |
| $\frac{7k}{11} = u$ the 3d Field = 539    | $385 + 847 = 1232 = 77 + 539 \times 2$             |
|                                           | $= 2d \text{ and } 3 \times 2.$                    |
|                                           | $539 + 847 = 1386 = 77 + 385 \times 3$             |
|                                           | $= 1st \text{ and } 2d \times 3.$                  |

Notes

*Notes on the three Operations to the last three Problems above.*

The middle Column has such plain Directions, as may suffice a docile Reader; but that I may render every thing plain, I shall observe that

*In the first Operation*, every Step is as easy as in common simple Equations, till you come to the sixth Step, which is formed thus:

Since  $u$  (an unknown Quantity in the 5th Step) is found  $= \frac{t-an}{k}$ , and since  $u+a$  (the 2 Quantities of the different Ingredients in the 2o) is equal to  $q=20$ ; therefore that I may get rid of  $u$ , I take what it is equal to, and say  $\frac{t-an}{k} + a = q$ , and then proceeding by

the common Rules laid down, I find  $a = \frac{t-kq}{n-k}$ , which are all known Quantities: and this Canon doth shew  $a=8$ .

Therefore  $u$  must be  $20-8=12$ . Or you may find it by a more artificial way, by having recourse to the 5th Step, where  $u = \frac{t-an}{k}$

ergo  $u = \frac{t-8n}{k}$ , which you'll find  $u=12$ .

*In the 2d Operation*, I proceed as in common simple Equations to the 4th Step, and there, because  $u = \frac{s-an}{k}$  I omit  $u$ , and take what it

is equal to,  $\frac{s-an}{k}$ , which according to the Tenor of the Question be-

ing multiplied by  $m$ , and made less by  $at$ , the Remainder is  $=d$ ; and so proceeding by the usual Rules, as before taught, I find

$a = \frac{ms-dk}{mn-kt} = 175$ : And if you have recourse to the 4th Step, and

instead of  $an$  there, put  $175n$  ( $a$  being found  $=175$ ) you will have  $u = \frac{s-175n}{k}$ , or  $u=230$ . And

*In the 3d Operation*, the three first Steps are composed according to the very Words of the Problem, and the 2d part of those Steps are the same, only having  $k$  transposed towards the right, there is shew'd what the 3 several unknown Quantities are simply equal to.

Now for the 4th Step I argue thus: Considering that in the 2d Step  $e = 2a + 2u - k$ , and that  $a$  is  $= e + u - k$ , therefore  $e = 2e + 2u - 2k$  (which is twice the first Step,  $= 2a$  in the 2d)  $+ 2u - k$ , which are the rest of the 2d Step.

K k k

Which



Which 4th Step is in the 5th reduced to  $3k - 4u$ .

Then I proceed to discover the Value of  $u$  by composing the sixth Step, thus:  $u$  being in the third Step towards the right hand  $= 3a + 3e - k$ ; and the 1st Quantity in what  $a$  is  $=$ , being  $e$ ; and  $e$  being in the 5th Step  $= 3k - 4u$ ; therefore I say  $3a$ 's is first 3 times  $3k - 4u$ , i.e.  $9k - 12u$ ; and the Remainder of what  $a$  is  $=$ , being  $u - k$ , ergo that  $\times$  by the 3, I say  $3a = 9k - 12u + 3u - 3k$ . Now the rest of what  $u$  is  $=$  being  $3e$ , and  $e$  in the said 5th Step being  $3k - 4u$ ; therefore (to make the Process plain) I repeat  $3e$  or  $9k - 12u$ . And there being farther  $-k$  to make up what  $u$  is  $=$ ; therefore I say in the 6th Step  $u = 9k - 12u + 3u - 3k + 9k - 12u - k$ , which contracted and reduced, is easily found (as per the middle Column in the Example)  $u = \frac{7k}{11}$ . Therefore  $e$  in the 5th Column; I put  $= 3k$ ,  $-4$  times what  $u$  is found equal to in the 8th Step.

Lastly,  $a$  being  $= e + u - k$ ; I say it is  $= e$  in the 9th Step,  $+ u$  (in the 8th Step)  $- k$ , or  $a = 2k - \frac{21k}{11}$ , which is  $= \frac{k}{11}$ .

#### SECT. XII. Concerning Cubical Equations.

A Cubical Equation may be known by having commonly (for the first Term) the Cube of the unknown Quantity; the 2d Term, being the Square of that Quantity multiplied in a Co-efficient or some known Quantity; the third is the unknown Quantity sought in a known, as thus and the like;  $uuu + cuu + du = k$ ,  $u$  being the Root or Quantity sought.

A Cubical Equation is formed from 1, 2 or 3 Binomials, &c. involved or multiplied in each other; representing either a compleat Cube or Paralleliped, whose Base is either a Geometrical Square, or a Parallelogram: in which last, if  $u + b$  be the greater Side,  $u - b =$  the lesser Side of the Base, and  $u + d =$  the Length; the Quantities multiplied produce this Cubical Equation, viz.  $uuu + duu - bbu - bbd = k$  the Content. To find the Dimensions, which depend on the Value of  $u$ , it must be done by the Rules following.

2. In a Quadratic Equation, by the Methods of Resolution foregoing, you gain 1 Root; and by dividing the Equation given by that Root you find the other, if there be 2 different Roots.

Thus in the Equation  $uu - 16u = -63$ , being resolved gives  $u = 9$ : And if the  $uu - 16u + 63$  (the 63 being transposed) be divided by  $u - 9$  (the 9 being transposed) the Quote will be  $u - 7$  or  $u = 7$ . So are  $u = 9$  and  $u = 7$  found the 2 Roots or Dimensions of that Equation, i.e. 9 and 7.

3. I. To resolve Cubic Equations by Tentation and Depression.

Having by tryal, as in the following Example, found 1 Root, you must divide the Cubic Equation given by that Root, and that will depress the Equation into a Quadratic; which Quadratic you may find the Roots of, as is shewn last above.

*Example 1.]* What are the real Roots of this three Dimension (or Cubical) Equation  $uuu - 57uu + 1040u = 6000$ ?

To answer this; I must first find by tryal a Root, which being multiply'd, &c. according to the given Equation, will be 6000.

*To which end I suppose 10 to be 1 Root = u.*

Then  $uuu$  (or  $10^3$ ) is = 1000 } Sum = 11400 }  
 The 3d Term  $1040u = 10400$  } Refts 5700  
 2d Term  $-57uu$  (or  $57 \times 10^2$ ) deduct. = 5700 }  
 The Remainder should be 6000, but 'tis only 5700; therefore the 10 supposed is too little.

2dly, I suppose therefore a Root to be  $12 = u$ .

Then  $uuu$  (or  $12^3$ ) is = 1728 } Sum = 14208 }  
 $+ 1040u = 12480$  } Refts 6000  
 $- 57uu$  (or  $57 \times 144$ ) = 8208 }

So that I find  $12 = u$  or  $u - 12$  to be 1 Root.

By which Root having divided the Cubic Equation given, I depress it to the Quadratic Equation  $uu - 45u = -500$ . Which Equation being resolved, gives  $u = 25$ , or  $u - 25$  for another Root. And dividing that Quadratic by this  $u - 25$ , the Quote or 3d Root is found  $u - 20$ , or  $u = 20$ . See the Operation.

$$\begin{array}{r|l}
 u-12) uuu-57uu+1040u-6000 (=uu-45u=500 = \text{the Quote or Quadratic.} \\
 \underline{uuu-12uu} & \text{or } u = \sqrt{6.25} + 22.5 = 25 \text{ Quadra-} \\
 & \text{(tic resolved, or} \\
 & \text{a 2d Root.} \\
 & u-25) uu-45u+500 (u-20 \\
 & \text{(= a 3d Root.} \\
 & \underline{uu-25u} \\
 & \underline{-20u+500} \\
 & \underline{-20u+500} \\
 & 0
 \end{array}$$

So that the 3 Roots being found 12, 20 and 25; you may prove the Truth by putting  $u$  equal to each, transposing the Roots, and multiplying the Quantities one in another: for the 2d Rectangle will be the Cubical Equation given, transposing the Member next the right hand of the last Product.

1. *Note*, That in a Cubic Equation if the single Root or Square of the unknown Quantity be multiply'd in more than one known Quantity, with Signs between the Rectangles, as several Members of the Equation; where the said unknown Root is found squared in 2 &c. such Members, they make but 1 Term: and so do all those where the unknown Root is found single make another; as  $u^3 - u^2b - u^2d + uc + ur + d^3 = 0$ . In this  $-u^2b - u^2d$  is but 1 Term of the Equation, and  $uc + ur$  is one other Term.

2. *Note*, That in the Cubic suppose  $u^3 = uc + ur + d^3 = 0$ ; the 2d Term here is said to be wanting, and  $uc + ur$  is the 3d Term.

*Example 2.*] Admit this Equation were to be resolved;  $uuu - 30.5uu + 309u = 1039.5$ : I suppose 1 Root = 12. and working as by the Equation, I find

$$\begin{array}{r} u^3 = 12^3 = 1728 \\ - 30.5u^2 = -4392 \\ + 309u = 3708 \\ \hline 1039.5 \end{array} \left. \begin{array}{l} \text{The Sum as per Addition of Algebra} \\ \text{is } 1044, \text{ which is more than the} \\ 1039.5 \end{array} \right\}$$

2dly, I therefore suppose a lesser Number, viz. 11 = a Root, then

$$\begin{array}{r} u^3 = 11^3 = 1331 \\ - 30.5uu = -3690.5 \\ + 309u = 3399 \\ \hline 1039.5 \end{array} \left. \begin{array}{l} \text{The Sum of which is } = 1039.5 = \text{the} \\ \text{Sum in the Equation given: so that} \\ 11 \text{ is 1 Root.} \end{array} \right\}$$

3dly, Dividing the Equation given (with the 1039.5 transposed) by  $u - 11$ , I depreß it to the Quadratic  $uu - 19.5u = -94.5$ .

4thly, This Quadratic being resolved is  $u = \sqrt{95.0625 - 94.5} + 9.75 = 10.5$  a 2d Root.

5thly, Dividing this Quadratic Equation by the 2d Root  $= u$  or  $u - 10.5$ , the Quote or 3d Root is found 9. And for Proof, the 3 Roots 9, 10.5, and 11, (putting each  $= u$ , and transposing it) multiplied one in another, will give the Cubic Equation proposed to have its Root extracted, viz.  $u^3 - 30.5u^2 + 309u = 1039.5$

From these 2 Examples it may be observed, That whether you happen to pitch upon the least Root (upon your Tryals) as in the first Example, or upon the greatest of the 3, as in the 2d Trial of the 2d Example; the Method of Process is the same; and so it would, if you happen first upon the middle Root. These

*The Proof of the foregoing Work.*

$$\begin{array}{r} u - 25 \\ u - 20 \\ \hline uu - 25u \\ - 20u + 500 \\ \hline uu - 45u + 500 \\ u - 12 \\ \hline uuu - 45uu + 500u \\ - 12uu + 540u - 6000 \\ \hline uuu - 57uu + 1040u - 6000 \\ \text{or } uuu - 57uu + 1040u = 6000 = \\ \text{(the Equation given, for Proof.)} \end{array}$$



These 2 Examples are of Regular Cubics having all their Terms.

4. I shall give one Example of an Irregular Solid Equation: as suppose  $uuuuu + .10uu = 20.61293$ .

To resolve this, I can easily see that 2 is more than the Root; for 10 times the Square of 2 (alone, without the Surfsolid) is 40. Therefore I take 1.5 for the Root, and (working with that according to the Equation) I find it too great; I therefore try 1.4, which is also a small matter too much: so that working according to the Equation with 1.3, I find it just  $= 20.61293$ . So that I discover the Root to be  $1.3 = u$ . for  $1.3^5 + 10 \times 1.3^2 = 20.61293$ .

5. Biquadratic Equations are resolved after the same manner as the Regular Cubics in the 2 first Examples: First, By supposing a Root till you pitch upon the right, and depressing the Biquadratic thereby into a Cubic; and then finding another Root by Tentation whereby to divide the Cubic to depress it to a Quadratic, &c. as in the said 2 first Examples above for all Regular Biquadratics (or such as have all their Terms.) Thus, for Example, to resolve or find the Roots of the Biquadratic  $uuuu - 29uuu + 308uu - 1420u = 2400$ .

(1.) By Tentation I find 8 to be 1 Root, and dividing the Equation given by  $u - 8$ , I depress it to the Cubical Equation  $uuu - 21uu + 140u = 300$ .

(2.) I make Tryal again, and find 10 to be another Root; so dividing the Cubic Equation by  $u - 10$ , I depress it to the Quadratic  $uu - 11u = -30$ , which being resolved, gives a 3d Root or  $u = 6$ .

(3.) Dividing the said Quadratic Equation by  $u - 6$ , I find the Quote to be  $= u - 5$  or  $u = 5$ , the 4th Root: so are 8, 10, 6 and 5 the 4 Roots required.

There are 3 other Ways of resolving or finding the Roots of all Cubic and Biquadratic Equations, done more Mathematically: as,

II. By Substitution, Deduction, and Division.

III. By way of Construction, with the help of the Parabola and Circle.

IV. By Approximation or Converging Series.

II. To resolve Cubic Equations by Substitution.

It has been shewed how to resolve Quadratics by Substitution, and much after the same manner is a Canon found whereby to resolve Cubic, &c. Equations.

Example 1.] Admit the Value of  $u$  is required in this following Equation, viz.  $uuu + duu - gu = k$ , or  $uuu + 120uu + 300u = 3714544$ : Here 'tis plain that  $d$  is put for the given 120;  $g$  for 300; and  $k$  for 3714544.

The first Step toward the Answer is to substitute  $b + c$  in the place of  $u$  in every Term of the Equation, in the Power that ( $u$ ) is put.

Then

Then will this be  $\left\{ \begin{array}{l} bbb + 3bbc + 3bcc + ccc \text{ (instead of } uuu) \\ bbd + 2bcd + ccd \dots \text{ (instead of } duu) \end{array} \right.$   
 Resolution.  $-bg - cg$  (in the place of  $-gu$ )

The whole Process and Rules are in the Margin, whereby  $u$  is found = 124.

1. Having your Canon before you, and also that  $d = 120$ ,  $g = 300$ , proceed as the Canon directs you, first pointing over every third after the first Digit of the given Number; the 3 Points shewing the Root sought, will consist of 3 places, and therefore the 1 (= the first  $b$ ) is 100.

2. You must compose a Number to deduct from that given, by taking those in the Canon which have not  $c$  found in them; as  $b^3 + bbd - bg = 2170000$ , which deducted reduceth the Number given to 1544544, which is your first Dividend, as by Step 7.

3. Take for a Divisor (in order to discover the Value of  $c$ ) the Sum of the Numbers answering to the Sumbals mixed with  $or \times$  by  $c$ , such are those in the 8th, 9th, 10th, 11th, and 12th Steps; which Sum is

54120

## An Example of the Resolution of a Cubic.

|       |    |                                               |
|-------|----|-----------------------------------------------|
| Step. | 1  | 3714544 (100 = $b$ )                          |
|       | 2  | 1000000 = $bbb$                               |
|       | 3  | 1200000 = $bbd$                               |
|       | 4  | 2200000 = $bbb + bbd = 1 + 2d$ Step.          |
|       | 5  | 30000 = $-bg$ deduct from the 4th.            |
|       | 6  | 2170000 refts, which deduct from the 1st Step |
|       | 7  | 1544544 ÷ = the 1st — the 6 Step ÷ by 13      |
|       | 8  | 30000 = $3bb$                                 |
|       | 9  | .. 300 = $3b$                                 |
|       | 10 | 24000 = $2bd$                                 |
|       | 11 | .. 120 = $d$                                  |
|       | 12 | — 300 = $-g$                                  |
|       | 13 | ÷ 54120 = the Sum, or Divisor.                |
|       |    | (20 = $c$ )                                   |
|       | 14 | 600000 = $3bbc$                               |
|       | 15 | 120000 = $3bcc$                               |
|       | 16 | .. 8000 = $ccc$                               |
|       | 17 | 480000 = $2bcd$                               |
|       | 18 | 48000 = $ccd$                                 |
|       | 19 | — 6000 = $-cg$                                |
|       | 20 | 1250000 = Sum deduct from the 7th Step.       |
|       | 21 | 294544 ÷, refts of $b$ , or 1st Step.         |
|       |    | Now 120 = $b$                                 |
|       | 22 | 43200 = $3bb$                                 |
|       | 23 | .. 360 = $3b$                                 |
|       | 24 | 28800 = $2bd$                                 |
|       | 25 | .. 120 = $d$                                  |
|       | 26 | — 300 = $-g$                                  |
|       | 27 | ÷ 72180 = Sum, by which to ÷ the 21 Step.     |
|       |    | (4 = $c$ )                                    |
|       | 28 | 172800 = $3bbc$                               |
|       | 29 | .. 5760 = $3bcc$                              |
|       | 30 | ... 64 = $ccc$                                |
|       | 31 | 115200 = $2bcd$                               |
|       | 32 | .. 1920 = $ccd$                               |
|       | 33 | — 1200 = $-cg$                                |
|       | 34 | 294544 = Sum, deduct from the 21 Step.        |
|       |    | 0 refts.                                      |

54120 your Divisor, which may be had in the 6 Figures next the left hand of the Dividend 2 times, (which 2 must be 20, because it is the middle of the 3 that will be in the Root, as is observed above) this 20 is =  $c$ .

4. You must not make any farther Use of the Quantities in the 1st, 2d, and 4th Steps, but having found the Value of  $c$ , you proceed to find your 2d Subtrahend in the 20th Step with the several Quantities in the Canon; which you see in the 14th, 15, 16, 17, 18, and 19th Steps.

5. And having found your 2d Divisor by the very same Quantities that you had the 1st, (and observing that here  $b = 120$ , and not to 100) you may easily finish the Operation by the Example, which is only a repetition of the Method above shewed, and you find the 3 several Roots put together is  $124 = u$  required: which I shall prove both from the Equation given, and by the Canon, to be the true Answer.

$$\begin{array}{r} 1. \text{ By the Equation } u^3 = 124^3 = 1906624 \\ \quad + 120uu = 1845120 \\ \quad - 300u = -37200 \\ \hline \end{array}$$

Sum = 3714544 =  $k$  in the Equation given.

2. The Proof by the Canon  $bbb + 3bbc + 3bcc + ccc + bbd + 2bcd + ccd - bg - cg$ .

|                   |                                                                    |
|-------------------|--------------------------------------------------------------------|
| $bbb = 1728000$   | } Sum = — — — 3751744<br>— $bg = 36000$<br>— $cg = 1200$ } — 37200 |
| $+ 3bbc = 172800$ |                                                                    |
| $+ 3bcc = 5760$   |                                                                    |
| $+ ccc = 64$      |                                                                    |
| $+ bbd = 1728000$ |                                                                    |
| $+ 2bcd = 115200$ |                                                                    |
| $+ ccd = 1920$    |                                                                    |

Note,  $b = 120$ ;  $c = 4$

$d = 120$ ;  $g = 300$

3714544 rests =  $k$

Biquadratics are resolved after the same Method, raising the Power of  $b + c$  (which is substituted) accordingly in making the Canon; and proceeding by that, as has been in the last Example observed.

Example 2.] Of an irregular Cubic Equation, as suppose  $uuu + 32334u = 823975 = k$ , or  $u^3 + fu = k$ .

1. Make your Canon as before, substituting  $b + c$  in the 2 Members of the Equation (as  $u$  is in power) in the place of  $u$ .

2. Where the Equation has not all its Terms, and the Co-efficient is large, you must put such a Number first in the Root, as being



ing multiply'd by the Co-efficient, and added to its own Cube, the Sum will be as near as possible to, and less than that given on the right side of the Equation; as in the Example finding  $50=b$  much too great (as in the 4th Step) I try  $30=b$ , which I still find too much, as in the 7th Step. Then putting  $20=b$ , I find its Cube as in the 8th Step, and the Co-efficient 32334 by 20, the Sum is deducible as in the 10th Step from the first, and then you proceed with the Remainder as in the last Example, and the Value of  $u=25$ .

3. It sometimes happens where the Co-efficient is very large, and the Root of the highest Power (or Value of  $u$ ) is small, that you must make that

Figure 1st put in the Root one place (or more) less than the Points over the Number given (as in the 1st Step above) denote. See the following Example.

Example 3.] Admit  $u^3 + 82334u = 2073975$ , what is the Value of  $u$ ? 82334 being  $=f$ , and  $b+c$  substituted in the room of  $u$ .

|       |                                                            |
|-------|------------------------------------------------------------|
| Step. | The Canon is $bbb + 3bbc + 3bcc + ccc + bf + cf = k$ .     |
| 1     | 823975 ( $50=b$ ) ( $30=b$ ) ( $20=b$ )                    |
| 2     | 125000 = $bbb$                                             |
| 3     | 1616700 = $bf$                                             |
| 4     | 1741700 = $bbb + bf > 1st$ Step, and cannot be deducted. } |
| 5     | 27000 = $30^3 = b^3$                                       |
| 6     | 970020 = $bf$                                              |
| 7     | 997020 = $b^3 + bf > 1st$ Step.                            |
| 8     | 8000 = $b = 20^3$                                          |
| 9     | 646680 = $bf$                                              |
| 10    | 654680 deduct from the 1st Step.                           |
| 11    | 169295 ÷ rests of the 1st Step.                            |
| 12    | 1200 = $3bb$ }                                             |
| 13    | 60 = $3b$ } Add.                                           |
| 14    | 32334 = $f$ }                                              |
| 15    | ÷ 33594 = the 3 last Steps a Divisor. ( $5=c$ )            |
| 16    | 6000 = $3bbc$ }                                            |
| 17    | 1500 = $3bcc$ } Add.                                       |
| 18    | 125 = $ccc$ }                                              |
| 19    | 161670 = $cf$ }                                            |
| 20    | 169295 Sum, deduct from the 11th Step. 0 rests.            |

Here

Here 'tis plain the Numb. given admits of 3 Cubical Points, but the Root  $u$  (or  $b+c$ ) is but 25.

1. For if I should put 1 as the Root of 2 next the left hand; that 1 (there being 3 Points) must be 100. Now if I multiply the Co-efficient by 100, *i.e.* if I suppose two Cyphers put towards the right hand of it, it will be 8233400, w<sup>ch</sup> (without the Cube of the 100, which must, as *per* the Examples above, be added) is much too great to be taken from the 2073975 in the first Step.

|                                                                      |                                       |
|----------------------------------------------------------------------|---------------------------------------|
| The Canon } = $b^3 + 3b^2c + 3bcc + c^3$<br>as before. } + $bf + cf$ |                                       |
| 1                                                                    | 2073975 (20 = $b$ )                   |
| 2                                                                    | 8000 = $b^3$                          |
| 3                                                                    | 1646680 = $bf$                        |
| 4                                                                    | 1654680 = Sum, from the 1st Step.     |
| 5                                                                    | 419295 refts of Step 1st.             |
| 6                                                                    | 1200 = $3bb$                          |
| 7                                                                    | 60 = $3b$                             |
| 8                                                                    | 82334 = $f$                           |
| 9                                                                    | 83594 = Sum or Divisor.<br>(5 = $c$ ) |
| 10                                                                   | 6000 = $3bbc$                         |
| 11                                                                   | 1500 = $3bcc$                         |
| 12                                                                   | 125 = $ccc$                           |
| 13                                                                   | 411670 = $cf$                         |
| 14                                                                   | 419295 = Sum, from the 5th Step.      |
|                                                                      | 0 refts.                              |

2. Seeing a lesser Digit than 1 cannot be put first, or in hundreds place in the Root; I must therefore conclude, that the Root will have but two places.

3. I therefore suppose 10 in the Root, by putting 1 Cypher towards the right hand of the Co-efficient (to represent  $bf$ ) which would be 823340, which (by doubling the 8 next the left hand I find 16) may easily be taken from 20, (next the same left hand of the Number given) I therefore put the double of 10 (or 20) in the Root and so proceed; all the difficulty in this case being to put the first true Figure in the Root: to which end I have not seen any Rules that contribute to do so easily and certainly as these above.

*Example 4.] Of Resolving Cubies by Substitution, &c.*

Admit  $uuu + 350u^2 = 1225125$ . Or that  $uuu + duu = k$ ; and that  $b+c$  is put =  $u$ .

This Example is of an irregular Cubic composed of 2 Members different from the former; where tho' there be 3 Cubical Points (or Ternaries) yet the Root has but 2 places, viz. 55.

For if we suppose  $b =$  but 1 in the 3d place (or 100) you will find  $bbb + bbd$  to be much more than can be taken from the Numb. pointed; and  $10 = b$  is much too little: therefore I try  $50 = b$ , and find that to be the Root, that produces (by its Cube added to  $bbd$ ) the next Number to and less than 1225125 given, and therefore I proceed as by the Canon, as directed in Example the 1st, last foregoing.

| Step. | The Canon is $bbb + 3bbc + 3bcc + ccc = u^3$<br>$bbd + 2bcd + ccd = duu.$ |
|-------|---------------------------------------------------------------------------|
| 1     | 1225125 (50 = $b$ )                                                       |
| 2     | 125000 = $bbb$                                                            |
| 3     | 875000 = $bbd$                                                            |
| 4     | 1000000 = Sum                                                             |
| 5     | 225125 = 1st — 4th Step, a Dividend.                                      |
| 6     | 7500 = $3bb$                                                              |
| 7     | 150 = $3b$                                                                |
| 8     | 35000 = $2bd$                                                             |
| 9     | 350 = $d$                                                                 |
| 10    | 43000 = Sum, a Divisor.<br>(5 = $c$ )                                     |
| 11    | 37500 = $3bbc$                                                            |
| 12    | 3750 = $3bcc$                                                             |
| 13    | 125 = $ccc$                                                               |
| 14    | 175000 = $2bcd$                                                           |
| 15    | 8750 = $ccd$                                                              |
| 16    | 225125 = 11 + 12 + 13 + 14 + 15th Steps<br>rests 0 5th — 16th Step.       |

### III. To resolve Cubic Equations by way of Construction.

Vid. Plate A. Fig. 27.

This being done by the Parabola and Circle, it is necessary to shew the making of the Parabola, &c.

A Parabola is one of the 5 Sections of a Cone, being made by cutting a Cone by a Plane parallel to the Side which is opposite to that wherein you cut; and it is Geometrically formed thus:

1. Draw the Line ( $tvF@dy$ ) the Axis of the Section. Then cross the same at right Angles with a Line at pleasure, as ( $ap$ ) and according as you would have the same (or if the  $ap$ , which is called the



the *Latus rectum*, or Parameter, be given) suppose 1 Inch, set half an Inch from  $F$  to  $a$ , and to  $p$ .

Then take half of one of those Halves (or  $\frac{1}{4}$  of the Parameter, or of 1 Inch) and set it from  $F$  to  $v$ , and from  $v$  to  $t$ , so is  $F$  the Focus, and  $v$  the Vertex of the Parabola.

Then draw as many Lines as you please parallel to  $ap$  (as  $ool$ , &c.) with a Ruler on purpose for drawing parallel Lines; and the Distance  $t$  to  $o$  in the Axis extends from  $F$  the Focus, to  $l$  in the same Parallel forming the Curve ( $vplll$ , &c.) which you may draw through the Points ( $l, l$ , &c.). And having found the Curve on one side of the Axis, as  $vl$ , set it off on the same Parallel, on the other side of the Axis, as  $vo, vo$ , &c. and draw the Curve ( $oov$ ) so is the Section finished, and  $aFp$  is the *latus rectum*,  $ool$  an Ordinat Applicate (or whole Ordinat)  $oo$  the Semi-Ordinat, (which is commonly called the Ordinat) any  $vo$  in the right Line is called an Abscissa.

Now suppose a Cubic Equation were 
$$\left. \begin{array}{l} uuu + buu - pu - q = a \\ \text{Or } uuu + 3uu - zu - 2 = 0 \end{array} \right\} \text{so } \begin{cases} b=3 \\ p=2 \\ q=2 \end{cases}$$

1st. To find the Value of  $u$ . From the same Scale by which you set off the *Latus rectum* of your Parabola, take 3 (the Co-efficient multiply'd in the Square of the Root sought) and applying it at right Angles to the Axis ( $vy$ ) towards the right hand, because  $+3$  it will cut  $b$  in the Curve.

2dly, Draw a Line at pleasure cutting a supposed Line ( $vb$ ) in the middle at right Angles, as  $qq$ .

3dly, Where that Line intersects the Axis, as at  $\odot$  set half  $p$  downward, because  $-p$ , viz. 1, from the same Scale extends from  $\odot$  to  $n$ .

4thly, On  $n$  erect a perpendicular Line  $nm$  which will cut the infinite Line  $qq$  in  $r$ .

5thly, From  $r$  set half  $q = 1$  towards the right hand because  $-q$ , and it extends (by the same Scale) from  $r$  to  $s$ , which is the Center of the Circle sought.

6thly, Extend the Compasses from  $s$  to  $b$ , and describe the Arch  $ab$ , which cutting the Curve at  $e$  the perpendicular Line  $ee$  measured on the same Scale, gives  $u = 1$ , which is  $+1$  or affirmative, because the Section  $e$  is in the right hand part of the Curve. See *Philosophical Transactions*, N<sup>o</sup>. 188 and 190; Dr Halley's Method.

## SECT. XIII. Approximation or Converging Series.

What is meant hereby, is shewn under the words *Converging Series* in the foregoing Alphabetical Account at the beginning of this Chapter, and will be farther explained by the Rules and Examples following: as  $u - ub + u^2b^2 - u^3b^3 + u^4b^4 - u^5b^5$ , &c. is a Series; which I multiply by  $u^2 - b^2$  as followeth.

$$\begin{array}{r} u - ub + u^2b^2 - u^3b^3 + u^4b^4 - u^5b^5 \\ u^2 - b^2 \\ \hline \end{array}$$

$$u - b + u^2b^2 - u^3b^3 + u^4b^4 - u^5b^5 + u^6b^6, \text{ \&c.}$$

$$-u^2b^2 + u^3b^3 - u^4b^4 + u^5b^5 - u^6b^6, \text{ \&c.}$$

$$u - b = \text{the Product.}$$

Here it is plain, that if the Series was continued never so far, the Negatives would destroy the Affirmatives: But my intent in this Section is not to shew the way of representing or expressing by a Series, a Product, Root, &c. for though that be a pretty Speculation, yet 'tis not so easy to discover the Roots, &c. therefrom. But I chuse rather here to shew the great Use of Converging, in finding with much ease and expedition the real Roots themselves.

## II. To find the Square Root, as suppose of 13.

Here I will put  $s$  for the Square or Surd Number given;  $r$  = the 1st two Figures in the Root;  $a$  = the 2d Approach to the true Root; and  $e$  = the 3d Approach; the several Approaches being the coming nearer and nearer to the Truth of the Root by so many Divisions, by 1 of which several places in the Root are gained, as is shewed under the fifth Head following.

If in the Example above it be considered what Number squared will be next to and less than 13, or what 2 Digits will be next to 13.00 and put  $r$  for those 2 places it will then be  $s - rr \div 2r = a$  the 2d Approach.

$$s - r + a^2 \div r + a \times 2 = e \text{ the 3d Approach, \&c.}$$

Or in Numbers  $13 - 3.6^2 \div 3.6 \times 2 = .00555 = a$  the 2d Approach,  
 $36. = r.$

$13 - 3.60555^2 \div 3.60555 \times 2 = .00000127546 = e$ ,  
 and  $r + a + e$ . Or  $3.6 + .00555 + .00000127546$  which is =  
 $3.605551275464 =$  the Square Root required to 13 places (but  
 see more under General Head the 5th following). Or  $e$  will be =  
 $\frac{s - rr - 2ra - aa}{2r + 2a}$  which facilitateth the Operation as to Squaring  
 $3.60555$ , &c.

Or instead of 3.6 at the 1st Approach, you may, as in the com-  
 mon Way of Extraction, put 3, whose square is next to and less  
 than that of any other Digit deducible from 13.

*Prop. 2. To find the Square Root of a larger Number, as of 371.00071*  
 $= s$ , &c. as before.

$371.00071 = s$ .  $19 = r$  the 1st Approach.

$$-361 = rr$$

$10.00071 \div 2r (= 31) = .26 = a$  the 2d Approach, and  
 $r + a = 19.26$ .

Again  $371.00071 - 19.26^2 = .05311 \div 19.26 \times 2 = .0013787 = e$   
 the 3d Approach.

And  $r + a + e = 19 + .26 + .0013787 = 19.2613787 =$  the  
 Root sought.

So by the same Rules of Converging Series the Square Root of  
 $4712345$  is found  $2170.7936 +$

### III. Approximation, or the Extraction of the Cube Root by the Method of Converging Series.

I shall also proceed in this with as much Plainness as I can, ob-  
 viating what only is to the purpose.

1. Suppose then I would know the Cube Root of  $13 = s$ , for the  
 first in the Root which is known here I put  $r$ . Then the Process,  
 as in common Extraction, (which is the best way in this Case, as  
 being most accurate and certain for the 2d in the Root) will be  
 $s - r^3 \div 3rr = R = .3$  the 2d in the Root.

Or supposing 2.3 put in the Root together (for I can easily see that  
 $2.3$  is the Root of the next Cube Number to and less than  $13.000$ )  
 it will be as before to find the 2d Approach,  $2.3 = r =$  the 1st  
 Approach.

Then  $s - r^3 \div 3rr = .05 =$  the 2d Approach, and now  $2.35 = R$ .

And  $s - R^3 \div 3RR = .001333 +$  the 3d Approach. So  
 is  $2.351333 +$  the Root.

Or



Or in Numbers more plainly thus ;

$= 13 - 2.3^3 (= 12.167) = .833, \div 2.3^2 \times 3 (= 15.87) = .05$  the 2d Approach.

And  $13 - 2.35^3 (= 12.977875) = .022125 \div 2.35^2 \times 3 (= 16.5675) = .001333$  the 3d Approach.

And the 1st Approach + the 2d + the 3d =  $2.351333$  +, which is the Cube Root of 13, done with near 200 Figures fewer than in the common Way of Extraction. — But see Head 8. following.

2. And if we put  $r$  = the known part of the Root, and  $a$  for each new Approach, you have

$r^2 + 2ra + aa$  = the Square of  $r + a$   
 $r^3 + 3r^2a + 3ra^2 + a^3$  = the Cube of  $r + a$

So likewise  $r^4 + 4r^3a + 6r^2a^2 + 4a^3r + a^4$  = the Biquadrate of  $r + a$

And  $r^5 + 5r^4a + 10r^3a^2 + 10r^2a^3 + 5ra^4 + a^5$  = the Sur-solid of  $r + a$

And as in the Square and Cube Roots it appears to be (putting  $s$  for the Number given to have its Root extracted)

$s - rr \div 2r = a$  the several Approaches in the Square Root.

$s - r^3 \div 3rr = a$  = the Approaches in the Cube Root.

So for the Biquadrate and Sur-solid Roots it will be

$s - r^4 \div 4r^3 = a$  = the Approaches to the Biquadrate Root.

And  $s - r^5 \div 5r^4 = a$  = the Approaches to the Sur-solid Root.

|                      |   |                                      |
|----------------------|---|--------------------------------------|
| Approaches<br>to the | { | Squared Cube = $8 - r6 \div 6r5 = a$ |
|                      |   | 2d Sur-solid = $8 - r7 \div 7r6 = a$ |
|                      |   | Squ. Biquad. = $8 - r8 \div 8r7 = a$ |
|                      |   | Cubed Cube = $8 - r9 \div 9r8 = a$   |

Hence it appears, That the known part of the 2d Quantity next the left hand in the abovementioned Powers of  $r + a$ , is your Divisor. I come next,

IV. To illustrate by Numeral Examples the Extraction of the Biquadrate and Sur-solid Roots by way of Converging Series.

1. To find the Biquadrate Root of 314. The Process is (the 1st  $r$  being = 4.)

1st,  $314 - r^4 (= 256) = 58 \div 4r^3 (= 256) = .2 = a$ . So is 4.2 a new  $r$ .

2dly,  $314 - r^4 (= 311.1696) = 2.8304 \div 4r^3 (= 296) = .009$ ; so is 4.209 = a new  $r$ .

And by repeating the same Method with that last  $r$ , you have 4.2099 + for the Biquadrate Root of 314.

2. To find the Sur-solid Root of 314 (the first  $r=3$ ) the Process will be

1<sup>st</sup>,  $314 - 3^3 (=243) = 71 \div 5r^2 (=45) = .1 = a$ . So is 3.1 a new  $r$ .

2<sup>dly</sup>,  $314 - 3.1^3 (=286.29) = 27.7185 \div .461.75 (=5r^2) = .05 = a$ , and 3.15 = a new  $r$ .

3<sup>dly</sup>,  $314 - 3.15^3 (=310.1364) = 3.8636 \div 5r^2 (=492.28) = .007 = a$ . So is 3.157 = a new  $r$ .

And proceeding in the same Method with that new  $r$ , the Sur-solid Root of 314 is found 3.1578 +

But there is yet

#### V. Another Way of Converging (for Example) to the Square Root.

For as in the 1<sup>st</sup> Example in the Square Root in this Section the Canon was (putting  $s$  = the Square Number or Surd given, and  $r$  for each Approach to the true Root)  $\frac{s-rr}{2r} = r$  the 2<sup>d</sup>, &c. (or here

to  $a$ ) so in this Method I am going to shew, the Canon is  $\frac{s+rr}{2r}$

=  $r$  the 2<sup>d</sup>, &c.

Now suppose  $s=13$ , it follows that the first  $r$  (or Figure in the Root) is 3, so that  $13 + 3^2 \div 6 = 3.66 = r$  the 2<sup>d</sup>.

Note, The Divisors are always the double of the  $r$  next before.

13 +  $3.66^2 \div 7.32 = 3.605955 = r$  3<sup>d</sup>.  
And  $13 + 3.605955^2 \div 7.21191 = 3.60555129 = r$  4<sup>th</sup>.

From the last Example you may note,

1. That though the preceeding Number be false, yet by every Repetition of Process according to the Canon, it convergeth or comes nearer to the true Root or Value of  $r$ .

2. That the Places of the Digits in the several Approaches are true, according to the Progression whose Ratio is 2, and the 1<sup>st</sup> Term = 1. So

In the 1<sup>st</sup> Approach *viz.* 3

|   |   |             |
|---|---|-------------|
| 2 | 2 | 3.6         |
| 3 | 4 | 3.605       |
| 4 | 8 | 3.6055512 + |

=  $r$  = the Square Root of 13 prop<sup>d</sup>.

} Are true, the rest not so

And if the Operation were repeated, as  $3.60555129^2 + 13 \div 3.60555129 \times 2$ , the Quote will give 16 places true in the Root; for

for every Approach affords double to that preceeding. But the Reader may chuse which of the two ways he likes as the best, this, or the first way, which I take to be somewhat easier and shorter in the Operations of Division.

*Example 2.] Of this Way of Converging; to extract the Square Root of 29507710.4132345.*

$$\begin{array}{r}
 29507710.4132345 \quad (5 = r \text{ the 1st.} \\
 \underline{25} \\
 2r = 10) 545 \quad \underline{\hspace{1cm}} \quad (54 = r \text{ the 2d.} \\
 \underline{45} \\
 2950 \\
 + 2916 = 54^2 \\
 \underline{\hspace{1cm}} \\
 2r = 108) 5866 \quad \underline{\hspace{1cm}} \quad (5432 = r \text{ the 3d.} \\
 \underline{466} \\
 347 \\
 \underline{237} \\
 29507710 = \text{of the Data.} \\
 + 29506624 = 5432^2 \\
 \underline{\hspace{1cm}} \\
 2r = 10864) 59014334 \quad \underline{\hspace{1cm}} \quad (5432.1000 = r \text{ the 4th.} \\
 \underline{46943} \\
 34873 \\
 \underline{22814} \\
 10864 \\
 29507710.41323450 \\
 + 29507710.41000000 \\
 \underline{\hspace{1cm}} \\
 2r = 10864.2) 59015420.82323450 \quad (5432.100000297721 = r \\
 \underline{469442} \quad \text{the 5th Answer.} \\
 \underline{348740} \\
 \underline{228148} \\
 108642 \\
 0323450 \\
 \underline{1061660} \\
 838820 \\
 \underline{783260} \\
 227660 \\
 \underline{103760}
 \end{array}$$

VI. But





bers) if one be divided into any Number of Parts, the Sum of the Rectangles of one by the several Parts of the other, is = the Rectangle of one whole by the other.

But it being my Design here, only to shew how the large Multiplications for producing the 4th, &c. Approaches may be contracted; I shall explain the said Canon, which is grounded on another *Prop.* of *Euclid*, i. e. any Line (or Number) divided into two Parts; the Sum of the Squares of the Parts, more the double Rectangle of those Parts is = the Square of the whole.

So in the last Example putting  $t$  the 3d = 5432  
 $a$  the 3d = .1000

$$\text{Ergo } aa + 2ta + tt = t + a^2$$

There arises agreeable to the Canon under the 2d Head of Converging toward the Square Root, this Equation.

Now whereas it would be a little tedious to square the whole of  $t$  the 4th = 5432.1000 (especially if the Cyphers were significant Figures) I therefore always divide it into 2 parts just at the meeting of the Approaches, as  $t$  the 3d, and  $a$  the 3d, viz. into 5432, and .1000, then as the Canon directs.

$$\begin{array}{rcl} .10 & = & aa \\ 1086.4 & = & 2ta \\ 29506624 & = & tt \text{ in the last Approach before, (ready done.)} \end{array}$$

$$29507710.41 = t + a^2 = \text{the Sum} = 5432.1^2 = tt \text{ in the last Example.}$$

In this Operation (this way performed) you save the trouble of working and inserting 12 Figures in 29.

And this later Method being used, after the 3d, &c. Approaches, does much facilitate the Operations. And thus I think I have made this matter shorter, more certain in the Approaches, and plainer than has been done before.

## VII. Another

VII. Another Example of Approximation to the Cube Root shall be to extract the Cube Root of  $33148648517294436663656267 = c$

$$\frac{c-s^3}{3s^2} = a = \text{the Theorem or Canon.}$$

Root sought =  $r$   
 Each Approach =  $a$   
 And the Sum of the Ap- }  
 proaches ————— } =  $s$

1. *Note*, You always bring down so much of  $c$ , as will admit of  $s^3$  to be deducted.
2. If any thing had remained upon deducting the last  $s^3$ , you must have put Cyphers, and divided again by  $3 s^2$ . See the Operation, and the Probation thereof.

$33148648517294436663656267$  ( $3 = a$  the 1st. =  $s$   
 $3s^2 = 27$ )  $61$  ( $\frac{\dots}{7} \dots$ )  $02 = a$  the 2d  
 $\vdash a$  the 1st =  $32 = s$ .

$$\begin{aligned} 33148 &= \text{part of } c. \\ 32768 &= j^3 \end{aligned}$$

$3s^2 = 3072$      $3806 - \frac{3806}{7344} = 0.51975 = a$  the 3d.  
 $\frac{7344}{12008} + 32 = 32.6123 = s$   
 $\frac{12008}{2792}$

$$33148648517294 = \text{part of } c.$$

$$33147310244867 = s^3$$

$$\begin{array}{r}
 3s^2 = 3095661387) \quad 13382724274 \text{---} 000004323 = a \text{ the 4th.} \\
 \quad 10000787263 \quad + s = 32123 \\
 \quad 7136031026 \quad = 321234323 \\
 \quad 9447082526 \quad = s = r.
 \end{array}$$

$$\begin{aligned} 33148648517294436663656267 &= c \\ 33148648517294436663656267 &= s \end{aligned}$$

o rests, *ergo* 321234323 = Root  
required.  
This



This might be proved by the common Way of Extracting, tho' with very many more Figures: I shall chuse therefore rather to prove it by Involution of the Root, multiplying by 2 Digits at once, as taught *Compend. 6. Chap. III.*

$$\begin{array}{r} 321234323 = r = \text{the Root} \\ \times 321234323 \\ \hline \end{array}$$

$$\begin{array}{r} 7388389429 \\ 13813075889 \\ 7388389429 \\ 6745920783 \\ 963702969 \\ \hline \end{array}$$

$$\begin{array}{r} 103191490273268329 = r^2 \\ 321234323 = r \\ \hline \end{array}$$

$$\begin{array}{r} 2373404276285171567 \\ 4437234081750538147 \\ 2373404276285171567 \\ 2167021295738634909 \\ 309574470819804987 \\ \hline \end{array}$$

$$33148648517294436663656267 = c = \text{the Proof.}$$

3. *Note*, That tho' you put a Figure or 2 too many or too few, or too little or too great, next the right hand of your Quotients, it will be rectify'd in the following Operations, if you proceed as *per the Theorm.* But the safest way is not to proceed farther at one Division than in the Example.

VIII. To illustrate what is said in *Note 3*: I shall shew how the first Example of Approximation to the Cube Root foregoing in this 13th Section, is done by different Approaches and Figures to what it is there: And how to manage when Negative Numbers fall in, as when the Cube of *a* or *s* exceed the Cube given from which they are to be subtracted.

Thus

Thus to extract the Cube Root of  $13 = c$  Approaches  $= a$

Sum of the Approaches  $= s$

Root  $= r$

The Theorem  $\frac{c - s^3}{3s^2} = a$  the 1st, 2d, &c.

$$\begin{array}{r}
 13.0 \text{ --- } (2 = a = s. \\
 3s^2 = 12) \ 50 \text{ --- } (.4 = a \ 2d. \ + a \ 1st. = s = 2.4 = s \\
 \hline
 13.000 = c \\
 13.824 = s^3 \\
 \hline
 3s^2 = 17.28) \text{ --- } .824000 \text{ refts } (-.0476 = a \text{ the } 3d. \ + a \text{ the } 2d. \\
 13280 \text{ --- } = 2.3524 = s. \\
 11840 \text{ --- } \\
 \hline
 13. \text{ --- } \\
 13.01767621824 = s^3 \\
 \hline
 3s^2 = 16.60135718) \text{ --- } .017676218240000 \text{ --- } (-.0010647 = a \text{ the } 4th. \\
 10748610600 \text{ --- } + 2.3524 = 2.3513353 = s. \\
 7877962920 \text{ --- } \\
 12374200480 \text{ --- } \\
 \hline
 13. \text{ --- } \\
 13.000010155464809237977 = s^3 \\
 \hline
 3s^2 = 1658633307907827) \text{ --- } .0000101554648092379770000 \text{ --- } (-.000000612278 = a \\
 \text{the } 5th. \\
 2036649617910150 \text{ --- } + 2.3513353 \\
 3780163100023230 \text{ --- } = 2.351334687722 \\
 4628964842075760 \text{ --- } (= s. \\
 13116982262601060 \text{ --- } = r \text{ the Root of the } \\
 15045491072462710 \text{ --- } (c \text{ given propd.}
 \end{array}$$

1. It may be observed from the last Operation, That there are different Digits in every Approach, from those in the Approaches under the first Example of the Cube Root, in this *Sett.* foregoing. But 2dly, That tho' the Digits towards the right hand in these Approaches be wrong, yet those  $a$ 's arising from the subsequent Operations proceed in each nearer and nearer to the Truth.

2. Although





Their other Instance of Converging, in the Example of 2, takes 25 Digits more in the Operation than the common Way of Extraction, and is not true in the 3d Decimal Place: Wonderful new Invention!

SECT. XIV. Concerning Figurate Numbers, with universal Series's to exhibit and give all the Figurate Numbers of any Order.

I have added what follows because the Series's will be found very useful in a great many Cases, as how the Unciæ of Powers of a Binome are found; they are useful likewise in determining the Numbers of Combinations, and the Laws of Chances, &c.

*The Table of Figurate Numbers.*

|                       |   |   |    |    |     |     |     |    |    |    |
|-----------------------|---|---|----|----|-----|-----|-----|----|----|----|
| Monads                | — | — | —  | 1  | 1   | 1   | 1   | 1  | 1  | 1  |
| Laterals              | — | — | —  | 1  | 2   | 3   | 4   | 5  | 6  | 7  |
| Triangulars           | — | — | —  | 1  | 3   | 6   | 10  | 15 | 21 | 28 |
| Pyramidals            | — | — | —  | 1  | 4   | 10  | 20  | 35 | 56 | 84 |
| Triangular Pyramidals | 1 | 5 | 15 | 35 | 70  | 126 | 210 |    |    |    |
| Pyramid Pyramidals    | 1 | 6 | 21 | 56 | 126 | 252 | 462 |    |    |    |

*The Construction and Use of the Table.*

The Monads, or Units, being added give the lateral Range, which are called Numbers of the 1st Order, as 1 and 1 is 2, and 1 is 3, and 1 is 4, &c. or 1, 2, 3, 4, 5, 6, 7.

The Laterals being added give the Triangulars: as 1 and 2 is 3, and 3 is 6, and 4 is 10, &c. so that these Triangulars 1, 3, 6, 10, 15, 21, 28, are called Numbers of the 2d Order.

The Triangular range being added give the Pyramidals or Numbers of the 3d Order: as 1 and 3 is 4, and 6 is 10, &c. giving 1, 4, 10, 20, 35, 56, 84.

The Pyramidals being added as 1 and 4 is 5, and 10 is 15, give 1, 5, 15, 35, &c. called triangular Pyramidals or Numbers of the 4th Order; and

The Triangular Pyramidals added as before, give the range called Pyramid Pyramidals which are Numbers of the 5th Order, &c.

The Names of these Numbers are analogous to the Cossic Numbers or Powers: For if in the side of an Equilateral Triangle there be

be 2 Points  $\therefore$ , the number of Points in the Triangle are  $3 = 1 + 2 =$  the triangular Number of the lateral 2 ; if there be 3 points in the Side as  $\therefore \therefore$  the Number of Points in the Triangle are 6,  $\therefore \therefore$  = the Triangular of 3 ; if 4 Points are in the Side as  $\therefore \therefore \therefore$ , the Number of all the Points are 10 = the Triangular of 4, and so in the rest. In the same manner, the Numbers in the Pyramidal Range will make the number of Points of which an equilateral triangular Pyramid may be supposed to consist, if it be imagined to be composed of Triangulars that are parallel to the Base, and that the Sum of all the Triangulars make up the Pyramid. That Number in the Lateral Range, which is the perpendicular Column over any Figurative Number, is called the Side ; thus 5 is the Side of the Triangular Number 15, and 7 is the Side of the Triangular 28, and of the Pyramidal 84.

**Problem 1.** The Laterals given, to find the respective Triangulars?  
**Note,** a Period, (in these Cases) between 2 Numbers, &c. is the same as  $\times$ , = multiplied by.

The Triangular of the Lateral 2 is  $2 + 1 = 3 = \frac{2 \cdot 3}{2}$  Answer.

Of 3, it is  $\frac{2.3}{2} + 3 = \frac{2.3 + 2.3}{2} = \frac{3.2 + 2}{2} = \frac{3.4}{2}$  Answer.

Of 4, it is  $\frac{3.4}{2} + 4 = \frac{3.4 + 2.4}{2} = \frac{4.3 + 2}{2} = \frac{4.5}{2}$  Answer.

Of 5, it is  $\frac{4.5}{2} + 5 = \frac{4.5 + 2.5}{2} = \frac{5.4 + 2}{2} = \frac{5.6}{2}$  Answer.

Univerſally therefore if the Side be  $l$ , the Triangular Number will be  $\frac{l.l+1}{2}$ , or if you ſuppoſe  $l = n - 1$ , the Triangular of  $n - 1$  will be  $n \cdot \frac{n-1}{2}$ .

*Problem 2.* The Laterals given to find the correspondent Pyramidals.

$$\text{The Pyramidal of the Lateral 2 is } = 4 = \frac{2 \cdot 3 \cdot 4}{2 \cdot 3}$$

$$\text{Of 3 is } = \frac{2 \cdot 3 \cdot 4}{2 \cdot 3} + \frac{3 \cdot 4}{2} = \frac{2 \cdot 3 \cdot 4}{2 \cdot 3} + \frac{3 \cdot 3 \cdot 4}{2 \cdot 3} = \frac{3 \cdot 4 \cdot 2 + 3}{2 \cdot 3} = \frac{3 \cdot 4 \cdot 5}{2 \cdot 3}$$

$$\text{Of 4, is } = \frac{3 \cdot 4 \cdot 5}{2 \cdot 3} + \frac{4 \cdot 5}{2} = \frac{3 \cdot 4 \cdot 5}{2 \cdot 3} + \frac{3 \cdot 4 \cdot 5}{2 \cdot 3} = \frac{4 \cdot 5 \cdot 3 + 3}{2 \cdot 3} = \frac{4 \cdot 5 \cdot 6}{2 \cdot 3}$$

$$\text{Of 5, is } = \frac{4 \cdot 5 \cdot 6}{2 \cdot 3} + \frac{5 \cdot 6}{2} = \frac{4 \cdot 5 \cdot 6}{2 \cdot 3} + \frac{3 \cdot 5 \cdot 6}{2 \cdot 3} = \frac{5 \cdot 6 \cdot 4 + 3}{2 \cdot 3} = \frac{5 \cdot 6 \cdot 7}{2 \cdot 3}$$

And universally if the Side be  $l$ , the Pyramidal of the Side will be  $\frac{l}{1} \cdot \frac{l+1}{2} \cdot \frac{l+2}{3}$ .

$$\text{Or making } n-2=l, \text{ it is } \frac{n}{1} \cdot \frac{n-1}{2} \cdot \frac{n-2}{3}.$$

*Problem 3.* The Lateral Numbers given to find the Triangular Pyramidals.

$$\text{The Triangular Pyramidals of 2 is } 5 = \frac{2 \cdot 3 \cdot 4 \cdot 5}{2 \cdot 3 \cdot 4}$$

$$\begin{aligned} \text{Of 3, it is } & \frac{2 \cdot 3 \cdot 4 \cdot 5}{2 \cdot 3 \cdot 4} + \frac{3 \cdot 4 \cdot 5}{2 \cdot 3} = \frac{2 \cdot 3 \cdot 4 \cdot 5}{2 \cdot 3 \cdot 4} + \frac{3 \cdot 4 \cdot 4 \cdot 5}{2 \cdot 3 \cdot 4} = \frac{3 \cdot 4 \cdot 5 \cdot 2 + 4}{2 \cdot 3 \cdot 4} \\ & = \frac{3 \cdot 4 \cdot 5 \cdot 6}{2 \cdot 3 \cdot 4} \end{aligned}$$

$$\begin{aligned} \text{Of 4, } & \frac{3 \cdot 4 \cdot 5 \cdot 6}{2 \cdot 3 \cdot 4} + \frac{4 \cdot 5 \cdot 6}{2 \cdot 3} = \frac{3 \cdot 4 \cdot 5 \cdot 6}{2 \cdot 3 \cdot 4} + \frac{4 \cdot 4 \cdot 5 \cdot 6}{2 \cdot 3 \cdot 4} = \frac{4 \cdot 5 \cdot 6 \cdot 3 + 4}{2 \cdot 3 \cdot 4} \\ & = \frac{4 \cdot 5 \cdot 6 \cdot 7}{2 \cdot 3 \cdot 4} \end{aligned}$$

$$\begin{aligned} \text{Of 5, } & \frac{4 \cdot 5 \cdot 6 \cdot 7}{2 \cdot 3 \cdot 4} + \frac{5 \cdot 6 \cdot 7}{2 \cdot 3} = \frac{4 \cdot 5 \cdot 6 \cdot 7}{2 \cdot 3 \cdot 4} + \frac{4 \cdot 5 \cdot 6 \cdot 7}{2 \cdot 3 \cdot 4} = \frac{5 \cdot 6 \cdot 7 \cdot 4 + 4}{2 \cdot 3 \cdot 4} \\ & = \frac{5 \cdot 6 \cdot 7 \cdot 8}{2 \cdot 3 \cdot 4} \end{aligned}$$

N n n

And



And univerfally if the Side be  $l$ , the Figurate Number of the 4th Order is  $\frac{l}{1}$ ,  $\frac{l+1}{2}$ ,  $\frac{l+2}{3}$ ,  $\frac{l+3}{4}$ . And if we

make  $n-3=l$ , the Figurate of  $n-3$  will be  $\frac{n}{1}$ ,  $\frac{n-1}{2}$ ,  $\frac{n-2}{3}$ ,

$$\frac{n-3}{4}$$

In the same manner we shall find, That in the Figurate Numbers of the 5th Order; if the Side be  $l$ , the Figurate Number answering is  $\frac{l}{1}$ ,  $\frac{l+1}{2}$ ,  $\frac{l+2}{3}$ ,  $\frac{l+3}{4}$ ,  $\frac{l+4}{5}$ . Or supposing  $l=n-4$

the Series will be  $\frac{n}{1}$ ,  $\frac{n-1}{2}$ ,  $\frac{n-2}{3}$ ,  $\frac{n-3}{4}$ ,  $\frac{n-4}{5}$ .

Hence the following Series will give us all the Figurate Numbers of any Order whatsoever: *i. e.*  $\frac{l}{1}$ ,  $\frac{l+1}{2}$ ,  $\frac{l+2}{3}$ ,  $\frac{l+3}{4}$ ,

$\frac{l+4}{5}$ ,  $\frac{l+5}{6}$ ,  $\frac{l+6}{7}$ ,  $\frac{l+7}{8}$ , &c. Or this Series.  $\frac{n}{1}$ ,  $\frac{n-1}{2}$ ,

$\frac{n-2}{3}$ ,  $\frac{n-3}{4}$ ,  $\frac{n-4}{5}$ ,  $\frac{n-5}{6}$ ,  $\frac{n-6}{7}$ ,  $\frac{n-7}{8}$ , &c.





# APPENDIX

*To an intire System of ARITHMETIC.*

By *EDWARD HATTON*, Gent.

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*Of the Mensuration of Superficies and Solids.*



Y Intent is to be as brief, and yet as copious as possible; not doubting but the Variety of Figures here measured, more than any other Treatise of Measuring contains, that I have seen or believe to be published, will meet with the Approbation of the Studios in this Geometrical Science.

The Dimensions I will suppose taken in Feet and Decimal Parts, as best answering that Accuracy and Brevity which will be expected from me: But at the Close of this Appendix I shall give my Advice what Instruments are most proper for the several kinds of Workmen; and also by what Denomination the several kinds of Work is valued.



## CHAP. I.

## To measure any Superficies.

**A** Superficies is properly defined to consist only of Length and Breadth or Girt, without taking any Notice of Thickness.

SECT. I. *The Geometrical Square and Parallelogram, Rhombus and Rhomboides.*

*Proposition 1. To measure the Geometrical Square.*

*Definition.* This is a Figure consisting of 4 equal Sides, and as many right Angles: as *Fig. I. Plate C.*

*Rule.* Take the Dimensions of one Side, as  $a b$ , and multiply it by itself, gives the Content.

*Example.* The Side of any Square being 11.28, the Content is found 127.238, as per Marg.

11.28  
11.28

*Prop. 2. To find the Area of the Parallelogram.*

*Defin.* This is a Figure consisting of, or bounded by, 2 longer equal parallel Sides, and 2 shorter equal and parallel Sides, having 4 right Angles: as *Fig. II. Plate C.*

31584  
12408

*Rule.* Multiply the Length by the Breadth, and the Product is the Area.

*Answ.* = 127.2384

*Example.* The Length  $a b = 23.53$ , Breadth  $b c = 2.15$ . See the Work.

23.53  
2.15

*Prop. 3. To find the Area of a Rhombus.*

*Answ.* = 50.5895

*Defin.* This Figure is bounded by 4 equal Sides, and hath 4 Angles, the 2 opposite ones being equal, and the Sides parallel: as *Fig. III. Plate C.*

*Rule.* The distance between 2 opposite Angles, as  $b d$ , multiplied by the Line  $e c$ , gives the Answer.

*Example.*



## SECT. II. To measure the Conic Sections.

*Example.* The Diagonal  $bd = 1.34$ ,  $ec = .85$ ;  
the Answer is 1.139

461

1.34

.85

*Prop. 4.* To find the Area of the Rhomboides.

*Defin.* This Figure is bounded with 4 right Lines,  
the opposite Sides equal and parallel, and the  
opposite Angles equal: as *Fig. 4. Plate C.*

1.1390

*Rule.* Multiply the Diagonal  $ab$  by the Perpen-  
dicular  $cn$  or  $mr$ , and the Rectangle answers.

12.47

4.127

*Example.* The Diagonal  $ab = 12.47$ , the Per-  
pendicular  $cn = 4.127$ , and the Answer is  
therefore, as *per Margin*, = 51.46369.

33669

51127

*Answ.* = 51.46369

## SECT. II. To measure the Conic Sections.

*Prop. 1.* To measure a Triangle.

*Defin.* Right-lined Triangles are of 3 kinds (besides the equilateral,  
whose Sides are equal) viz. Right angled, as  $arn$  or  $xrn$  right  
angled at  $r$  in *Fig. 5.* 2dly, Scalenoous Triangles, as  $oro$ , the Sides  
of which are all unequal. And, 3dly, Ifoceles Triangles as  $anx$   
appears; 2 of whose Sides are equal. This last is one of the  
Sections of a Cone, made by cutting the Cone ( $ewagxna$ ) down  
the Axis, as  $op$ , *Fig. 5. Plate C.*

*Rule.* Multiply the Base (as  $xa$ ) by half the Perpendicular (as  $nr$ )  
or half the Base by the Perpendicular, or the Perpendicular by  
the Base; and taking half the Product, will give the Area of any  
Triangle as above mentioned: for all Triangles are half a Square  
of the same Base and Perpendicular.

*Example.* The Base  $ax$  being = 9.35, the  
Perpendicular  $nr = 12.42$ , the Area is  
58.0635, as *per Margin*.

9.35

6.21

*Prop. 2.* To measure the Circle

19635

*Defin.* This is formed by cutting the Cone  
( $xwagxna$ ) parallel to the Base ( $xwagx$ )  
as  $mc$ , *Fig. 5. Plate C.*

5610

*Rule.* There are several ways of finding the  
Area of a Circle. As

*Answ.* = 58.0635

1st,

1<sup>st</sup>, As 7 to 22; so the Diameter (as  $rs$ , Fig. 6) to the Circumference.

2<sup>dly</sup>, As 1 to 3.1416, so the Diameter to the Circumference.

Then multiply the Circumference by a 4<sup>th</sup> of the Diameter, or a 4<sup>th</sup> of the Circumference by the Diameter, gives the Answer.

Or the Area of a Circle is found without the Circumference thus:

As 1 is to .7854 :: so is the Square of any Diameter to the Area.

*Example.* Admit the Diameter  $rs$ , Fig. 6. be

14.32

= 14.32, what is the Area?

14.32

By the last Method

1. .7854 :: 205.0624. 161.056 = Answ.

45824

*Prop. 3.* To measure the Ellipsis, or Oval.

20048

*Defin.* This is a Section of a Cone, made by cutting it through both its Sides, but not parallel to the Base; as the Section  $le$  Fig. 5.

Plate C. which produceth Fig. 7. Plate C.

□ = 205.0624

*Rule.* Multiply the *Latus transversum* (or transverse Ax)  $lt$ , by the Diameter Conjugate  $cd$  and the Product is = the Square of the Diameter of a Circle equal to the Oval: then work as in the last.

*Example.* Admit  $lt = 15.34$ , and  $cd = 12.28$

 $lt = 15.34$ 

the Answer is found 147.95 fere: for

 $cd = 12.28$ 

1. .7854 :: 188.3752. 147.9499

*Prop. 4.* To find the Area of the Parabola.

42952

*Defin.* This Section is formed by cutting the Cone parallel to the opposite Side, as in the Line  $ts$ , Fig. 5. which is parallel to the Side  $an$  and produceth a Parabola; as Fig. 8. Plate C.

18408

&lt; = 188.3752

*Rule.* This Figure being 2 Thirds of a Parallelogram made of the whole Ordinat  $pa$  and the Abscissa  $ss$  therefore multiplying those 2 Lines together, 2 Thirds of the Rectangle is the Area.

3.84

2.54

20736

768

*Example.* Admit  $ap = 3.84$ , and  $ss =$

Rectangle = 9.7536

2.54; 2 Thirds of the Rectangle is

 $\frac{2}{3} = 3.2512$ 

= the Answer = 6.5024.

 $\frac{2}{3}$  or Answer = 6.5024

*Prop.*

### SECT. III. To measure any of the Regular Polygons. 463

*Prop. 5. To find the Area of the Hyperbola (or Hyperbolic Space).*

*Defin.* This Section of a Cone is formed by cutting it (not parallel to the Side, as in the Parabola, but) so that if the Axis of the Section were continued upward, it would intersect the opposite Side of the Cone produced: the Section's Axis is represented by the Line  $yz$ , Fig. 5. and produceth an Hyperbola, as Fig. 9. Plate C.

To measure this Figure, there is no general Rule that can be given for Practice: But the Investigation is performed by the help of the Asymptotes, an Infinite Series, and partly by the Method of Fluxions; which 'tis not proper here to insert.

### SECT. III. To measure any of the Regular Polygons.

*Defin.* These are Figures consisting of above 4 equal Sides, and as many equal Angles. As,

The Pentagon having 5 equal Sides, and as many Angles.

Hexagon 6

Heptagon 7

Octagon 8

Nonagon 9

Decagon 10

Undecagon 11

Dodecagon 12

*Rule.* To measure any of these, draw 2 Lines from any 2 Angles that include a Side to the Center: then find the Content of that Triangle, as taught *Prop. 1. Sect. 2.* above. Lastly, Multiply the Area of that Triangle by the Number of Sides, and the Product is the Content of the whole Polygon. Or multiply half the Sum of the Sides by the nearest Distance from the Center to one of the Sides, gives the Answer.

*Example.* The Side of the Pentagon (*sd*, &c.) Fig. 10. is = 15.34, the Perpendicular (or Radius of a Circle inscribed) is  $rc = 10.5$ : So that 15.34 multiplied by 5, the Number of Sides, gives = 76.7 = Sum of the Sides; 38.35  
half of that is = 38.35, which multiplied by 10.5  
 $rc = 10.5$ , gives the Area = 402.675.

*Answer* = 402.675

SECT.



SECT. IV To measure the Trapezium, the Parallelogramic, the Polygram, (or Multangle, or irregular Polygon) the Area or Superficies of the Cylinder, Cone, and Sphere.

Prop. 1. To measure the Trapezium.

Defin. This is a Figure of 4 Sides and 4 Angles, uncertain whether parallel or equal, but is commonly neither; as Fig. 11. Plate C.

Rule. Multiply the Diagonal ( $an$ ) by half the Sum of the Perpendiculars  $mp$  and  $cp$ ; or the Sum of the Perpendiculars by half the Diagonal, and either of the Products is the Answer.

Example. The Diagonal  $an = 17.42$ ; the Perpendicular  $mp = 4$ , and  $cp = 2.6r$ , half the Diagonal is  $= 8.71$ , and  $mp$  more  $cp = 6.6r$ , the Product of which 2 last is  $58.06666$ , &c.

8.71  
6.6r

5226

Or half the Product of the Diagonal in the Perpendiculars gives the Answer.

Answer  $= 58.06r1$

Prop. 2. To find the Area of the Parallelogramic, or Parallelopleuron.

Defin. This Figure hath 4 Sides, 2 of which opposite are parallel, and of 4 Angles the 2 at each end being equal: as Fig. 12. Plate C.

Rule. Multiply half the Sum of the 2 Sides ( $vr$ ) and ( $st$ ) by the Length ( $tw$ ) gives the Answer. Or a Diagonal ( $rt$ ) multiplied by half the Sum of 2 Perpendiculars let fall on that Diagonal from  $s$  and  $v$ .

Sum  $ru + st = 1.49$

Example. The Ends  $ru = 0.79$ ;  $st = 0.7$ ; Length  $wt = .8888$ , &c. half the Sum of  $.79$  and  $.7$  multiplied in  $.8r = .662r1$ .

.745  
.8r

Prop. 3. To find the Area of the Polygram, (sometimes called an irregular Polygon, or Multangle.)

5960

Defin. This is any Geometrical Figure having above 4 Sides, which are uncertain, and generally unequal; as are also its Angles: as Fig. 13. Plate C.

.662r1

Rule. First divide your Figure into Trapeziums and Triangles, and then measure the same as under Prop. 1. Sect. 2. and Sect. 4.

Example.

SECT. IV. To measure the Cylinder, &c. 465

Example. Figure 13 is divided into,

1. The Trapeziums (*utmqv*) | In the 1st,  $qt = 18.26$ , the  $\frac{1}{2} = 9.13$
2. ———— and *qmprq* |  $ux = 5.17$ , more  $bm = 3.5 = 8.67$
3. And the Triangle *qra q*.

|                                                        |                 |                  |
|--------------------------------------------------------|-----------------|------------------|
|                                                        | 61171           |                  |
|                                                        | 7304            |                  |
|                                                        | Prod. = 79.1571 |                  |
| In 2d Trapez. $qp = 15.73$ , the half = 7.865          | }               | Prod. = 53.56065 |
| $mn = 4.5$ , more $rs = 2.31 = 6.81$                   |                 |                  |
| In 3d or the $\Delta$ , $qa = 10.72$ ; the half = 5.36 | }               | Prod. = 17.5272  |
| $ro =$ the Perpend. = 3.27                             |                 |                  |
| The Sum = the Area of the whole Fig. 13. = 150.24495   |                 |                  |

Prop. 4 To find the Area of a Cylinder.

Defin. This Figure is defined under Prop. 1. Chap. 2.

Rule. Multiply the Ambit or Circumference of the Cylinder by its Length, and to that Rectangle add the double Area of the Base or End, and the Sum is the whole Area.

|                                                                |   |        |
|----------------------------------------------------------------|---|--------|
| Area of the curved<br>part of the Cylin-<br>der is therefore — | } | 3.1416 |
| Area of the Base = .7854                                       |   |        |
| Ditto = .7854                                                  |   |        |
| Whole Area (or Sum) = 4.7124                                   |   |        |

Example. Ambit of the Cylinder = 3.1416; Altitude as also the Diameter of the Base = 1.

But there are 2 other ways of doing this, where the Diameter and Altitude are equal.

As, 2dly, Multiply the Area of the Sphere having the same Diameter by 3, and divide by 2.

Or, 3dly, 6 times the Area of the Base, or End, is the Answer. Thus 6 times .7854 is = 4.7124, as above.

Prop. 5. To find the Convex Superficial Content of a Sphere.

Defin. This Body is defined where its Solidity is measured, Prop. 4. Chap. II. and 'tis represented Fig. 6. Plate C.

Rule. The Area is 4 times that of a great Circle; therefore having by the Diameter found the Area of a great Circle of any Sphere, multiply it by 4, and you have the Answer.

|                               |
|-------------------------------|
| 1. .7854 :: 144.              |
| 144                           |
| Prod. = 113.0976 Area Circle. |
| 4                             |
| (the Sphere.                  |

Enam. The Diameter of a Sphere being 12, the Area of the same is 452.39, as per Margin.

O o o

Prop. 6.

|                    |
|--------------------|
| 452.3904 = Area of |
|--------------------|

466 *To find the Area of the Parts of a Circle.* CHAP. I.

*Prop. 6. To find the Superficies (or Area) of a Pyramid.*

*Defin.* This is a solid Body, having (generally) a Square or some regular Polygon for its Base, being tapering like the Spire of a Steeple, &c.

*Rule.* Multiply the Circumference of the Base (or Sum of the Sides) by half the Altitude, and it gives the Superficial Content.

*Example.* The Side of the Base of a Pyramid (being an Octogon) is = 5.65, that multiplied by 8 = the Number of Sides, and that Product by half the Altitude = 30.7, the last Product is the Area sought.

$$\begin{array}{r} 5.65 \\ \times 8 \\ \hline \end{array}$$

*Prop 7. To find the Area of a Cone.*

*Defin.* The Cone is a Solid which hath a Circle for its Base, from whence it is gradually tapering upward till it terminate in a Point directly over the Center of its Base; as Fig. 5. Plate C.

$$\begin{array}{l} \text{The Total Sides} = 45.20 \\ \text{Half the Altitude} = 30.7 \\ \hline \end{array}$$

$$\text{Area} = 1387.64$$

*Rule.* As the Radius of the Base : Is to the Side of the Cone :: So is the Area of the Base to the Area of the Cone.

$$\begin{array}{l} 1. .7854 :: 36 \\ \hline 36 \end{array}$$

*Example.* Diameter of the Base = 6; Side of the Cone = 15 : So the Area of the Base of the Cone is found = 28.2744; and the Area of the Cone, as by the 2d Analogy = 141.37.

$$28.2744 = \text{Area of the Base.}$$

|      |      |       |          |          |
|------|------|-------|----------|----------|
| Rad. | Bas. | Side. | Ar. Base | Ar. Cone |
| 3.   | 15   | ::    | 28.2744. | 141.37   |

Or Note farther, That the Area of a Cone is = that of a Triangle, whose Perpendicular is the Altitude of the Cone, and the Base = the Circumference of the Cone's Base: For in Example,  $1. 3.1416 :: 6. 18.8496$  = the Base or Circumference; half of which multiplied by the 15, gives the Answer as before = 141.37.

SECT. V. *To find the Area of the Parts of a Circle; as the Semicircle, Quadrant, Sector, Segment, Lune or Crescent, and the Trochoid or Cycloidal Space.*

*Prop. 1. To measure the Semicircle, Quadrant, and Sectors of a Circle, also the Segment.*

*Defin.* The Semi- or Half-Circle is the Space between  $dmbd$ . Fig. 14. Plate C; the Quadrant or Quarter of the Circle is  $dmc d$ , the Sector is  $cbrec$  or the like, less or more than a Quadrant.

*Rule.*



SECT. V. *To find the Area of the Parts of a Circle.* 467

*Rule.* Multiply 1 fourth of the Ark, or Curve Line, by the Diameter of the Circle, and the Product is the Area required.

*Example.* The Circumference is = 3.1416, the Diameter of that Circle = 1.

The Ark of the Half-Circle  $dcbmd$  as  $bmd = 1.5708$

of the Quadrant  $cmdc$  as  $cmd = .7854$

of the Sector  $cbrec$  as  $erb = 1.05$

So that 1 fourth of 3.1416 = .7854  $\times$  by 1, gives .7854 the Area of the Circle.

1 fourth of 1.5708 = .3927  $\times$  by 1, gives .3927 the Area of the Half-Circle.

1 fourth of .7854 = .1963  $\times$  by 1, gives .19635 the Area of the Quadrant.

1 fourth of 1.05 = .2625  $\times$  by 1, gives .2625 the Area of the Sector.

And the Area of the Sector  $cbrec$ , less the Area of the Triangle  $cbnec$ , gives the Area of the Segment  $enbre$ ; Fig. 14. Plate C.

*Prop. 2. To find the Area of the Lune.*

*Defin.* This is a Figure like a Crescent or increasing Moon; as  $arnea$ , Fig. 15.

*Rule.* Find first the Area of the Semicircle  $ncan$ , and from that deduct that of the Segment  $ncan$ , and the Remainder is the Area of the Lune.

*Example.* The Diameter  $na = 1$ , and consequently the .3927  
Ark  $arn = 1.5708$ , and Area = .3927. .121

The Area of the Sector  $eaone$  is = .256, from which  
deducting the Area of the Triangle  $nean = .135$ , .2717  
the rest is the Area of the Segment =  $noan$   
= .121; which taken from the Area of the Semicircle, there  
resteth the Answer = .2717.

*Prop. 3. To find the Area of the Trochoid, or Cycloidal Space.*

*Defin.* This Figure (as 16, Plate C.) is generated by the Circle ( $lbar$ ) which turneth round on the Line  $arc$ , the Point at  $b$  beginning at  $a$ , and when it touches the Line again at  $c$ , the Cycloidal Curve  $abc$  is finished, and the Space from that to the Line  $arc$  contains 3 times the said generating Circle.

*Rule.* Therefore having found the Area of a Circle, whose Diameter is ( $rb$ ) multiply that by 3, and it produceth the Area of the Cycloidal Space.

*Example.* The Line  $rb = 1$ ; Area of the Circle = .7854, and of the Cycloidal Space = 2.356.

## C H A P. II.

*The Mensuration of Solids.*

SECT. I. *To find the solid Content of a Cylinder, Parallelopiped, Prism, Cone, Pyramid, and Sphere.*

**P**ROP. 1. *To measure the Cylinder.*

*Defn.* A Cylinder is a solid Body having a Circle for its Base, and is of equal Circumference from one end to the other, like a Rolling-Stone for Walks, &c. as the *Figure 17. Plate C.*

*Rule.* Multiply the Area of the Base (found as under *Prop. 2. of Sect. 2. Chap. 1.*) by the Length, and the Product is the Answer.

*Example.* The Diameter of the Base, as  $(ab)$  or  $(cd) = 14.32$ , the Area of that Circle will be found 161.056; which multiplied by the Length  $(bc) = 14.32$ , or  $(ad)$  the solid Content, is =

|            |  |
|------------|--|
| 161.056    |  |
| 14.32      |  |
| 2306.32192 |  |

as per Margin.

*Prop. 2. To find the solid Content of the Parallelopipedon and of the Prism.*

*Defn.* This Solid is bounded by 4 Parallelograms, equal and parallel, for the Sides; and 2 Squares or Parallelograms, equal and parallel, for the Ends: as *Fig. 18. Plate C.*

*Rule.* Multiply the Area of the Base or End by the Length, and the Rectangle is the Answer.

*Example.* The End is a Square whose Side is 3.72, and the Length = 31.44. See the Operation, where (according to the Method of working Case 5. of *Sect. 5. of Ch. 3.*) the Solidity is found

|                 |          |      |                 |
|-----------------|----------|------|-----------------|
| Area at the end | 138384   | 3.72 |                 |
| Length inverted | 44.13    | 3.72 |                 |
|                 | 415152.  |      | 26784           |
|                 | 138384   |      | 1116            |
|                 | 55354    |      |                 |
|                 | 5535     |      | 13.8384 = Area  |
|                 | 435.0793 |      | = Solid Content |

*Note,*

SECT. I. To find the solid Content of the Cone, &c. 469

*Note*, That a *Prism* is measured also by multiplying the Area of the Base (which is either a Triangle or a Polygon) by the Length of the Prism. This or the last Figure are three times a Pyramid of the same Base and Altitude.

*Prop. 3. To find the solid Content of the Cone: And 2dly, of a Pyramid.*

*Defin.* This Figure hath a Circle for its Base, and is defined *Prop. 7 Sect. 4. of Chap. 1. of this Appendix.*

*Rule.* Multiply the Area of the Base by one third of the perpendicular Altitude of the Cone, and that Rectangle is the Answer.

*But Note*, That to find the Height (it being a solid Body) must be done by this Rule, as *Euclid 47. 1.*

From the Square of the slant Height (*ml* Fig. 19.) deduct the Square of the Semidiameter of the Base as (*ma*), and the Square Root of the Remainer is the perpendicular Altitude (*xl*).

*Example.* The Diameter *md* is = 6, the slant Height *ml* = 5; and consequently the true Altitude (*xl*) = 4: for 5 squared less  $\frac{1}{2}$  = *ma* squared, = 16 = the Square of *xl*; that is, 5 squared less 3 squared, is = 16; whose Root is 4 = *xl*. So that as 1. .7854 :: 36. 28.274 = the Area of the Base: Which multiplied by 1 third of 4 = 1.3r, the Product is = 37.6986r1.

To measure the Pyramid, you must also multiply the Area of the Base by a Third of the perpendicular Altitude: for as a Cone is 1 Third of a Cylinder of the same Base and Altitude, so is a Pyramid 1 Third of a Prism or Parallelopiped. 'Tis defined *Prop. 6. Sect. 4. Chap. 1. of this Appendix.*

*Prop. 4 To find the solid Content of a Sphere.*

*Defin.* This is a solid Body, every way perfectly round, so that all Lines drawn from the Convex Area to the Center are equal. It is 2 Thirds of a Cylinder of the same Diameter of the Base and Altitude with the Diameter of the Sphere, or of a Cylinder circumscribed.

*Rule.* Find the Area of a great Circle, as *Prop. 5. of Sect. 4. Chap. 1.* which multiply by the Diameter; and 2 Thirds of the Rectangle is the solid Content required.

*Example.* The Diameter of a Sphere being 12, the Area of a great Circle (as Fig. 6. Plate C.) is = 113.0976; which multiplied by the

$$\begin{array}{r}
 28.274 \\
 1.3r1 \\
 \hline
 84822 \\
 \hline
 94246 \\
 28274 \\
 \hline
 37.6986r1 = Ans.
 \end{array}$$



the Diameter (or Length of a Cylinder) 12, the Rectangle is = 1357.171, the Solidity of the Cylinder; 2 Thirds of which is the solid Content of the Sphere = 904.7808.

SECT. II. *To find the Solidity of the five Platonic, or Regular Bodies, viz. the Tetrabedron, Hexabedron, Octabedron Dodecabedron, and Icosibedron.*

*Prop. 1. To measure the Tetrabedron.*

*Defin.* This is a solid Body bounded with 4 equal equilateral Triangles; and being measured as a Pyramid, it needs no other Rule than as under *Prop. 3. of Sect. 1. Chap. 2.* See *Fig. 20. Plate C.*

*Prop. 2. To find the Solidity of the Hexabedron or Cube.*

*Defin.* This Solid is bounded with 6 equal Geometrical Squares.

*Rule.* Multiply a Side in itself, and that Rectangle by the Side gives the Answer. See *Fig. 21. Plate C.*

*Prop. 3. To find the solid Content of the Octabedron.*

*Defin.* This Figure is bounded with 8 equal equilateral Triangles, which are the Bases of as many Pyramids, which meet in the Center of a Sphere, (out of which this Body is cut) so that it being 8 Pyramids. (See *Fig. 22. Plate C.*)

*Rule.* First find the Area of one of the Triangles, then take with Callipers, &c. the Distance between the Center of that Triangle and that of the opposite; by one sixth part of which multiply the Area of any one Triangle; and that Product multiply'd by 8, this last Rectangle gives the Solidity required.

*Prop. 4. To find the solid Content of the Dodecabedron.*

*Defin.* This is a Solid bounded by 12 equal Pentagons, which are the Bases of so many Pyramids that meet in the Center of this Solid, or of a Sphere out of which this Body is cut. See *Fig. 23. Plate C.*

*Rule.* Therefore take with a Pair of Callipers the Distance from the Center of one Pentagon to that of its opposite, and by one sixth of that multiplying the Area of any of the Pentagons; 12 times that Rectangle gives the Solidity. Or twice the Distance of the opposite Pentagons multiplied by the Area of 1 Pentagon, gives the Answer.

*Prop. 5. To find the Solidity of the Icosibedron.*

*Defin.* This is the last of the 5 Platonic or Regular Bodies: it is bounded with 20 equal equilateral Triangles, which are Bases to so many Pyramids, whose small Ends terminate in a Point in the Center of a Sphere which circumscribeth this Solid and out of which it is cut. (See *Fig. 24. Plate C.*)

*Rule.*

SECT. III. *Solid Content of a Parabolic Conoid, &c.* 471

*Rule.* Multiply the Area of one Triangle by 1 sixth of the Distance of the Centers of 2 opposite Triangles (taken with Callipers) and that Rectangle multiplied by 20 (the Number of Pyramids in the Body) gives the Solidity sought.

I have not thought it necessary to give Examples to these 5 Rules because there is not much more therein besides the repeated Mensuration of a Pyramid, which is taught above. But I have chiefly inserted them for the sake of the Definitions, which I do not remember to be in any Tract of Measuring, no more than many other things contained in other Propositions of this Appendix.

SECT. III. *To find the solid Content of the Parabolic Conoid, the Parabolic Spindle, the Cyliindroid, the Spherooid, and the Hyperbolic Conoid.*

*Prop. 1. To find the solid Content of the Parabolic Conoid.*

*Defn.* This Solid is generated by the Rotation of a Semi-Parabola, as (*pss* Fig. 8. Plate C.) round its Axis *ss*; and is = half of the circumscribing Cylinder *pqsxasp* (or 1 and half of the Cone inscribed). Therefore

*Rule.* Multiply the Area of the Base by the Axis or greatest Absciss, and half the Rectangle is the Answer.

*Example.* The Diameters of the Figure (as *ap*) = 14.32, the Area of the Base will be found 161.056; which multiplied by the Absciss (*ss*) = 14.32, the Rectangle is = 2306.32192; the half of which is = 1153.16096 the Answer.

*Prop. 2. To find the Solidity of the Parabolic Spindle.*

*Defn.* This Figure is generated by the Rotation of the Semi-parabola round its Ordinat, and is  $\frac{1}{3}$  of the circumscribing Cylinder. Therefore

*Rule.* Multiply the Content found as a Cylinder by 8, and divide by 15, the Quote is the Answer, which needs no Example.

*Prop. 3. To find the solid Content of the Cyliindroid.*

*Defn.* This Figure only differs from that under *Prop. 1. Sect. 1. of Chap. 2.* inasmuch as that hath a Circle, this hath an Ellipsis for its Base: so that to measure it,

*Rule.* Multiply the Base found as *per Prop. 3 Sect. 2. Chap. 1.* by the Length, and that Product is the Answer.

*Example.* Admit the *Latus transversum* (as *lt*, Fig. 7. Plate C.) by *cd* = the Diameter Conjugate, (i. e. suppose *lt* = 15.34, and *cd* =

$cd = 12.28$ ) the Area is  $= 147.95$ , which multiply'd by the Length  $= 1.871$ , the Answer is  $= 279.45$ .

Prop. 4. To find the solid Content of the Sphæroid.

Defin. This Figure is generated by the Rotation of the Semi-Ellipsis round its Axis, or *Latus transversum*; and 'tis by some called the Problate Sphæroid, to distinguish it from that which is generated by the Rotation of a Semi-Ellipsis, which turns round its Diameter Conjugate, and is called an Oblate Sphæroid, (of which Figure, 'tis asserted by the most Learned, our Earth is, the Diameter at the Equator being greater than between the Poles.) This Problate Sphæroid is 2 Thirds of a Cylinder, whose Base's Diameter is  $=$  the greatest Diameter of the Sphæroid, and its Altitude  $=$  the *Latus transversum* of the Ellipsis.

Rule. Multiply the Area of a Circle, whose Base is the Conjugate Diameter, (or here the greatest Diameter of the Sphæroid) by the Length, and 2 Thirds of the Product is the Content of the Sphæroid.

Example. The Diameter in the Middle  $= 14.32$ , as  $cd$  Fig. 7. the Area of that Circle  $161.056$ , which multiply'd in the Length  $11 = 14$ , gives  $200.48$ ; 2 Thirds of which  $= 133.65$ , the Answer.

Prop. 5. To find the Solidity of the Hyperbolic Conoid.

Defin. This Figure is generated by the Rotation of the Semi-Hyperbola ( $zbr$ ) Fig. 9. round the Abscissa  $zi$ ; which formeth the Hyperbolic Conoid  $brinz$ .

Rule. The *Latus transversum*  $= 10$ , multiply'd by 6; more the Axis or the Abscissa  $iz$  multiply'd by 6 is  $=$  the Divisor.

2dly, Multiply  $baisnz$   $=$  the Content of the Cylinder (whose Diameter is equal to the whole Ordinat  $bn$ , and its Length  $=$  the Abscissa  $zi$ ) in 3 times the *Latus transversum*, more 2 times the Abscissa, and that Rectangle is  $=$  a Dividend; which divided, there ariseth the Quotient, which is the Solidity required.

Example. The whole Ordinat (or Diameter)  $bn = 3.51$ , the Abscissa or Axis  $iz = ba = 2.43$ , and the *Latus transversum*  $10 = 2.16$ .

The Content of the Cylinder  $baisnz$ , is therefore  $= 23.513$  } mult.

The *Latus transversum* mult. in 3  $= 6.48$  }

The Axis or Abscissa mult. in 2  $= 4.82$  }

Sum  $= 11.3$  }

The *Latus transversum* mult. in 6  $= 12.96$  |

The Axis multiplied in 6 is  $= 14.46$  |

$=$  Sum Divisor  $= 27.42$

265.6969  $=$  Prod.

18916 } 9.69  $=$

24649 the Ans.



SECT. IV. *To find the Solidity of the Frustums of a Cone, Sphere, and Spheroeid.*

*Prop. 1. To find the solid Content of the Frustum of a Cone or Pyramid.*

*Defin.* This may be done several ways: as to find the Content of that part of the Cone (*drrmad*) *Fig. 19. Plate C.*

*Rule. 1.* Find the Content of the whole Cone (*dldm*) then deducting the Content of the upper part (*rlr*) the rest is the Content of the Frustum (*drrmd*.)

All the difficulty (more than as *Prop. 3. Sect. 1.* of this *Chap.*) is to find the Height (*al*) which is done by this Rule, That the homogeneous Sides of similar Triangles are in proportion: so that as *mn. mr :: ma. ml*; whence a Perpendicular let fall to *a*, is the true Altitude of the Cone.

*Rule 2. But more general for a Cone, Pyramid, &c. without the upper part:* Multiply the Areas of the greater and lesser Ends together, extract the Square Root of the Product; then multiply the Sum of the said Root, and 2 Areas of the Ends by one third of the Altitude of the Frustum, and the Product is the Content.

*Example.* In *Fig. 19. Plate C.* the Area of the greater Base (*modxm*) is = 36.72; of the lesser End (as *rsrsr*) = 15.5; the Product of which is = 569.16, whose Square Root is 23.857; which added to the 2 Areas, gives = 76.077; which multiply'd by 1 third of the Altitude *nr* = 6.6*r*, the Product is the solid Content of the Frustum = 507.179*r*1.

*Prop. 2. To find the solid Content of the Frustum of Sphere or Globe.*

*Defin.* By a Segment is meant a part of the Sphere cut off perpendicular by a Plain: as *bnerb*, *Fig. 14. Plate C.*

*Rule.* Find the Area of the Segment's Base, which multiply by the Altitude, reserving the Product. Then

To half the Altitude of the greater Frustum add 1 sixth of the Altitude of the lesser, and multiply the Sum by the reserv'd Rectangle, and divide this last by the greater Frustum's Altitude, and the Quote is the lesser Frustum's Solidity required.

*Example.* Diameter of the Frustum = 11.832 = *be*, Area = 179.46

Its Altitude = *ns* . . . . = 5

Alt. of the greater Frustum = *no* = 7 | Reserv'd Rectan. = 897.3

Half that Altitude is — — = 3.5

1 Sixth of the Alt. of the lesser = .83*r*1

} Sum multipl. = 43*r*1

Product = 3885.309

1 Seventh

1 Seventh of which last Product is  $= 555.044 =$  the Solidity of Frustrum required.

Note, That if you divide the Square of half the Diameter of the Frustrum's Base by the Altitude of either Frustrum, the Quote is the other Frustrum's Altitude.

Prop. 3. To find the solid Content of the middle Frustrum of a Spheroid.

Defin. This is represented by *raiiicr*, Fig. 7. like a Cask,

Rule. The easiest Rule, and what is near the Truth, is this: Multiply the Difference of the Bung and Head Diameters by .7, and add the Product to the Head Diameter; to that Sum find the Area of a Circle, and multiply it by the Length of the Cask, and that Product is the Answer.

Example. The Bung Diameter  $20 = cd$ , Head  $15 = ra$  or  $ii$ , Length  $00 = 12$ . The Difference between the Bung and Head  $= 5$ , which multiply'd by .7 is 3.5; which added to the Head 15, gives 18.5 for a mean Diameter, whose Area is  $= 268.803$ ; which multiplied by the Length  $= 12$ , the Solidity is  $= 3225.638 =$  the Content of the said Frustrum (*radioir*).

Note, That if the Spheroid be cut in the middle of (*tm*) by a Plain (*ra*) that part (*rstaor*) is 5 Ninths of a Cylinder, whose Base is (*roa*) and the Altitude (*ot*.)

I believe I have, in the Pages preceeding of this Appendix, shewn the best Ways of Measuring a greater Variety of Superficies and Solids than any Book wrote purely on that Subject contains: and to close this Chapter of Solid Measure, I may farther add, There is no Body, tho' never so irregular, but what may have its Content discovered by the help of Water For Example, I would know the solid Inches in a Faggot.

First, I put Water into a Vessel (suppose 36 Inch. by 20, and is a Parallelopipedon; and taking the Depth, I find it 30 Inches deep) into which I immerse the Faggot, so as to cover it with Water; and taking the Depth again, I find it 33 Inches deep, or 3 more than before the Faggot was in: so the said 36 by 20  $= 720$  multiply'd in this 3, produceth 2160, the Inches solid contained in the Faggot.

Brief

*Brief and Useful Observations on the most Proper Instruments to be used in the Mensuration of several things, &c.*

**B**ricklayers Work is best measured by the Foot and 100 Parts: for the Divisor being 272: and a quarter, (which is the Feet in one Rod, or the Square of 16 Foot and half) to reduce Feet into Rods, by which Denomination they value their Work: it is necessary that the Superficies of the Walls being the Dividends, should be Feet and Decimal Parts, the Divisor or 1 Rod, Brick and half thick is  $\text{---} = 272.25 \text{ Feet}$

Or without the Rod, the shortest Way is to multiply any Number of Feet by the Value of 1 Foot at the Rate agreed on per Rod, as at  $\text{---} = 5.4450$   
 $l. 5 : 10 : 0$ , the Foot is in Value  $l. .020202$ , &c.

Here the Multiplier being  $.020202$ , &c. or  $.02r2$   $\text{---} = 5.499r1$

I only multiply the Feet on the flat Wall by 2, and add every other Figure of the Product till there are but 2 places left, which put down as in

*Example 2.* What is the Value of 1361.25 Foot, Brick and half thick, at  $l. .0202020202$ , &c. or  $.02r2 l.$  per Foot, or  $l. 5 : 10$  per Rod. See the Operation.

Foot 1361.25  
 Mult.  $= .02r$

As per Rule, add every other of  $= 27.2250$

Sum or Answer  $= 27.4999$ , &c. or  $l. 27.5$

Note, That all Walls must be reduced to Brick and half thick by Multiplying the Superficial Feet by the half Bricks which the Wall is thick, and Dividing by 3.

*Carpenters Work.* The common two Foot serves as well as any other Instrument to measure Flooring, Partitioning, and Roofing; they being each valued by the Square of 100 Foot: so that Feet multiply'd by Feet, and 2 Figures cut off towards the right hand, give the Squares towards the left hand.

*Painters and Plaisterers* giving in and valuing their Work by the Yard, I think the best Instrument for them to take their Dimensions with, to be a Yard divided into 100 equal Parts.

*Glasiers*



*Glaifiers Work* and *Glass* being valued by the Foot, and Crown-Glass being to be done accurately, ought to be measured by the Foot, and Decimal, or 100 Parts of a Foot.

*Joiners Work* (as Wainscoat, especially Oak) being valued by the Yard, ought to be measured by the Foot divided into 100 Parts, (as being of considerable Value) and the Sum of the Feet divided by 9. (the Square Feet in a Yard.)

*Land.* The best Instrument to measure it with, is a 4 Pole Chain, divided into 100 Parts or Links: for 40 single Pole long and 4 broad making 160 square Perches or 1 Acre; if the Chain be 4 Pole long, then 10 long and 1 broad make an Acre. So that whatever the Dimensions be in square Chains and Links (or hundred Parts) instead of dividing them by 160, if they were measured by a one Pole Chain, here you have nothing to do but to cut off 5 places from any Product to give the Acres. As suppose a Field be 40 Chains and 30 Links long, and the Breadth 30.40 Chains; the Content, by dividing the Product by 10, (or cutting off 1 place besides the 4 which are Decimals) is 122.512 Acres.

Chains.

40.30

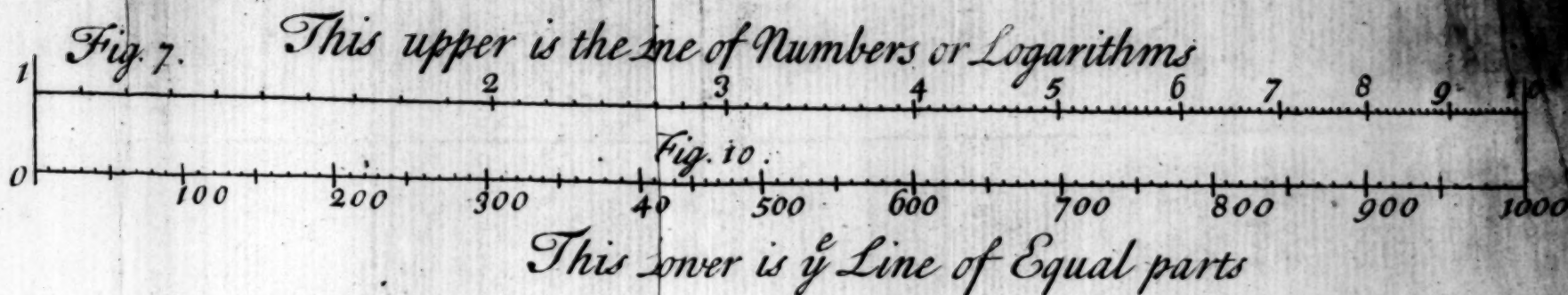
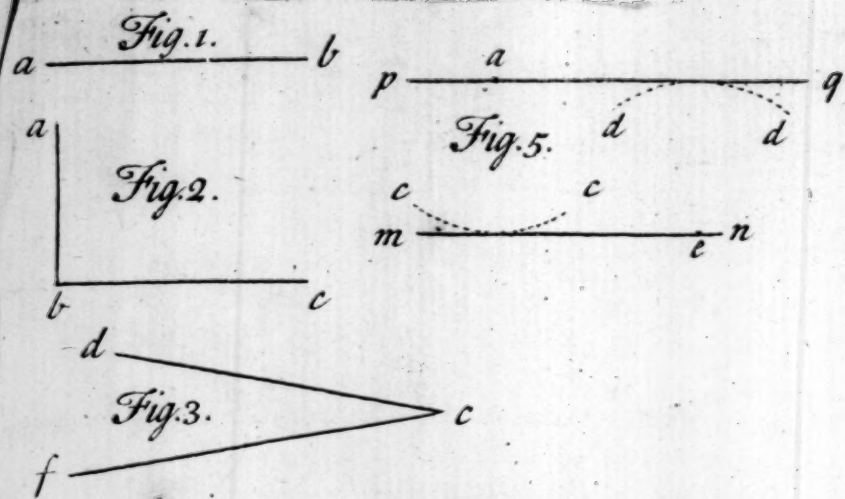
30.40

122.51200 Acres

F I N I S.







Place this plate A, —  
at the end of the Book,  
next to y<sup>e</sup> last leaf thereof.

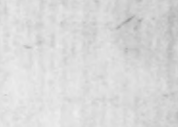
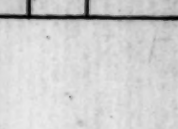
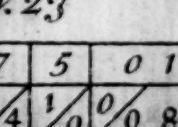
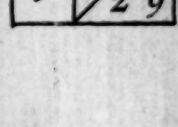
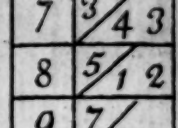
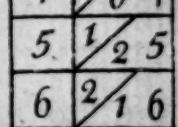
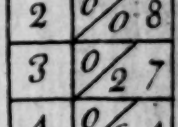
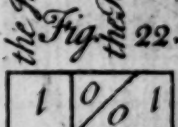
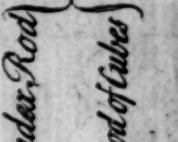
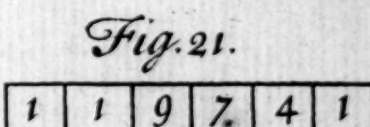
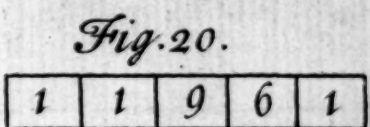
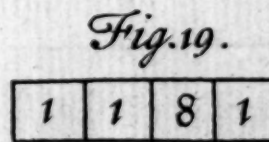
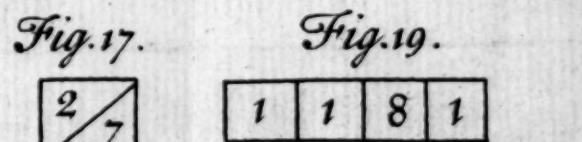
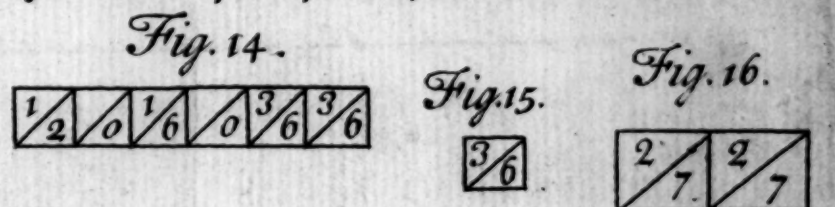
**Fig. 13.** there stands on y<sup>e</sup> other 5 rods these figures

|   |   |   |   |   |
|---|---|---|---|---|
| 3 | 4 | 7 | 2 | 9 |
| 5 | 1 | 2 | 3 | 1 |
| 9 | 2 | 3 | 6 | 3 |
| 7 | 2 | 1 | 2 | 8 |
| 3 | 9 | 1 | 2 | 1 |
| 2 | 6 | 8 | 1 | 4 |

Which Added by y<sup>e</sup> Rule above are these Numbers

|        |
|--------|
| 173645 |
| 312561 |
| 243103 |
| 104187 |
| 69458  |

Summ. or Product required = 82637655



**Fig. 18.** Dividend Quotient

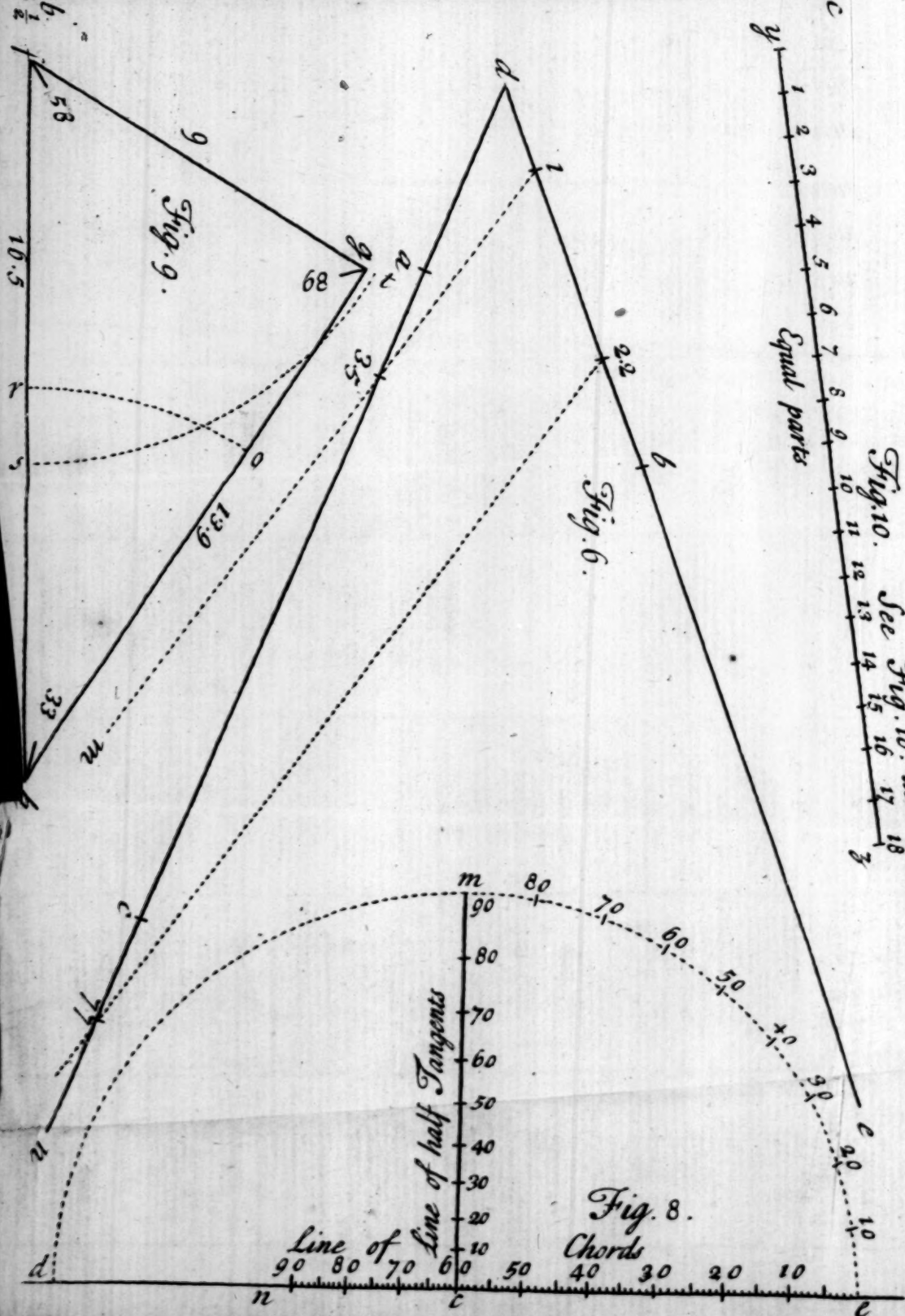
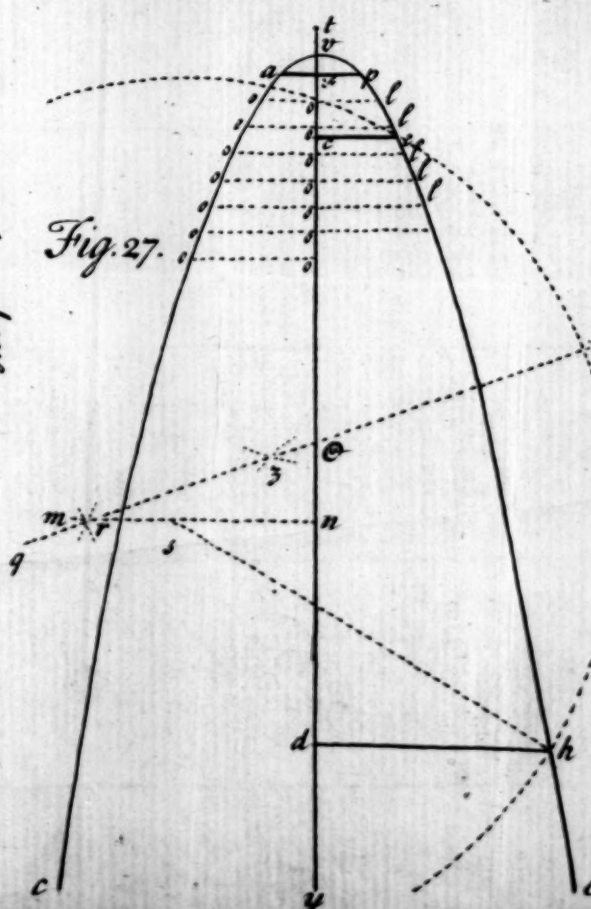
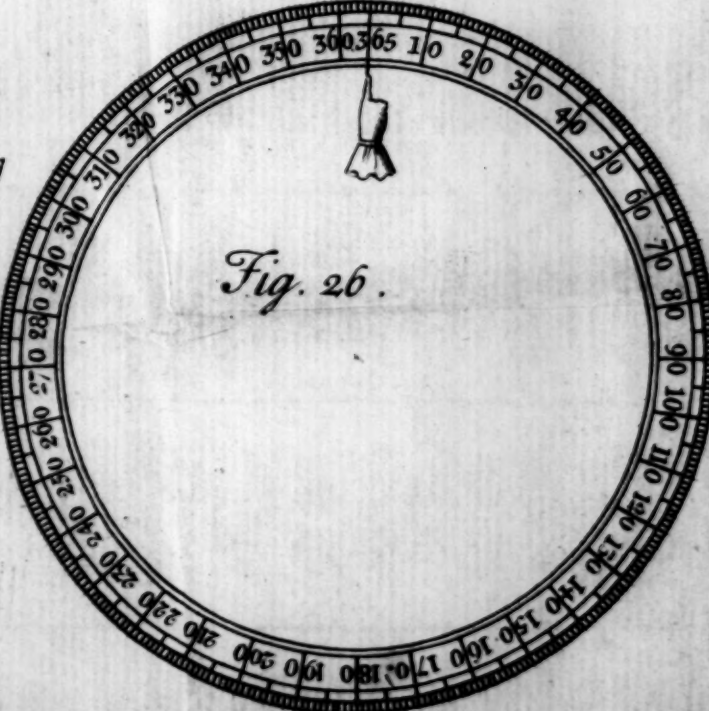
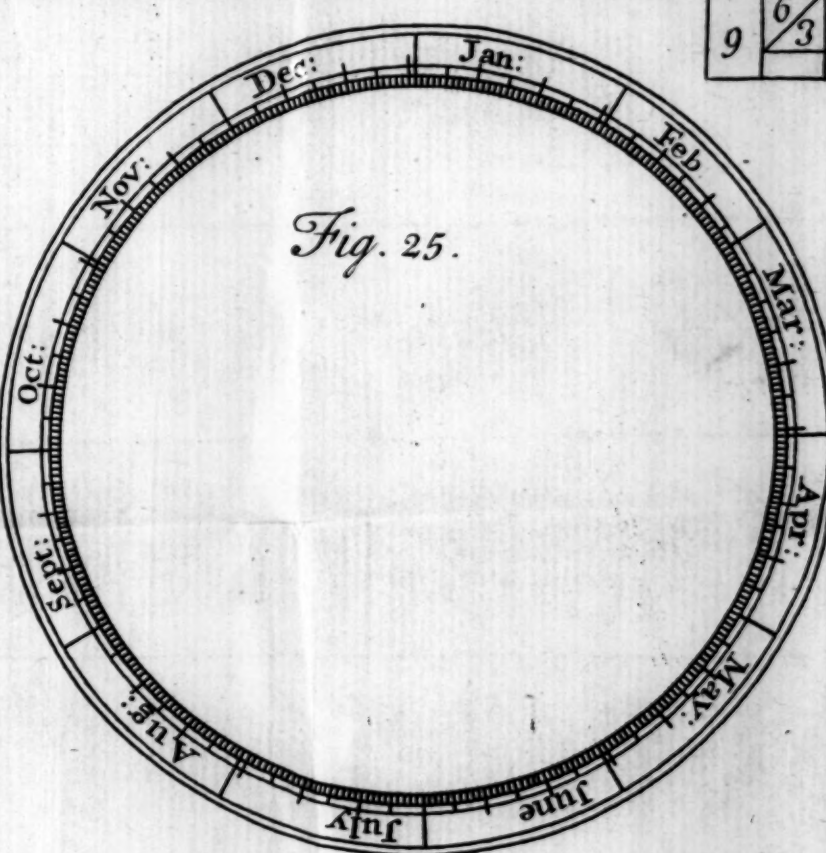
2 5 3 2 4 = Divisor ) 23734691084 ( 937241.

|   |   |   |   |   |   |
|---|---|---|---|---|---|
| 9 | 1 | 4 | 2 | 1 | 3 |
| 3 | 8 | 5 | 7 | 8 | 6 |
| 7 | 6 | 5 | 9 | 6 | 2 |
| 2 | 1 | 3 | 2 | 1 | 2 |
| 4 | 4 | 5 | 1 | 4 | 8 |
| 1 | 4 | 0 | 6 | 4 | 8 |
|   | 8 | 0 | 2 | 8 | 6 |
|   | 2 | 5 | 3 | 2 | 4 |

Or... 227916 deduct  
94309 ..... 3  
Or... 75972 deduct  
183371 ..... 7  
Or... 177268 deduct  
61030 ..... 2  
Or... 50648 deduct  
103828 ..... 4  
Or... 101296 deduct  
25324 ..... 1  
Or... 25324 deduct  
(0) Rests

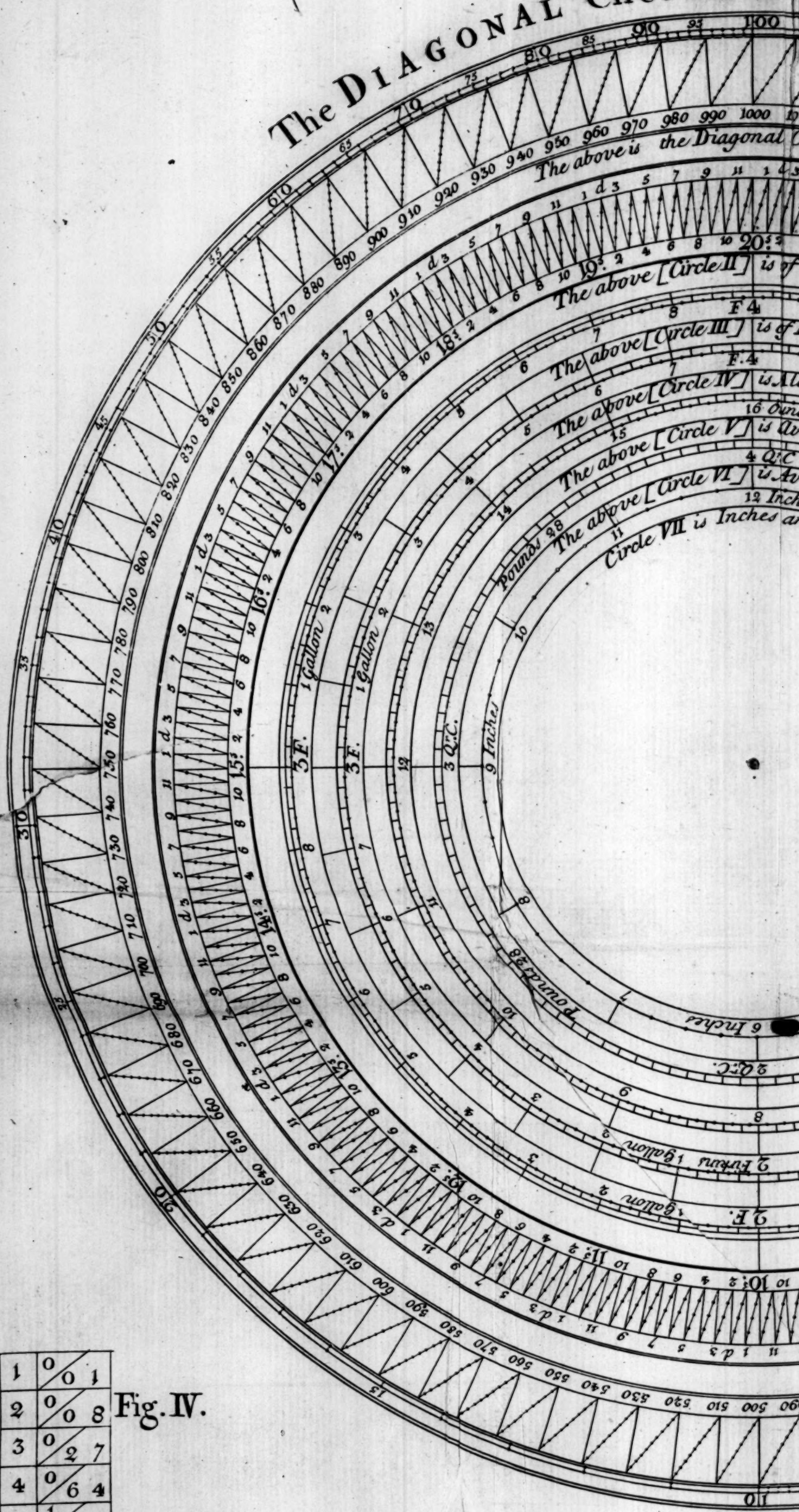
**Fig. 24.** Divisor Tabulated

|   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|
| 1 | 9 | 4 | 0 | 8 | 0 | 0 | 1 |
| 2 | 1 | 8 | 8 | 0 | 1 | 6 | 0 |
| 3 | 2 | 7 | 2 | 0 | 2 | 4 | 0 |
| 4 | 3 | 6 | 1 | 6 | 0 | 3 | 0 |
| 5 | 4 | 5 | 0 | 0 | 0 | 1 | 2 |
| 6 | 5 | 4 | 2 | 4 | 0 | 4 | 2 |
| 7 | 6 | 3 | 2 | 8 | 0 | 5 | 6 |
| 8 | 7 | 2 | 3 | 2 | 0 | 6 | 4 |
| 9 | 8 | 1 | 3 | 6 | 0 | 7 | 2 |





# The DIAGONAL Circular I



|   |   |   |   |
|---|---|---|---|
| 1 | 0 | 0 | 1 |
| 2 | 0 | 0 | 8 |
| 3 | 0 | 2 | 7 |
| 4 | 0 | 6 | 4 |
| 5 | 1 | 2 | 5 |
| 6 | 2 | 1 | 6 |
| 7 | 3 | 4 | 3 |
| 8 | 5 | 1 | 2 |
| 9 | 7 | 2 | 9 |

Fig. IV.

A Rod of Roots  
A Rod of their  
Cubes

Fig. I. A Set of the Bones or Rods.

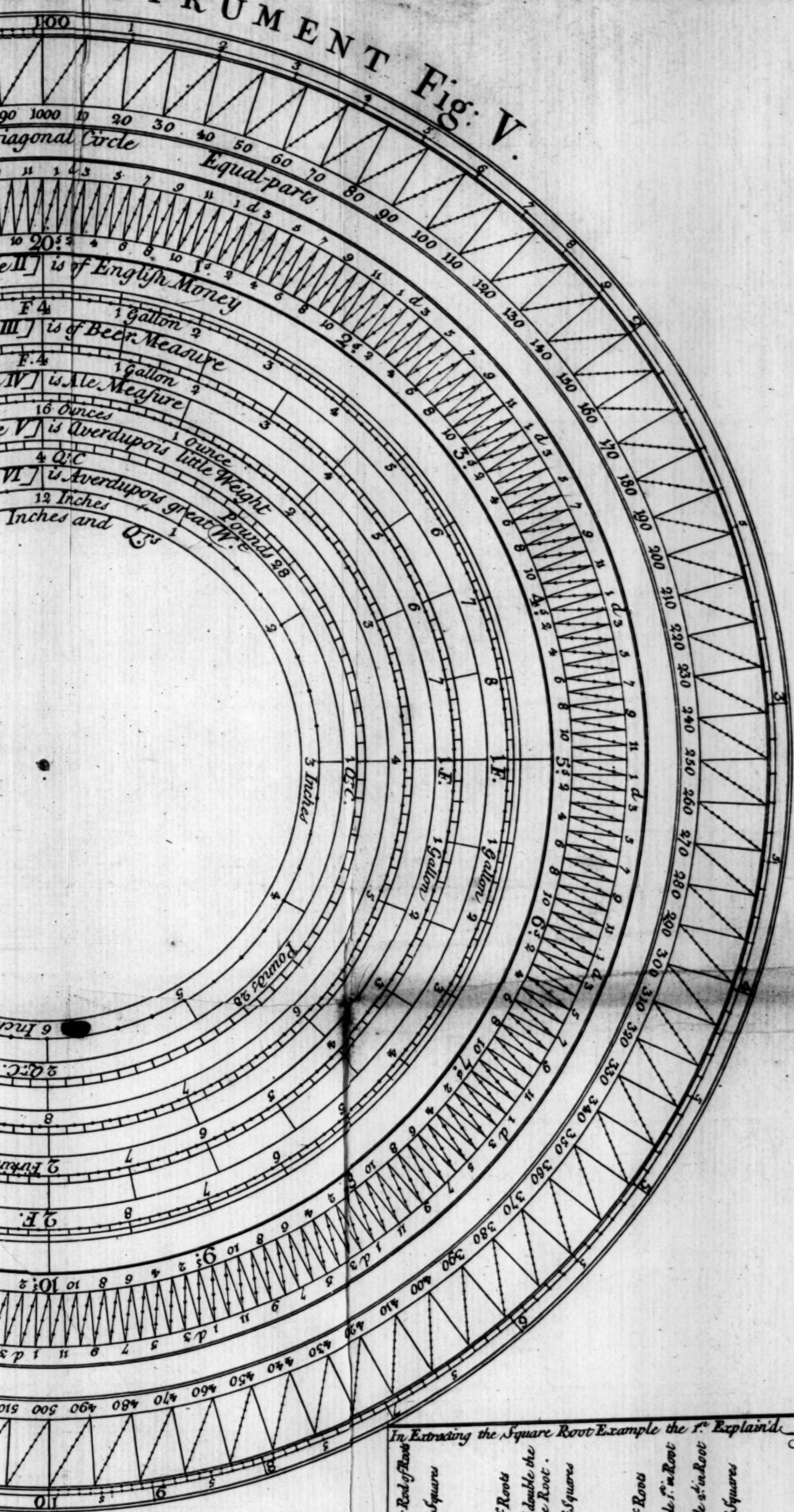
|   |   |   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 2 | 0 | 2 | 4 | 6 | 8 | 1 | 1 | 1 | 1 | 1 |
| 3 | 0 | 3 | 6 | 9 | 1 | 1 | 1 | 1 | 1 | 1 |
| 4 | 0 | 4 | 8 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 5 | 0 | 5 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 6 | 0 | 6 | 2 | 8 | 4 | 0 | 6 | 2 | 8 | 4 |
| 7 | 0 | 7 | 4 | 1 | 8 | 5 | 2 | 9 | 6 | 3 |
| 8 | 0 | 8 | 6 | 4 | 2 | 0 | 8 | 6 | 4 | 2 |
| 9 | 0 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 |

In Multiplication

|   |   |   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 2 | 0 | 2 | 4 | 6 | 8 | 1 | 1 | 1 | 1 | 1 |
| 3 | 0 | 3 | 6 | 9 | 1 | 1 | 1 | 1 | 1 | 1 |
| 4 | 0 | 4 | 8 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 5 | 0 | 5 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 6 | 0 | 6 | 2 | 8 | 4 | 0 | 6 | 2 | 8 | 4 |
| 7 | 0 | 7 | 4 | 1 | 8 | 5 | 2 | 9 | 6 | 3 |
| 8 | 0 | 8 | 6 | 4 | 2 | 0 | 8 | 6 | 4 | 2 |
| 9 | 0 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 |



# INSTRUMENT Fig: V.



Question 1  
Tabulated.

In Multiplication

|   |   |   |   |   |   |
|---|---|---|---|---|---|
| 1 | 3 | 4 | 7 | 2 | 9 |
| 2 | 6 | 8 | 1 | 4 | 1 |
| 3 | 9 | 1 | 2 | 1 | 6 |
| 4 | 1 | 2 | 6 | 8 | 3 |
| 5 | 1 | 5 | 0 | 5 | 0 |
| 6 | 1 | 8 | 4 | 2 | 1 |
| 7 | 2 | 1 | 8 | 9 | 1 |
| 8 | 2 | 4 | 3 | 2 | 6 |
| 9 | 2 | 7 | 3 | 6 | 1 |

Fig: II.

In Extracting the Square Root Example the 1<sup>st</sup> Explained

| Index or Root of Root | Root of Squares | Root of Roots | Root of double the 1 <sup>st</sup> in the Root | Root of Squares | Root of Roots | Root of double the 1 <sup>st</sup> in Root | Root of double the 1 <sup>st</sup> in Root | Root of Squares |
|-----------------------|-----------------|---------------|------------------------------------------------|-----------------|---------------|--------------------------------------------|--------------------------------------------|-----------------|
| 1                     | 1               | 1             | 4                                              | 1               | 1             | 4                                          | 6                                          | 1               |
| 2                     | 4               | 2             | 8                                              | 4               | 2             | 8                                          | 1                                          | 4               |
| 3                     | 9               | 3             | 1                                              | 2               | 9             | 1                                          | 2                                          | 8               |
| 4                     | 1               | 6             | 1                                              | 6               | 1             | 6                                          | 2                                          | 1               |
| 5                     | 2               | 5             | 2                                              | 0               | 5             | 2                                          | 0                                          | 2               |
| 6                     | 3               | 6             | 2                                              | 4               | 6             | 2                                          | 4                                          | 3               |
| 7                     | 4               | 9             | 2                                              | 8               | 9             | 2                                          | 8                                          | 4               |
| 8                     | 6               | 4             | 3                                              | 2               | 4             | 3                                          | 2                                          | 8               |
| 9                     | 8               | 1             | 3                                              | 6               | 1             | 3                                          | 6                                          | 4               |

Fig: III

200 30 4 = 234 the Root



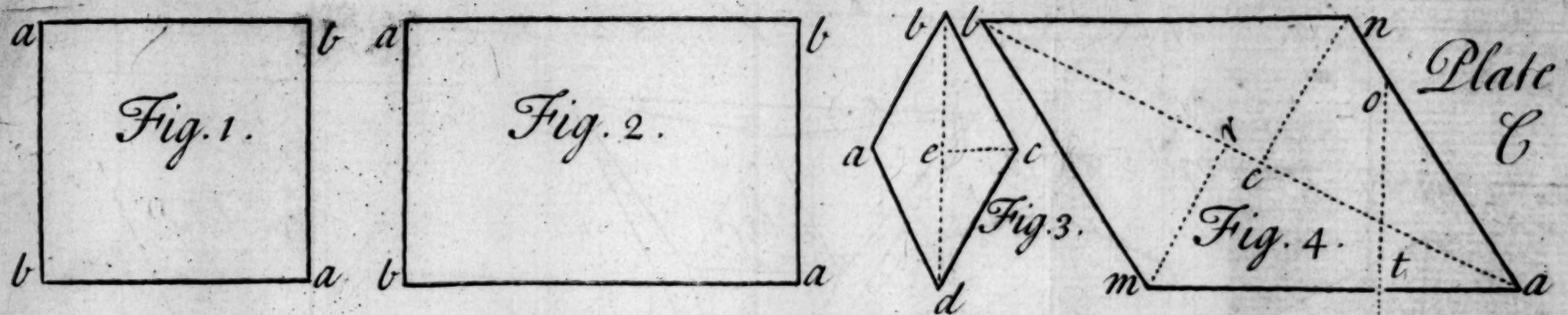


Plate C

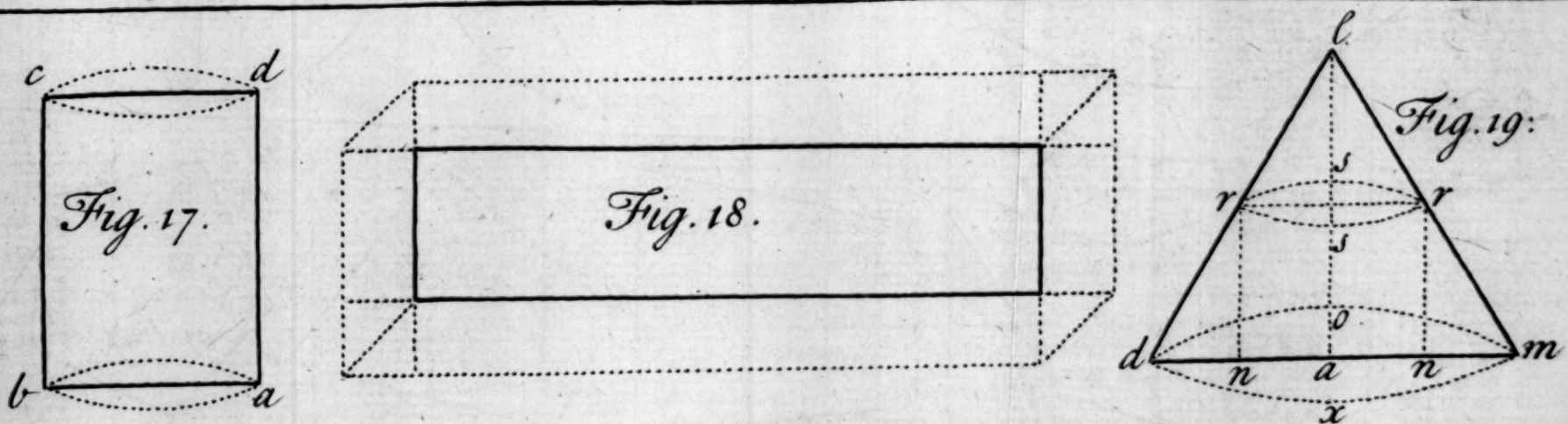
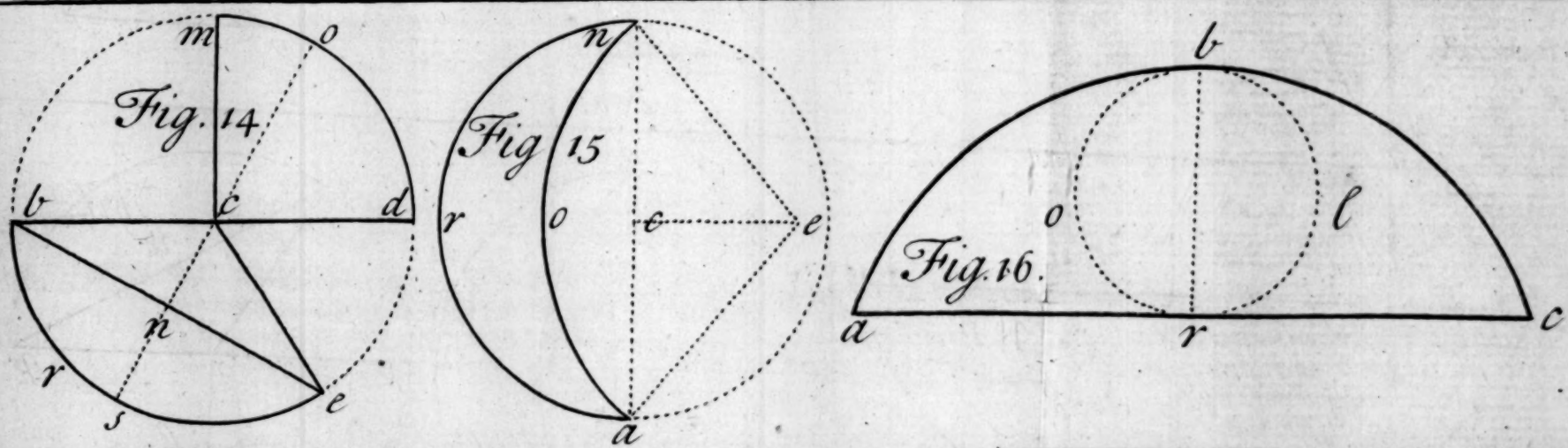
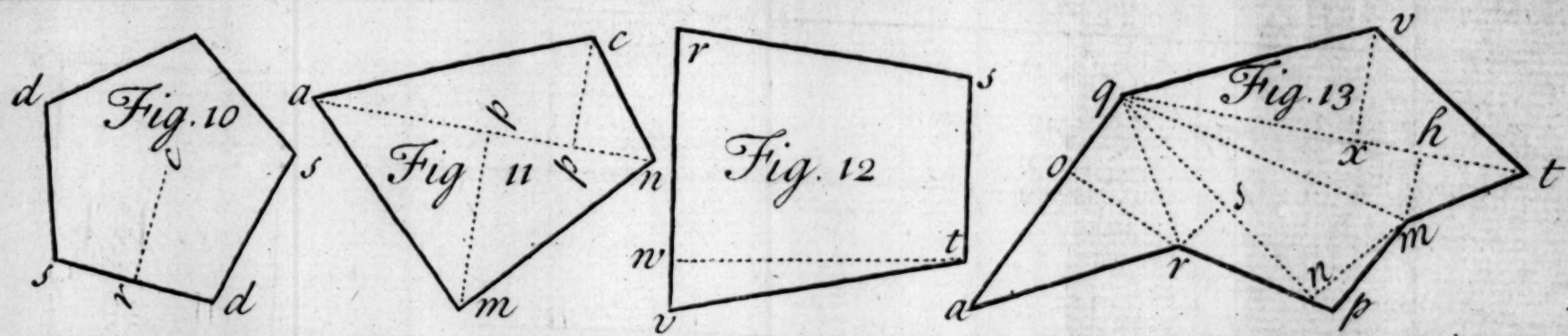
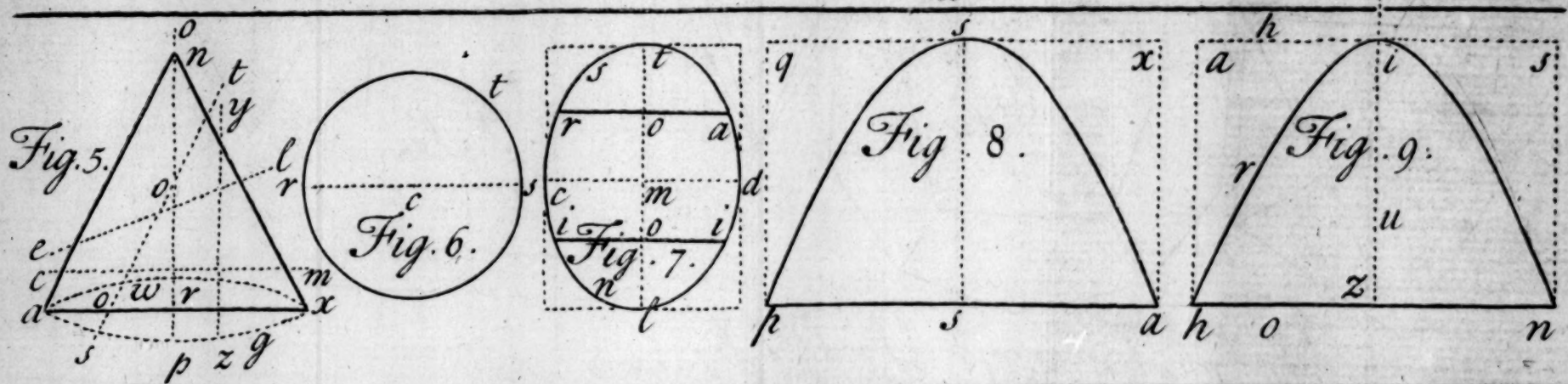
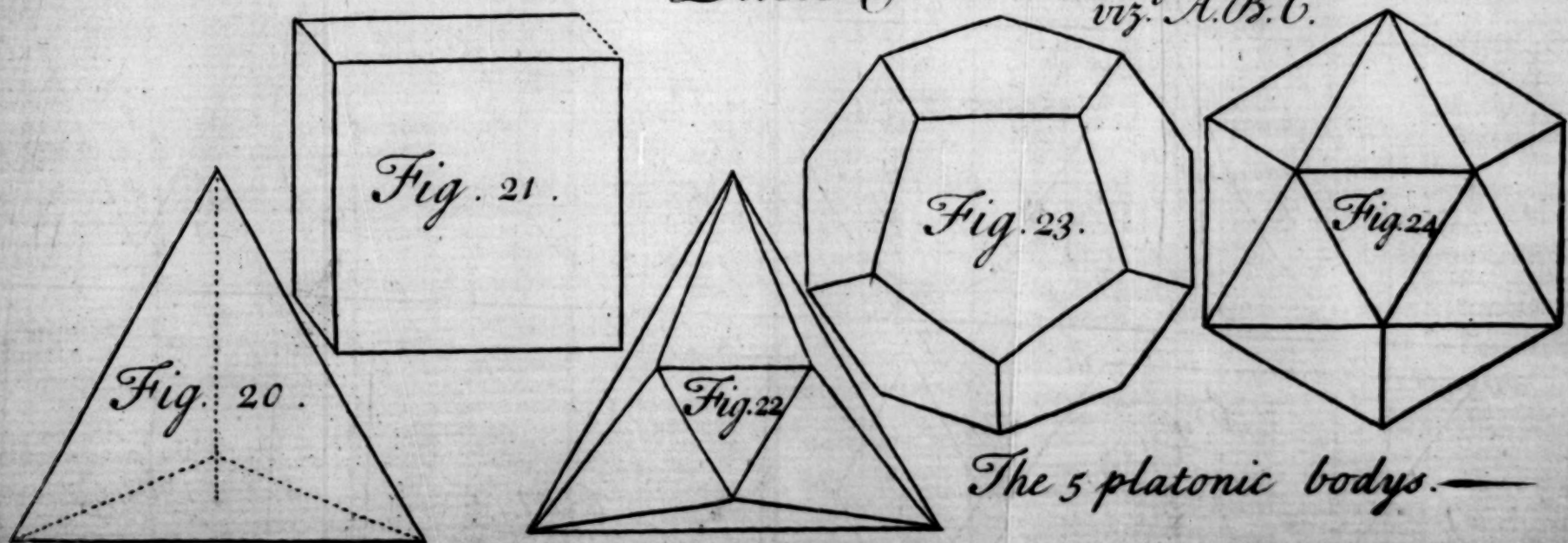


Plate C. place at y end of the Book next B.  
viz. A. B. C.



The 5 platonic bodies.